Spinor Bose gas in optical lattices

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Outline:

- Motivation
- Spin Hamiltonians
  - (differences from electronic systems)
- Ground states
  - mean-field
  - 1D

Main collaborators:

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Mott Insulating phase has been achieved in cold atoms:

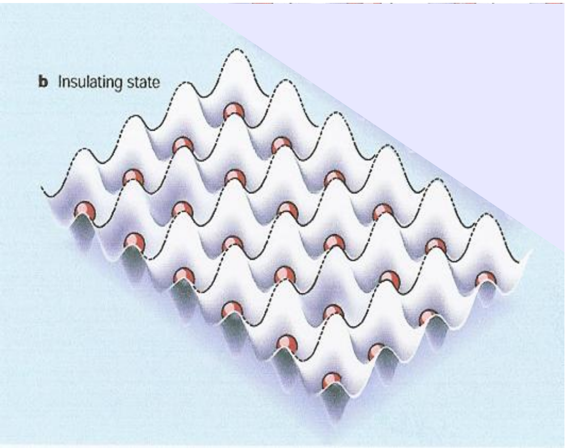
Greiner, Esslinger, Bloch, Porto, Chin, Takahashi ..

Lattice potential from optical standing waves

Particles -- Bosons (here)

Integral number of particle per site

Hubbard repulsion prevents particles from hopping



But: Bosons have (hyperfine) spins $F = I \oplus S$

spins frozen in earlier experiments (spin-polarized Bosons)

Spin-1 superfluids have already been studied:

^{23}Na F=1 (MIT)
^{87}Rb F=1 (Georgia Tech, Berkeley ..)

Q: Mott states for spin-1 atoms in optical lattice

(arrangements of spins, i.e., magnetic states)

↑ 0 ↓

shall consider one particle per well (stability)

one orbital per well (assuming sufficiently low energy)

First consider bulk gas (superfluid; no lattice)

dilute gas: effective short-ranged (delta-function) interaction

two spin-1 Bose particles, total spin 0 or 2 (1 non-interacting)

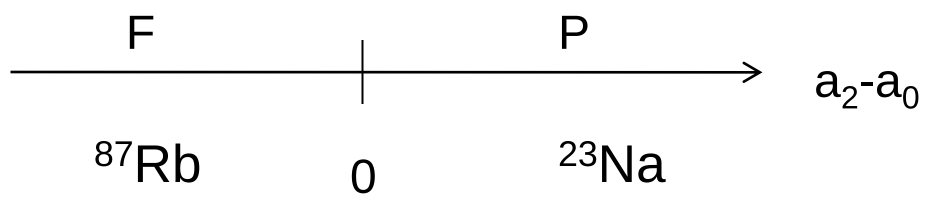
two interaction constants g0, g2

equivalently two scattering lengths a0, a2

$$g_0 = \frac{4\pi\hbar a_0}{M} \qquad g_2 = \frac{4\pi\hbar a_2}{M}$$

Phase diagram (mean-field):

$$\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$



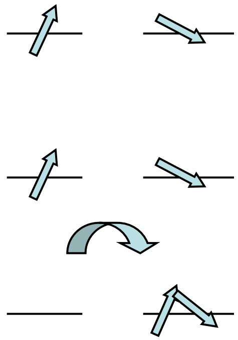
$$\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \leftrightarrow \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$$

z y

With lattice, no tunneling, spin random:

Small tunneling amplitude t:

mixes virtual states with two particles per site



on-site energy spin dependent

$$H = H_{\text{hopping } t} + H_{\text{on-site Hubbard } U}$$

$U_0 \propto 4 \pi \hbar^2 a_0/m$ $U_2 \propto 4 \pi \hbar^2 a_2/m$ [spin-1 forbidden]

^{23}Na atoms: $a_2 > a_0 > 0$; $U_2 > U_0 > 0$

^{87}Rb atoms: $a_0 > a_2 > 0$; $U_0 > U_2 > 0$

Spin Hamiltonian:

two sites:

classify states according to total spin:

$E_{S=0} = -4\,t^2 / U_0$ $\qquad U_0 \propto 4\pi\hbar^2 a_0/m$

$E_{S=1} = 0$

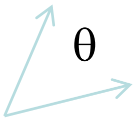
$E_{S=2} = -4\,t^2 / U_2$ $\qquad U_2 \propto 4\pi\hbar^2 a_2/m$

^{23}Na atoms: $a_2 > a_0 > 0 \;;\; E_0 < E_2 < E_1 = 0$

^{87}Rb atoms: $a_0 > a_2 > 0 \;;\; E_2 < E_0 < E_1 = 0$

Very different from two classical spins

E (θ) monotonic with θ



$$E_0 \sim E_2 < E_1 = 0$$

$$H^{int}_{ij} = \varepsilon_0 + J\,(S_i \cdot S_j) + K\,(S_i \cdot S_j)^2$$

very different from Heisenberg (K=0)

[Yip 03; Imambekov 03]

(electronic systems, typically K small [4^{th} order in hopping])

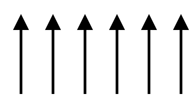
$$H = \sum H_{ij}$$

$$H_{ij} = \varepsilon_0 P_{ij}^{(0)} + \varepsilon_2 P_{ij}^{(2)}$$

$$\varepsilon_0, \varepsilon_2 < 0$$

Case 1: $\varepsilon_2 < \varepsilon_0 \leq 0$

Favor parallel spins



Similar to Heisenberg with J < 0

Case 2: $\varepsilon_0 < \varepsilon_2 \leq 0$

very different from Heisenberg with J > 0

two sites

$|\uparrow\downarrow>$

bad

$|\downarrow\uparrow>$

$|\,00>$ much better (symmetric) [Bosons]

Lattice:

$...\uparrow\downarrow\uparrow\downarrow\uparrow\downarrow\ ...$ Neel ordering: bad

$...0000000000...$ much better

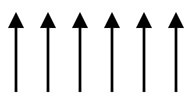
exact if $\varepsilon_0 = \varepsilon_2 < 0$

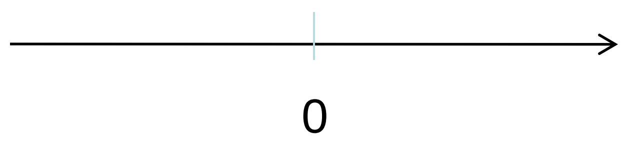
$$H = \sum H_{ij}$$

$$H_{ij} = \varepsilon_0 P_{ij}^{(0)} + \varepsilon_2 P_{ij}^{(2)}$$

$$\varepsilon_0, \varepsilon_2 < 0$$

Mean-field, direct analogy with superfluid



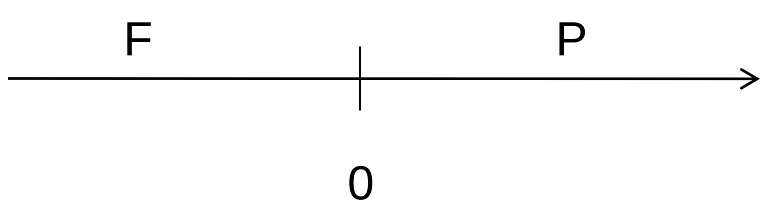


$\varepsilon_2 - \varepsilon_0$

0000

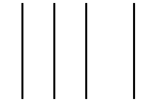
bulk BEC:

$$\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$



a_2-a_0

$$\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$$

...0000000000... 

nematic order / quadrupolar order

$$< S_z^2 > \neq < S_x^2 > \neq \frac{2}{3}$$

rotational symmetry broken,

no moment

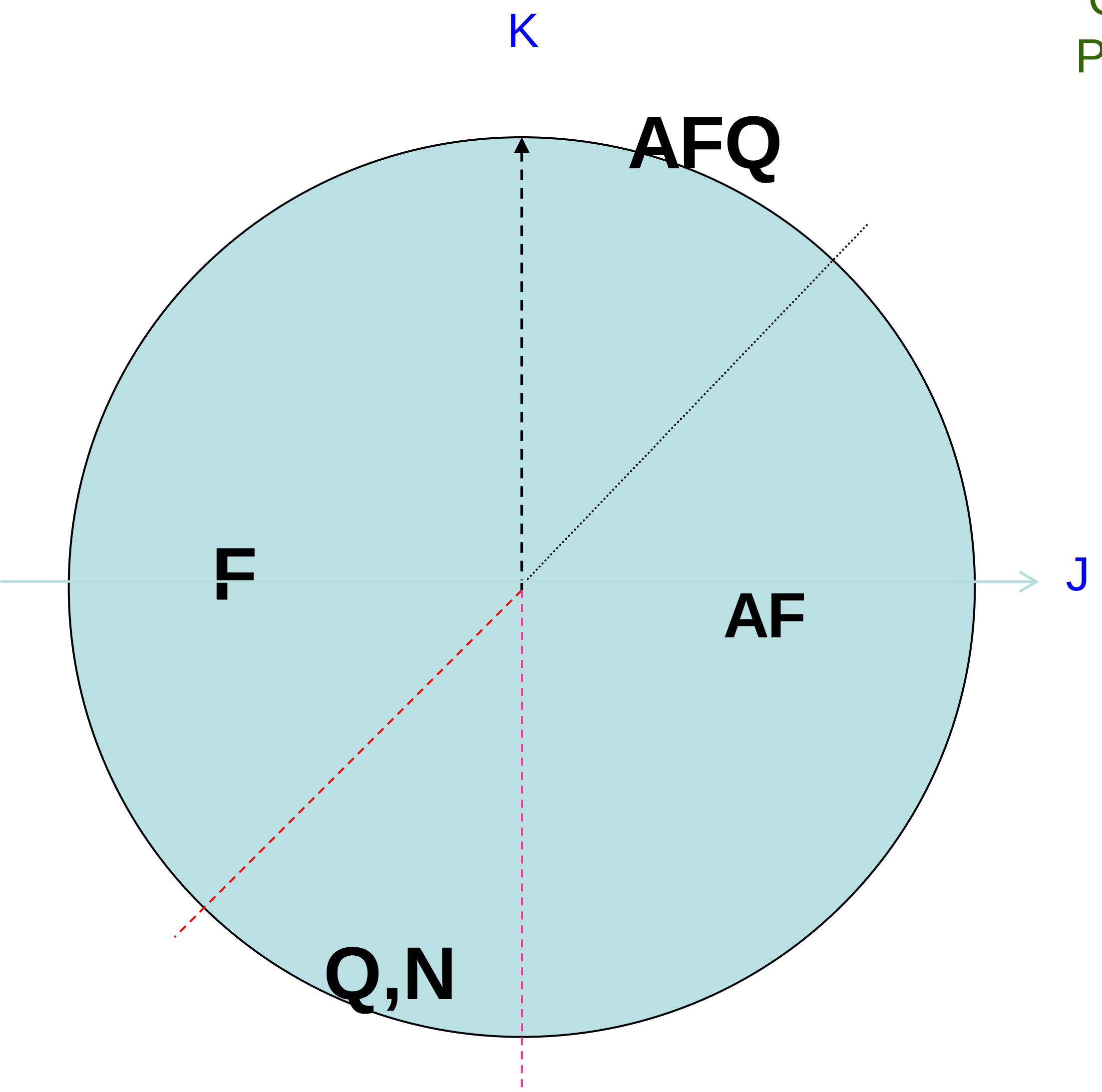
time-reversal not broken

$$H^{int}_{ij} = \varepsilon_0 + J\,(S_i \cdot S_j) + K\,(S_i \cdot S_j)^2$$

$$H^{int}_{ij} = \varepsilon_0 + J\,(S_i \cdot S_j) + K\,(S_i \cdot S_j)^2$$

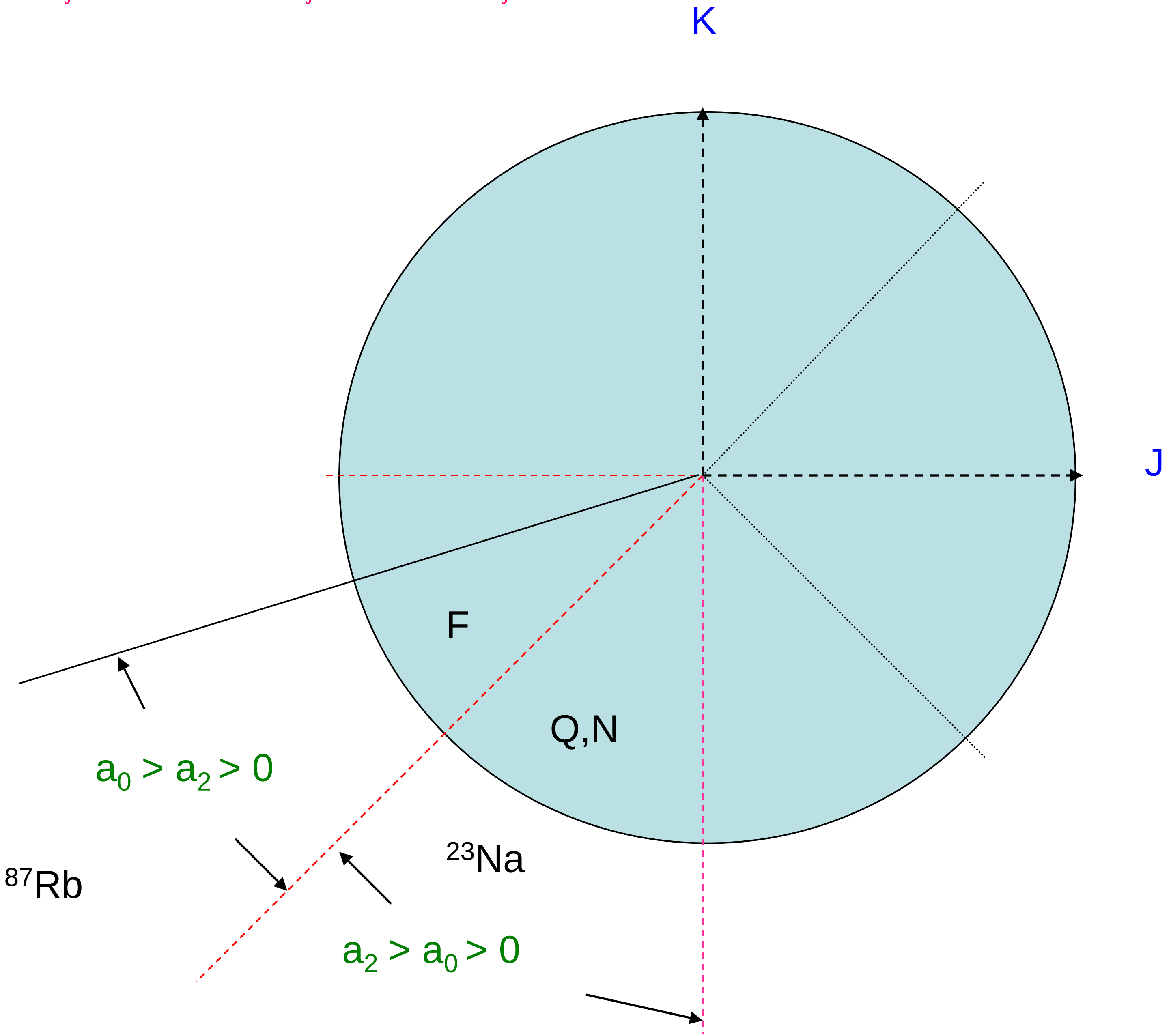
Chen+Levy 73
Papanicolaou 88

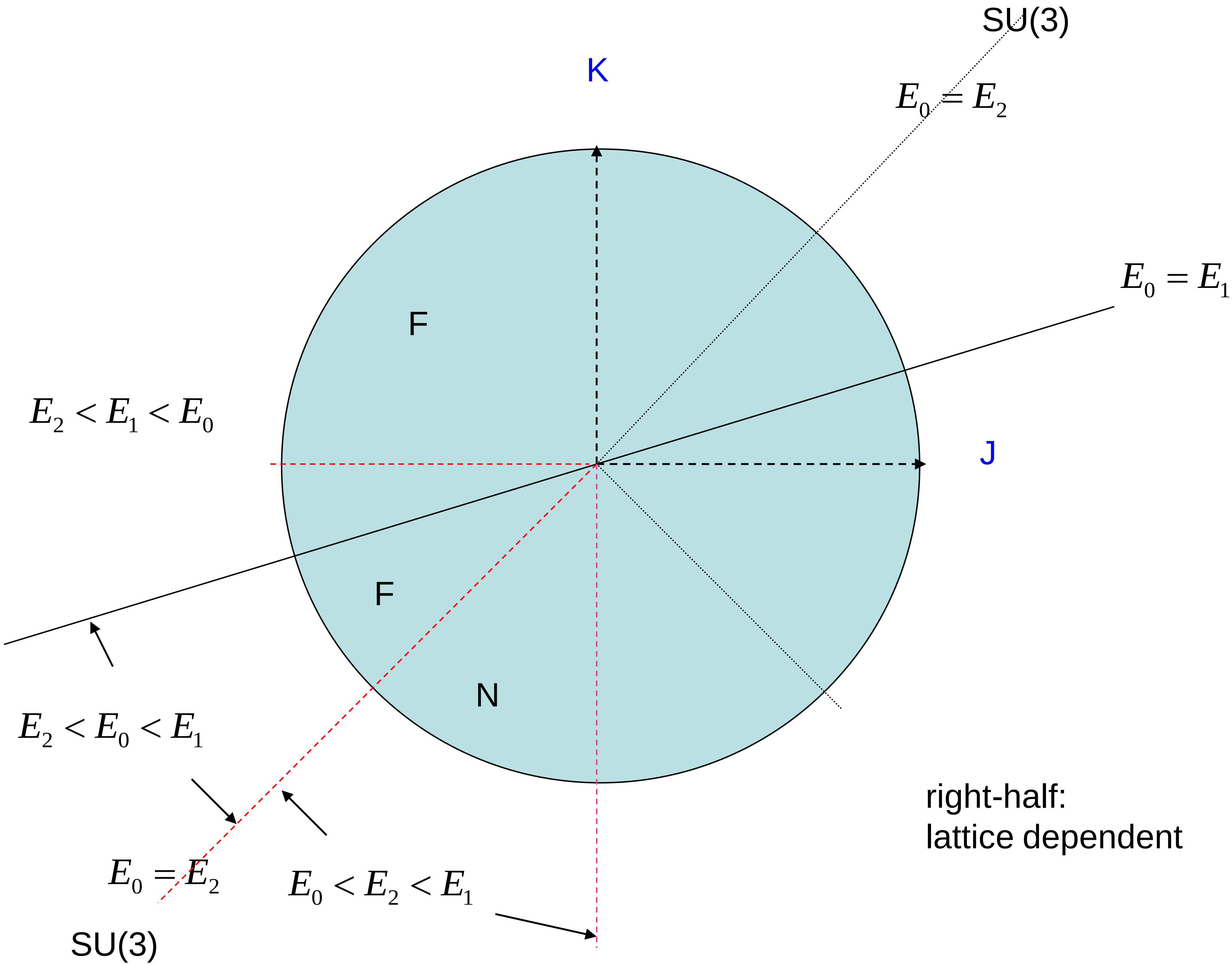
Mean-field:



lattice type dependent

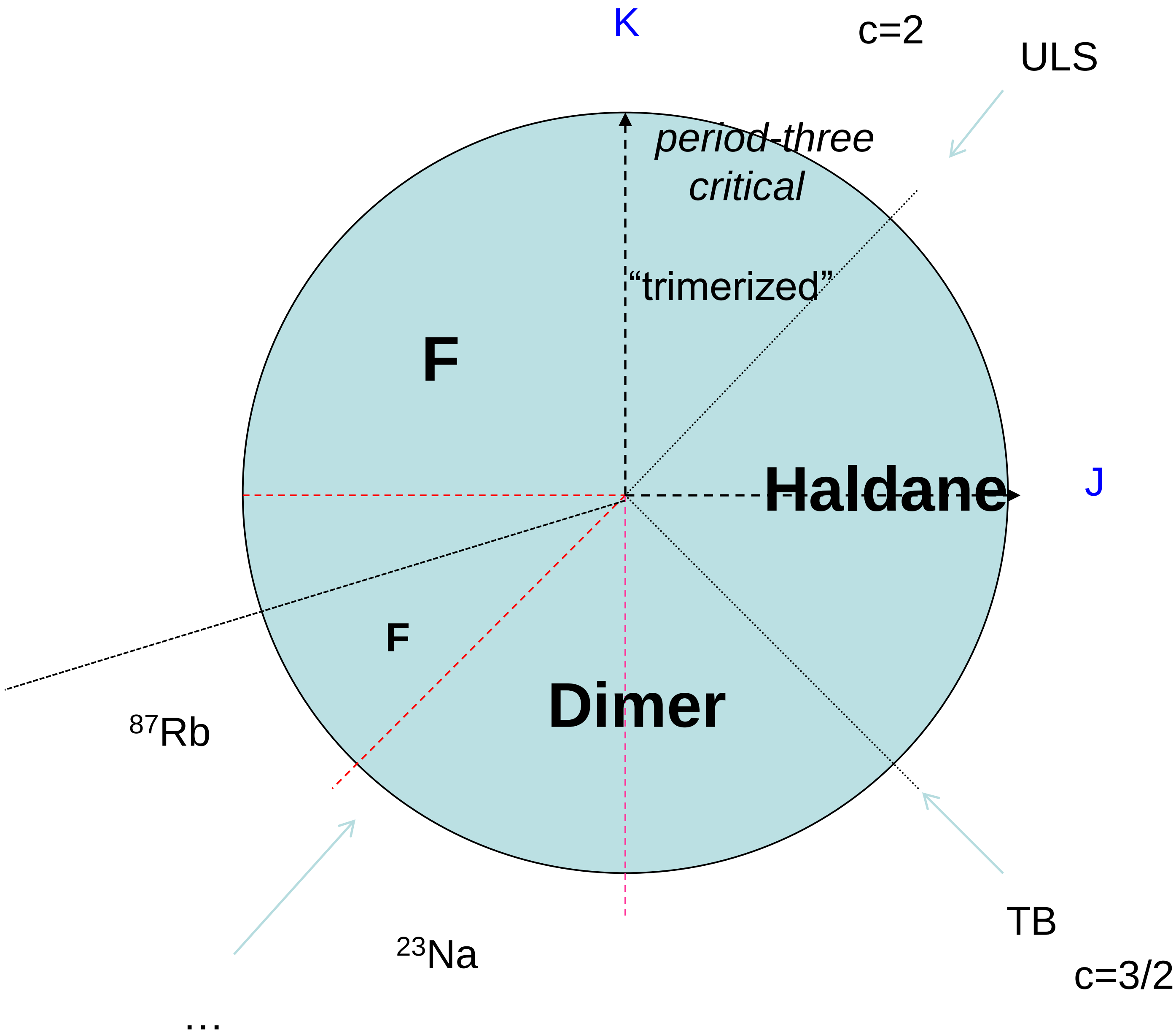
$$H^{int}_{ij} = \varepsilon_0 + J\,(S_i \cdot S_j) + K\,(S_i \cdot S_j)^2$$

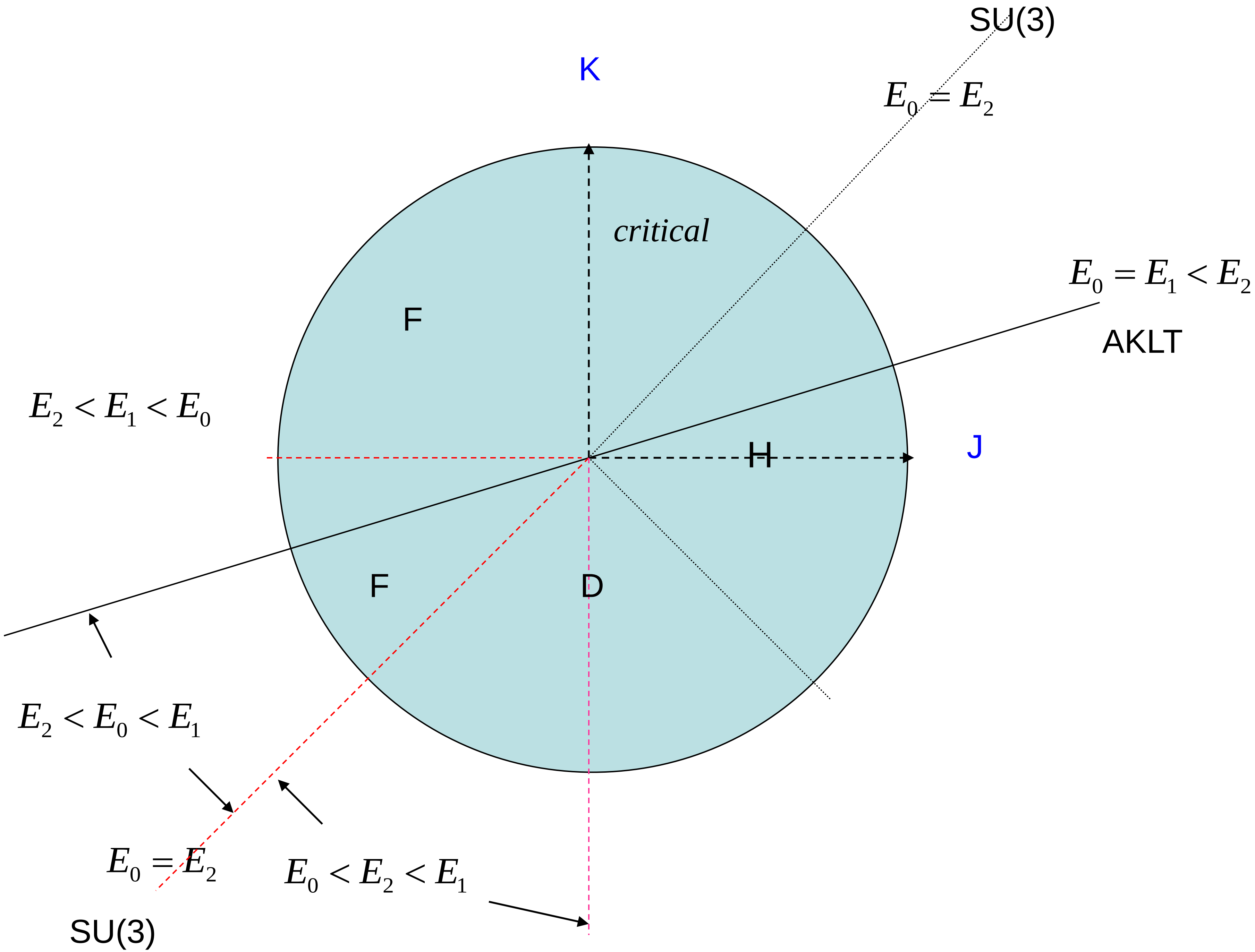




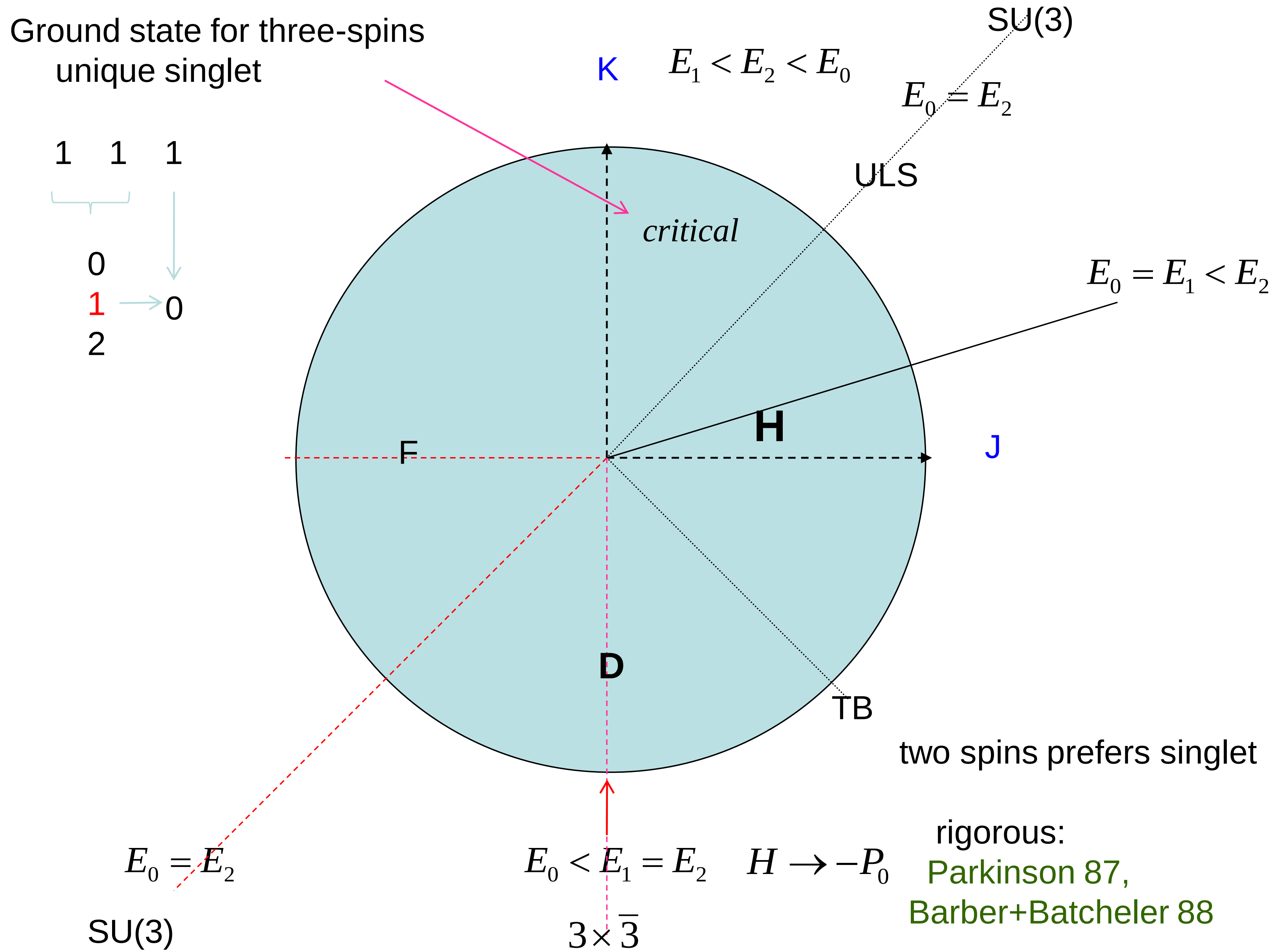
SU(3)
K
$E_0 = E_2$
$E_0 = E_1$
F
$E_2 < E_1 < E_0$
J
F
N
$E_2 < E_0 < E_1$
right-half:
lattice dependent
$E_0 = E_2$
$E_0 < E_2 < E_1$
SU(3)

1D:


K
c=2
ULS
period-three
critical
"trimerized"
F
Haldane
J
F
Dimer
87Rb
TB
23Na
c=3/2
...


SU(3)
K
$E_0 = E_2$
critical
$E_0 = E_1 < E_2$
F
AKLT
$E_2 < E_1 < E_0$
H
J
F
D
$E_2 < E_0 < E_1$
$E_0 = E_2$
$E_0 < E_2 < E_1$
SU(3)

Ground state for three-spins
unique singlet



Spin-2

Available systems:

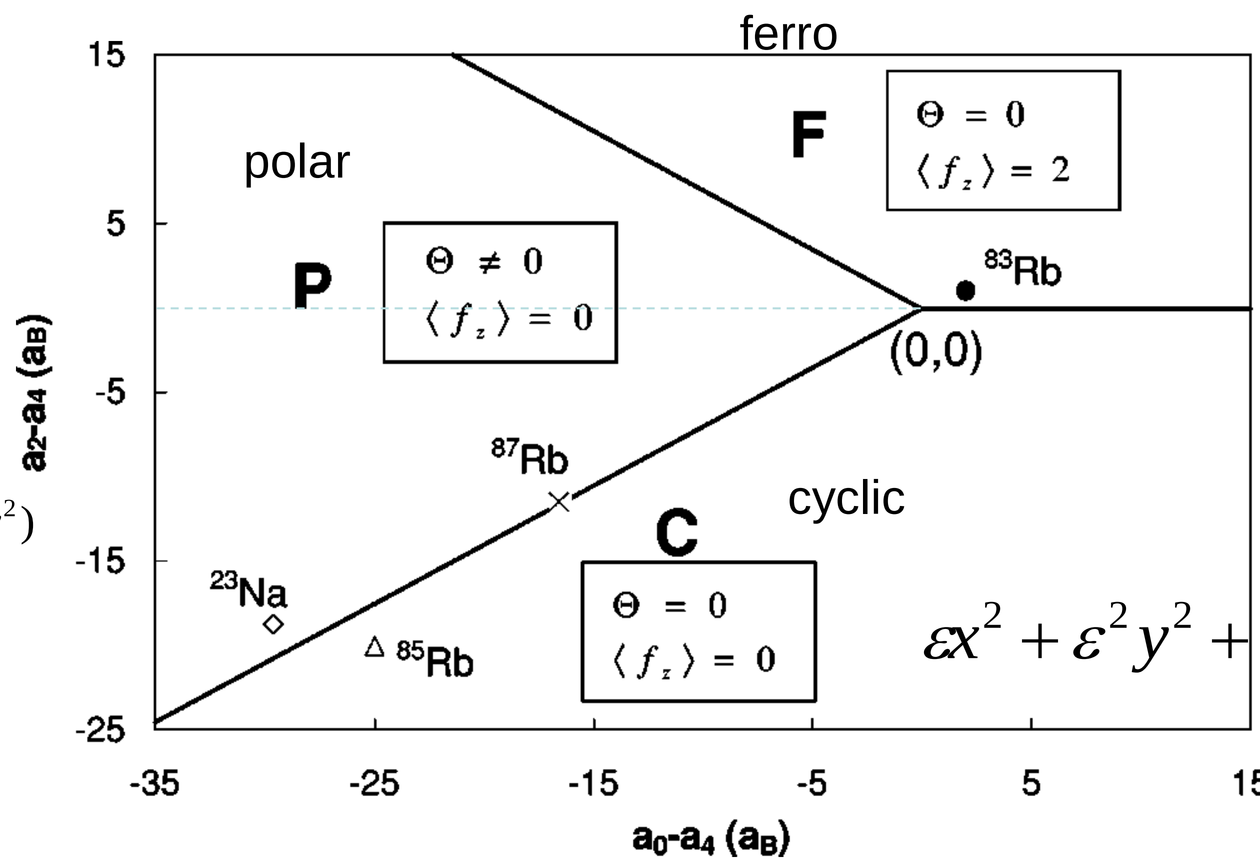
^{23}Na	F=2		upper
^{87}Rb	F=2	(Hamburg, Mainz, Tokyo.)	
^{83}Rb	F=2		lower /86 days
^{85}Rb	F=2		lower / $a < 0$

$2 \oplus 2 = 0, 1, 2, 3, 4$;   scattering lengths a0, a2, a4

Bulk phase diagram (BEC), mean-field   [Ciobanu, Yip, Ho; 2000]

$$\begin{pmatrix}1\\0\\0\\0\\1\end{pmatrix} \quad x^2 - y^2$$

$$\begin{pmatrix}0\\0\\1\\0\\0\end{pmatrix} \quad 2z^2 - (x^2 + y^2)$$



$$\begin{pmatrix}1\\0\\0\\0\\0\end{pmatrix}$$

$$\varepsilon x^2 + \varepsilon^2 y^2 + z^2$$

$$\varepsilon = e^{i\frac{2\pi}{3}}$$

Lattice, insulating state, one Boson per site

$$H = \sum H_{ij}$$

$$H_{ij} = \varepsilon_0 P_{ij}^{(0)} + \varepsilon_2 P_{ij}^{(2)} + \varepsilon_4 P_{ij}^{(4)}$$

$$\varepsilon_0 = -\frac{4t^2}{U_0}, \ldots$$

$$U_0 \propto a_0, \ldots$$

$$\varepsilon_0, \varepsilon_2, \varepsilon_4 < 0$$

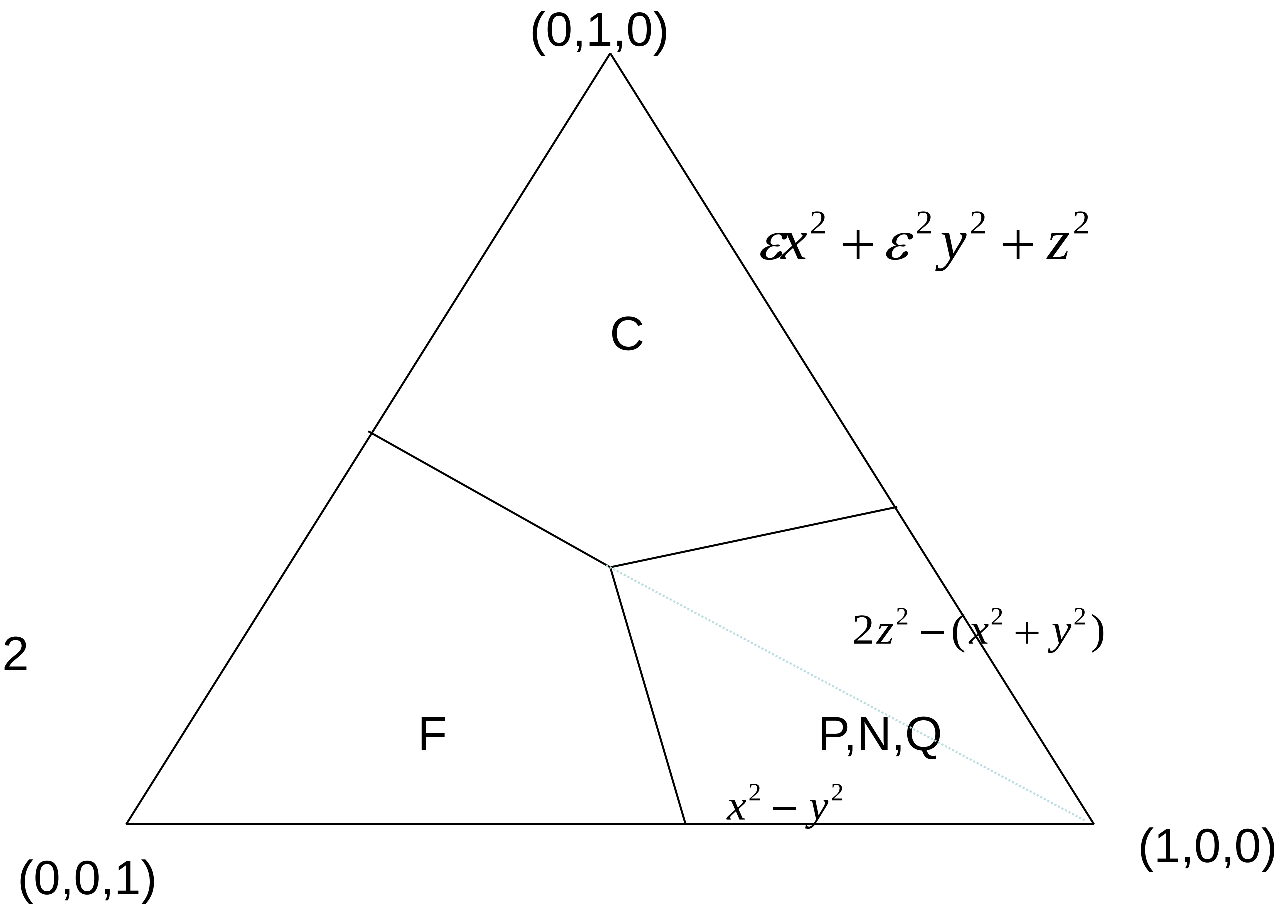
Mean-field:

$$(x_0, x_2, x_4) = (\varepsilon_0, \varepsilon_2, \varepsilon_4)/(\varepsilon_0 + \varepsilon_2 + \varepsilon_4)$$

$$x_0 + x_2 + x_4 = 1$$

$$x_{0,2,4} \geq 0$$

2222



$$H_{eff} = \sum H_{ij}$$

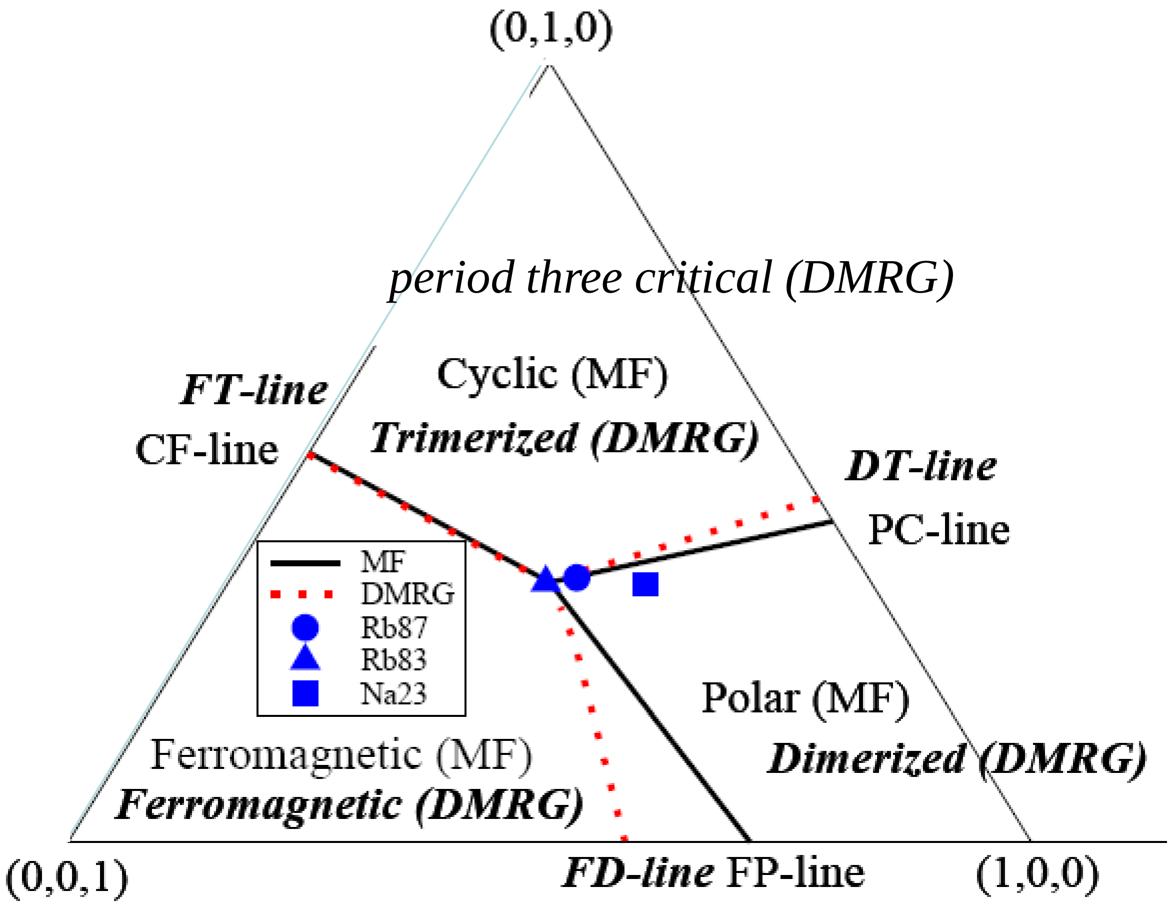
$$H_{ij} = J_1(\vec{S}_i \bullet \vec{S}_j) + J_2(\vec{S}_i \bullet \vec{S}_j)^2 + J_3(\vec{S}_i \bullet \vec{S}_j)^3 + J_4(\vec{S}_i \bullet \vec{S}_j)^4$$

Complete phase diagram

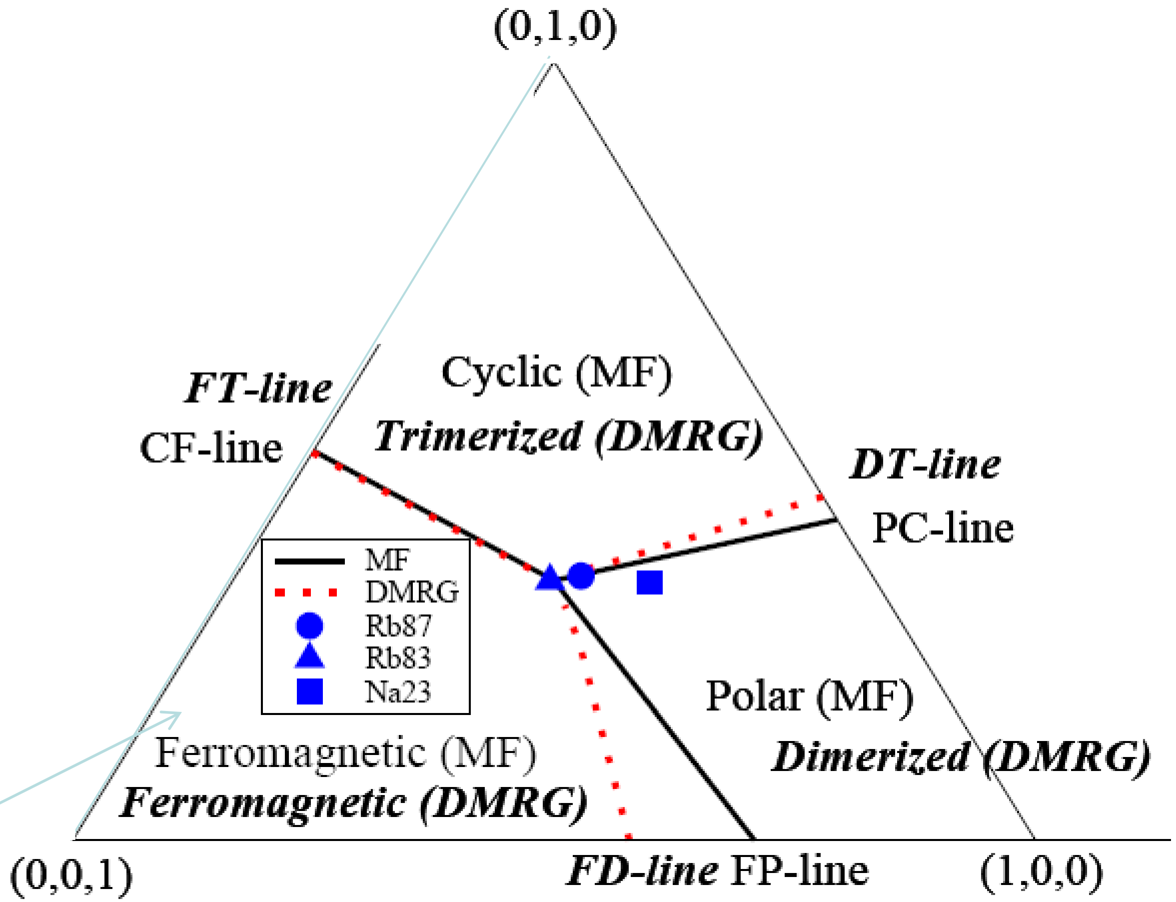
e.g. S^3 surface of a sphere in 4D

here: small subset

1D (DMRG)

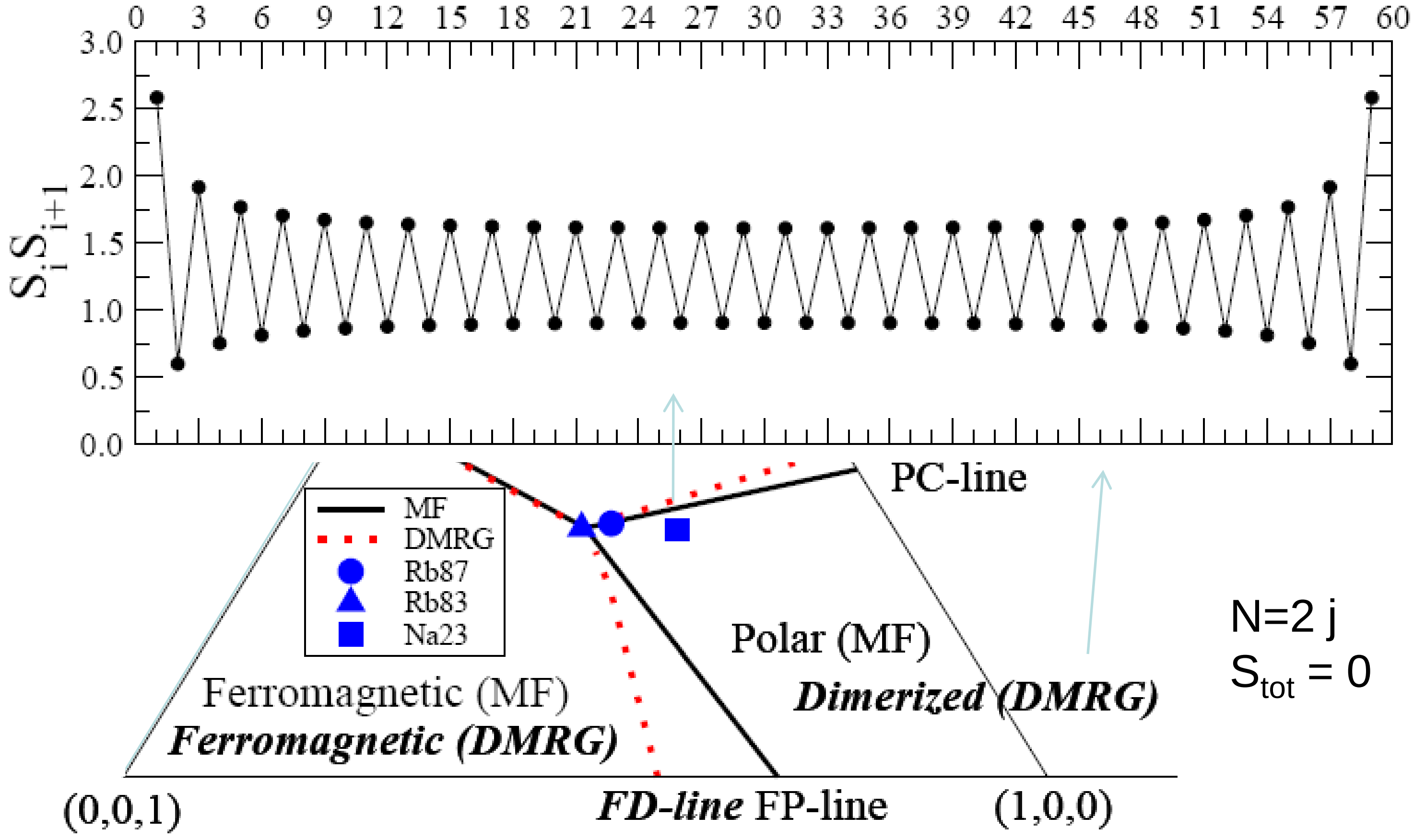


(0,1,0)
period three critical (DMRG)
Cyclic (MF)
Trimerized (DMRG)
FT-line
CF-line
DT-line
PC-line
MF
DMRG
Rb87
Rb83
Na23
Polar (MF)
Dimerized (DMRG)
Ferromagnetic (MF)
Ferromagnetic (DMRG)
(0,0,1)
FD-line FP-line
(1,0,0)



$$\varepsilon_4 < \varepsilon_{0,2}$$

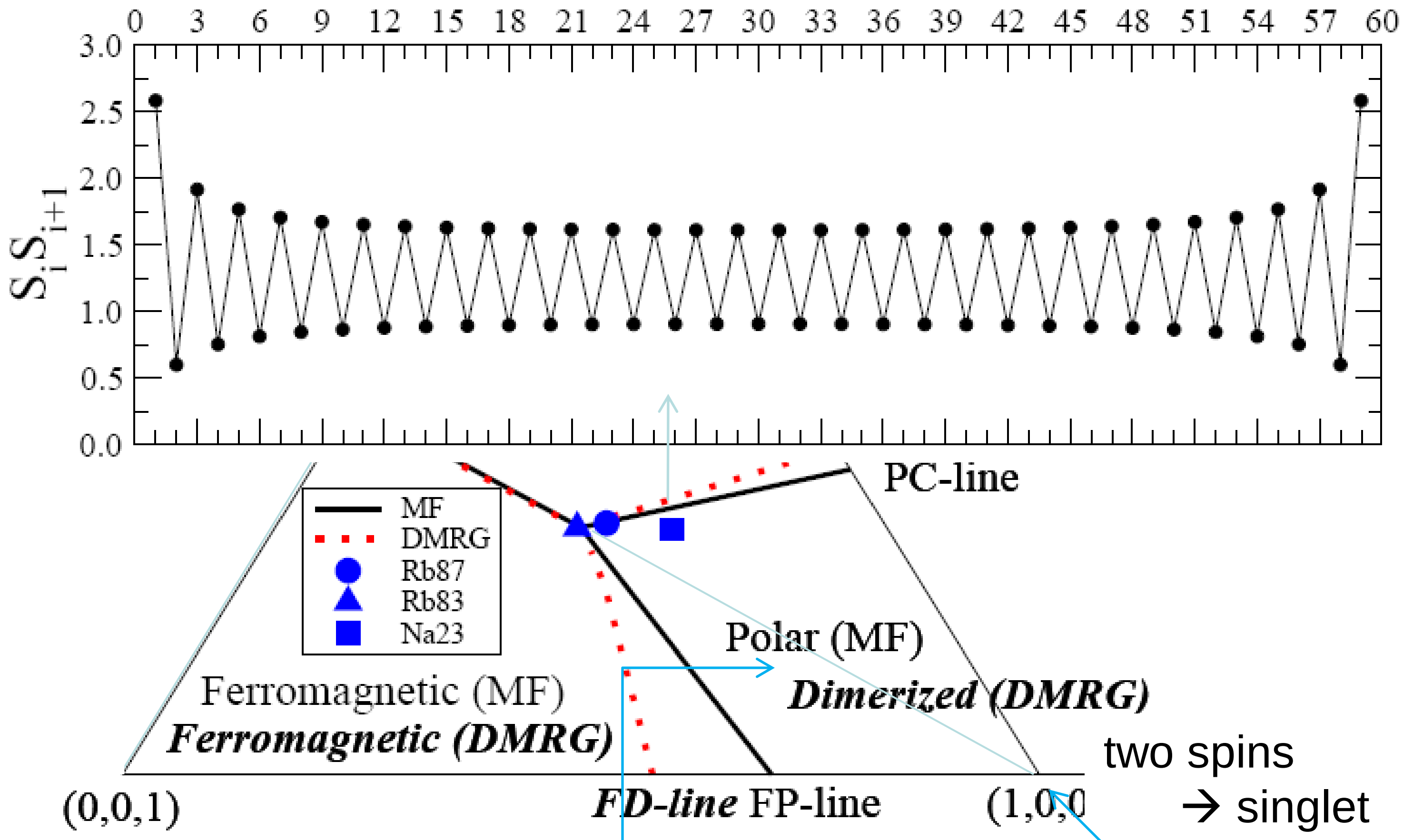
Na23



N=2 j

$S_{tot} = 0$

Na23

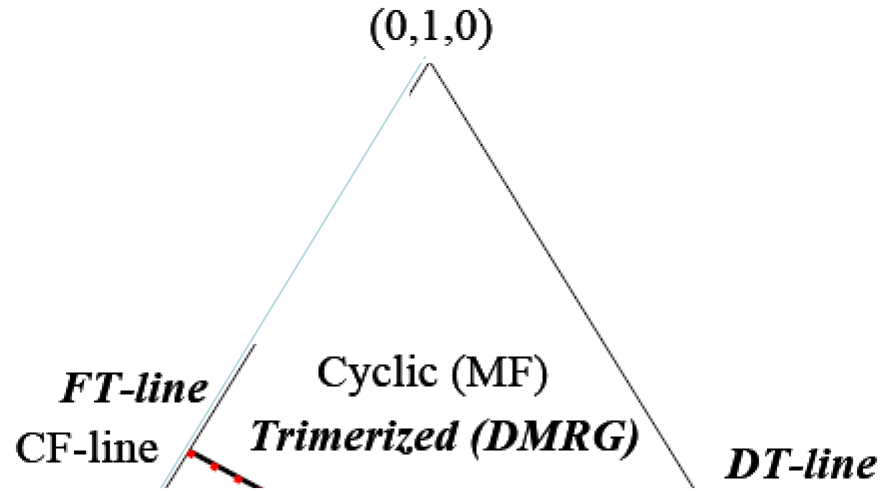


two spins → singlet

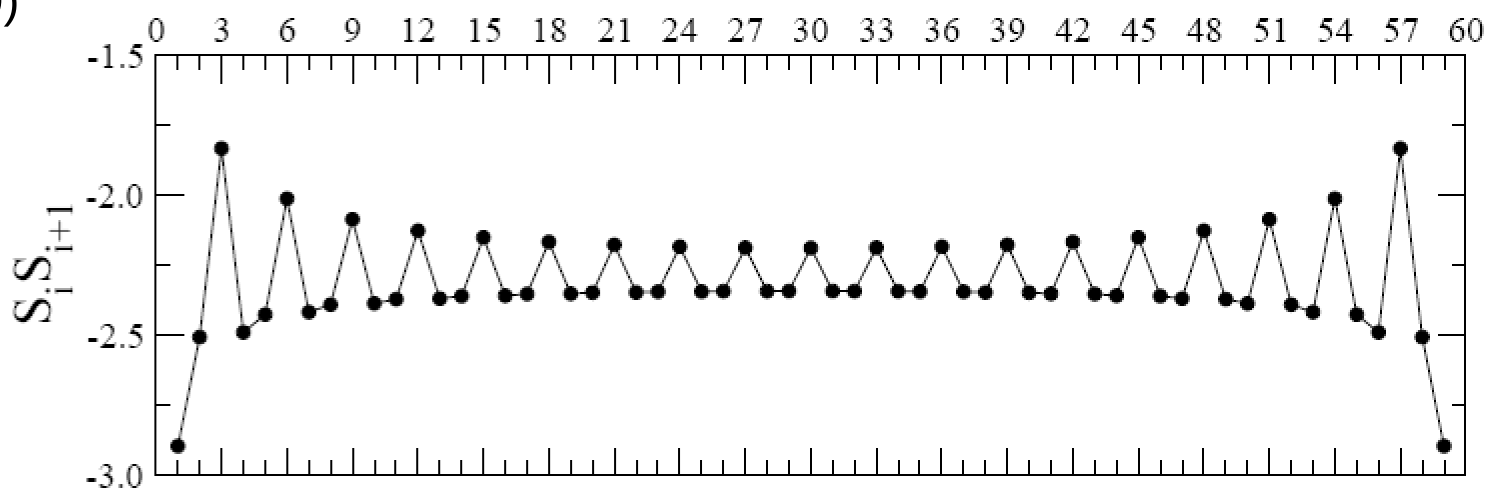
$H \to -P_0 \quad 5 \times \bar{5}$

SO(5); Tu,Zhang,Xiang 08

rigorous: Affleck 90


(0,1,0)
FT-line
Cyclic (MF)
CF-line
Trimerized (DMRG)
DT-line

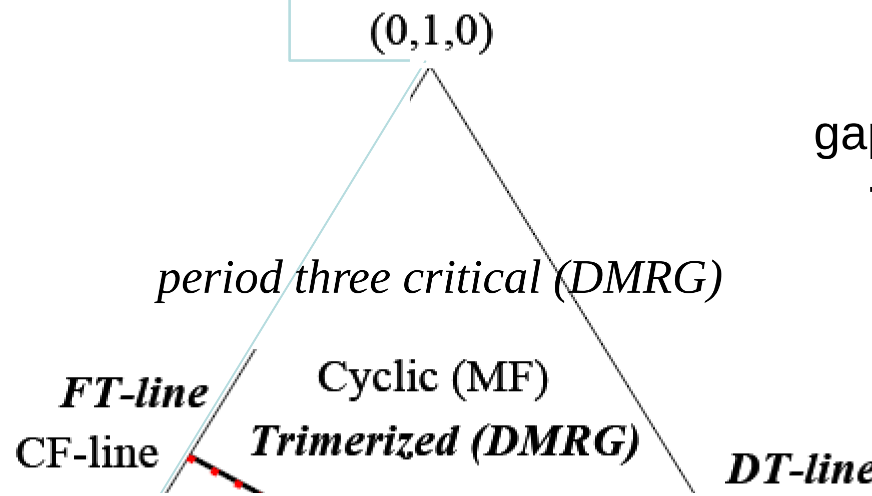
(0,1,0)


0
3
6
9
12
15
18
21
24
27
30
33
36
39
42
45
48
51
54
57
60
-1.5
-2.0
-2.5
-3.0
$S_i S_{i+1}$

Ground state of three spins
= unique singlet

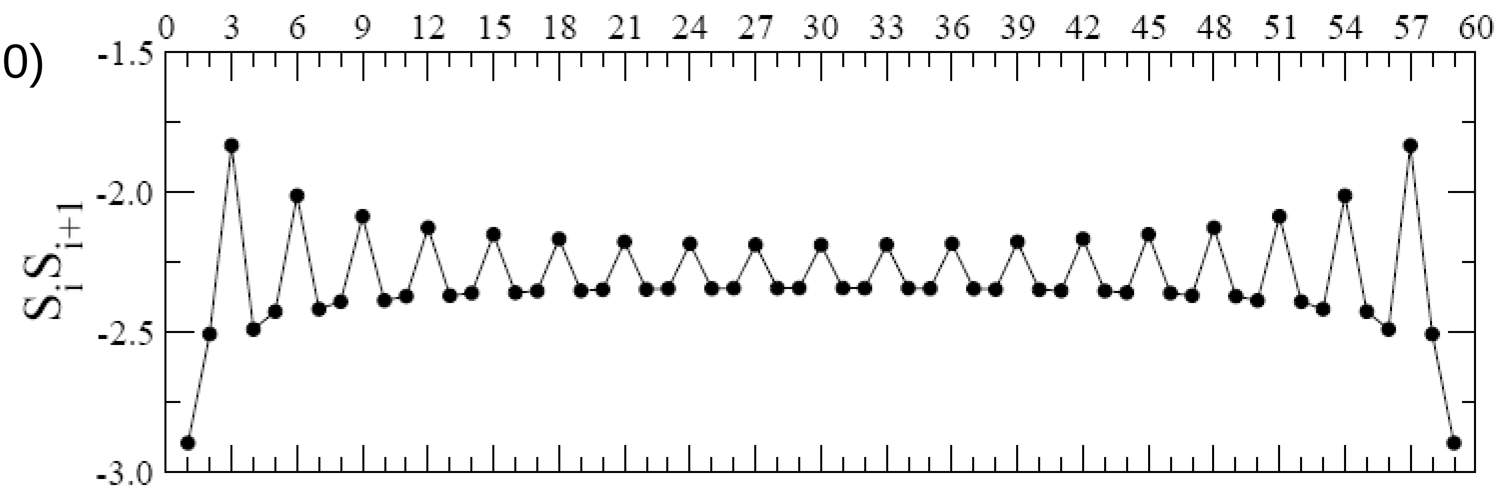
2 2 2

0
1
2 → 0
3
4



gap, order parameter
→ 0 as N→ 0

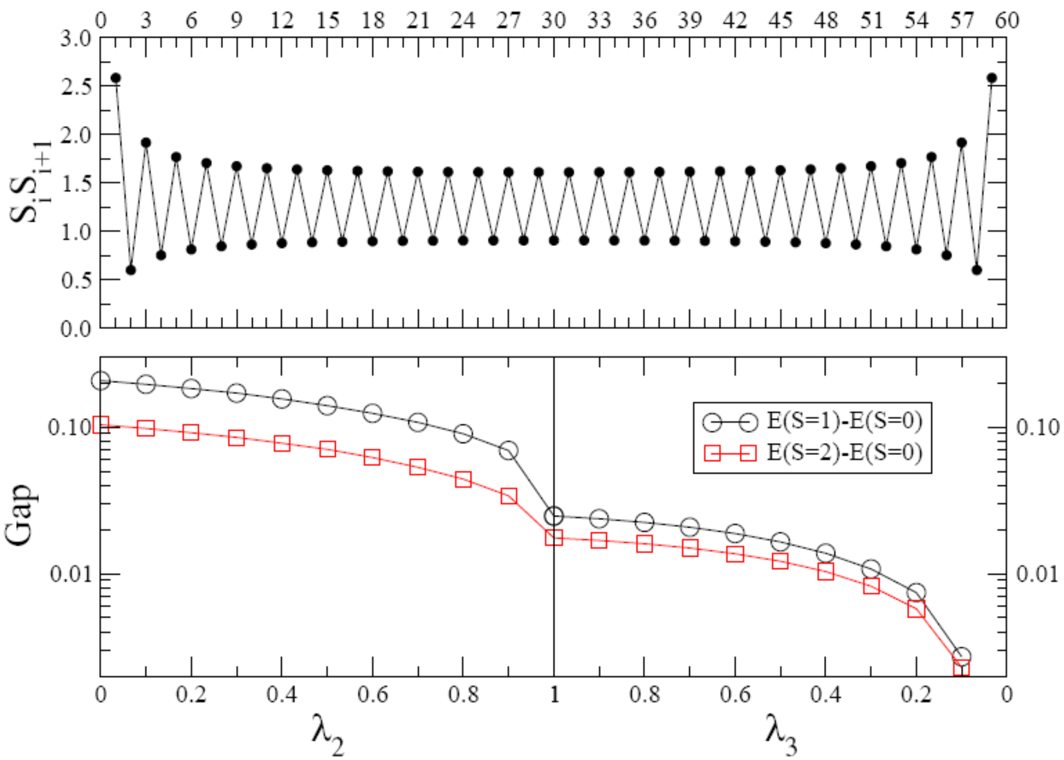
(0,1,0)



N=3j; $S_{tot} = 0$

Adiabatic connection between ground states:

Na23 (dimerized)

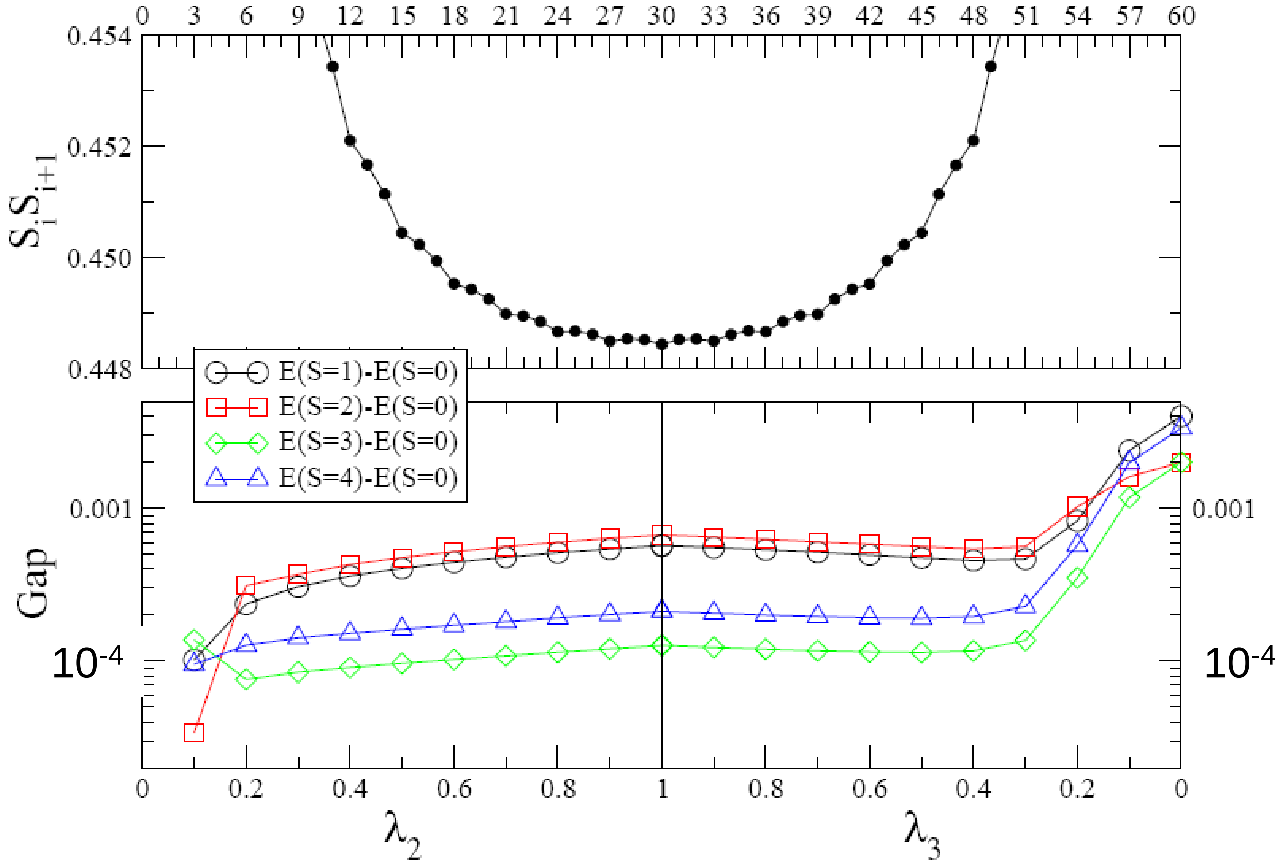


1 λ_2 1 λ_2

1 1 λ_3 1 1 λ_3

Adiabatic connection between ground states:

Rb87 (trimerized)



1 λ_2 1 λ_2

1 1 λ_3 1 1 λ_3

Summary:

Spinor Bosons in optical lattice

realize spin Hamiltionians usually not available in electronic systems

Spin 1

Mean-field: ferro, nematic/quadrupolar

1D: ferro, dimer

Spin 2:

Mean-field: ferro, nematic/quadrupolar, cyclic

1D: ferro, dimer, trimer (period-three critical)

superlattice helps distinguishing phases