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Effects of particle-hole channel on the behavior of BCS-BEC crossover

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Outline

- Motivation
- Pairing fluctuation theory with the particle-hole channel included.
- Calculations and Discussions
- Summary

QC, Kosztin, Janko, and Levin, *Phys. Rev. Lett.* 81, 4708 (1998).

QC, Stajic, Tan, and Levin, *Physics Reports* 412, 1 (2005).

Motivations

- Fermionic superfluidity concerns pairing -– particle-particle channel.
- Particle-hole channel causes chemical potential shift, often neglected.
- Gorkov -- Melik-Barkhudarov (GMB) [Sov. Phys. JETP 13, 1018 (1961)] showed a big effect: Suppression of both Tc and $\Delta(0)$ by a factor of $(4e)^{1/3} \approx 2.22$. -- lowest order effect.
- Berk and Schrieffer studied the “effects of ferromagnetic spin correlations on superconductivity” [PRL 17, 433 (1966)].
- GMB effect neglected until atomic Fermi gases [Heiselberg et al., PRL 85, 2418 (2000)].
- A few other groups:
  - Torma, Pethick, et al, PRL 102, 245301 (2009); 103, 260403 (2009)
- Yin et al, PRA 79, 053636 (2009); 82, 013605 (2010).
  - …

Motivations (cont'd)

- Only lowest order (GMB, Heiselberg et al, Torma et al)

  or
- summation without pseudogap (feedback) effects included: Yin et al.
- Treatment of the particle-hole channel effects only at an average level
- Effects of even higher order ?
- Where to stop?

Theory without particle hole channel -- Grand canonical Hamiltonian

$$H - \sum_{\sigma} \mu_\sigma N_\sigma = \sum_{\mathbf{k},\sigma} (\epsilon_{\mathbf{k}} - \mu_\sigma) a^\dagger_{\mathbf{k},\sigma} a_{\mathbf{k},\sigma}$$
$$+ \sum_{\mathbf{q},\mathbf{k},\mathbf{k}'} U_{eff}(\mathbf{k},\mathbf{k}') a^\dagger_{\mathbf{q}/2+\mathbf{k},\uparrow} a^\dagger_{\mathbf{q}/2-\mathbf{k},\downarrow} a_{\mathbf{q}/2-\mathbf{k}',\downarrow} a_{\mathbf{q}/2+\mathbf{k}',\uparrow}$$

BCS keeps only q=0 terms

Fermi gases: Take contact potential $U_{eff}(\mathbf{k},\mathbf{k}') = U$

Cuprates: Separable potential: $U_{eff}(\mathbf{k},\mathbf{k}') = U \varphi_{\mathbf{k}} \varphi_{\mathbf{k}'}$

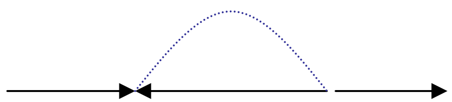
$$\varphi_{\mathbf{k}} = \cos k_x - \cos k_y$$

Pairing fluctuation theory -- Physical Picture

PRL 81, 4708 (1998)

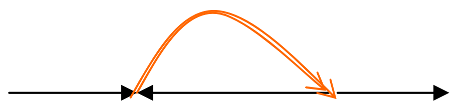
- Fermionic self energy has a pairing origin.
- Pairs can be either condensed or fluctuating.

$\Sigma =$



Σ_{sc}

$+$



Σ_{pg}

T-matrix Formalism

Q, K -- 4-momentum

- T-matrix

$t_{pg}(Q) =$ 

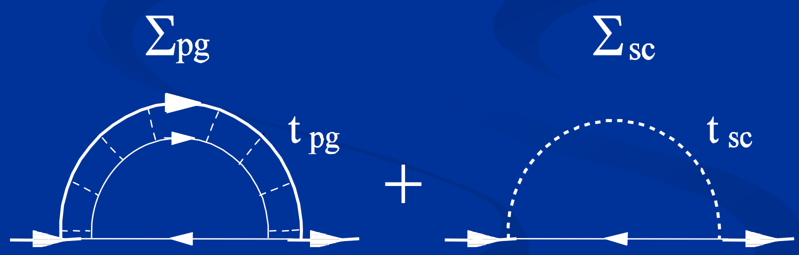
$$t_{pg}(Q) = \frac{U}{1+U\chi(Q)} \approx \frac{Z}{i\Omega - \Omega_q + \mu_{pair}}$$

- Fermion self-energy:

$$\chi(Q) = \sum_K G_0(Q-K)G(K)$$

$$1 + U\chi(Q) = 0 = \mu_{pair}$$

$(T \leq T_c)$

$\Sigma =$ 

$$\Delta^2 = \Delta_{pg}^2 + \Delta_{sc}^2$$

Self Energy

$$\Sigma_{sc}(K) = \longrightarrow\!\!\blacktriangleright\!\!\blacktriangleleft\!\!\overset{G_0(-K)}{\longleftarrow}\!\!\longrightarrow$$

$$= \frac{\Delta^2_{sc,\mathbf{k}}}{i\omega_l + \xi_{\mathbf{k}}}$$

$$\xi_{\mathbf{k}} = \epsilon_{\mathbf{k}} - \mu$$

$$\Sigma_{pg}(K) = \sum_Q t_{pg}(Q) G_0(Q-K)$$

$$\begin{aligned}
\Sigma_{pg}(K) &= \sum_Q \frac{t_{pg}(Q)}{i\Omega_n - i\omega_l - \xi_{\mathbf{q}-\mathbf{k}}} \\
&= -\sum_Q \frac{t_{pg}(Q)}{i\omega_l + \xi_{\mathbf{k}}} + \delta\Sigma \\
&= \frac{\Delta^2_{pg}}{i\omega_l + \xi_{\mathbf{k}}} + \delta\Sigma
\end{aligned}$$

$$\Delta^2_{pg} = -\sum_Q t_{pg}(Q)$$

$$\Delta^2 = \Delta^2_{pg} + \Delta^2_{sc}$$

$$\boxed{\Sigma = -\Delta^2 G_0}$$

$$Q \equiv (i\Omega_n, \mathbf{q})$$

Self consistent equations

PRL 81, 4708 (1998)

$$\Sigma = -\Delta^2 G_0$$

BCS form self energy → BCS form of gap equation, with total gap Δ.

$$E_{\mathbf{k}} = \sqrt{\xi_{\mathbf{k}}^2 + \Delta_{\mathbf{k}}^2}$$

$$1 + U \sum_{\mathbf{k}} \frac{1 - 2f(E_k)}{2E_k} \varphi_{\mathbf{k}}^2 = 0$$

$$\Delta_{sc}^2 \sim n_{cond}$$

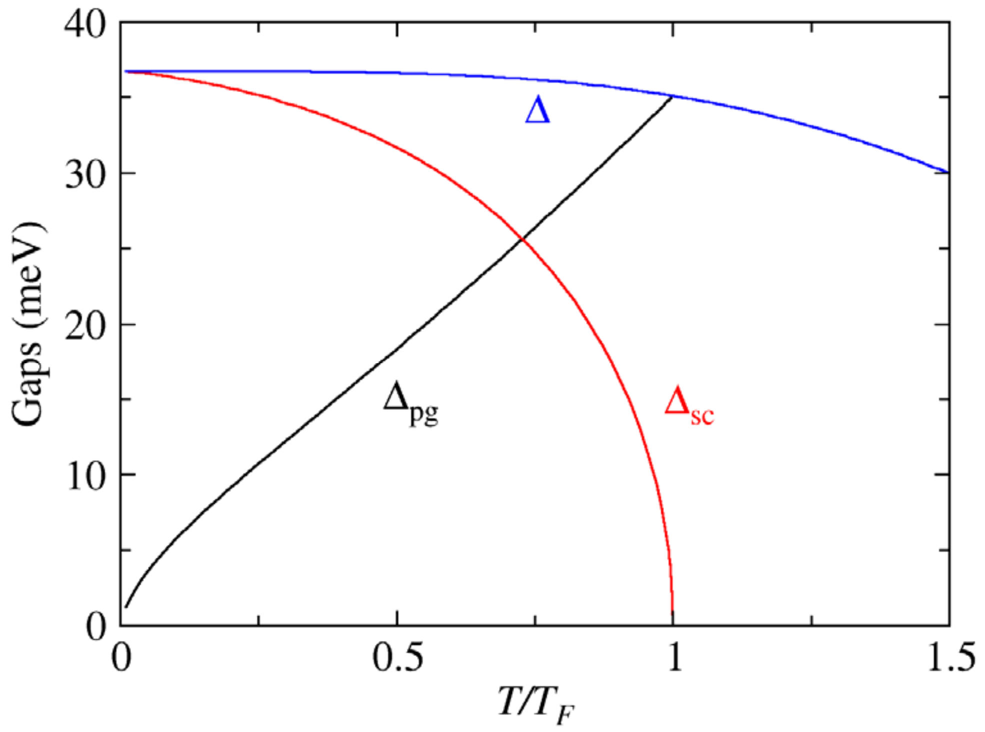
$$\Delta_{pg}^2 \sim n_{nc,pair}$$

$$n = 2 \sum_K G(K)$$

$$\Delta_{pg}^2 = -\sum_Q t_{pg}(Q)$$

$$t_{pg}(Q) \approx \frac{Z}{i\Omega - \Omega_q + \mu_{pair}}$$

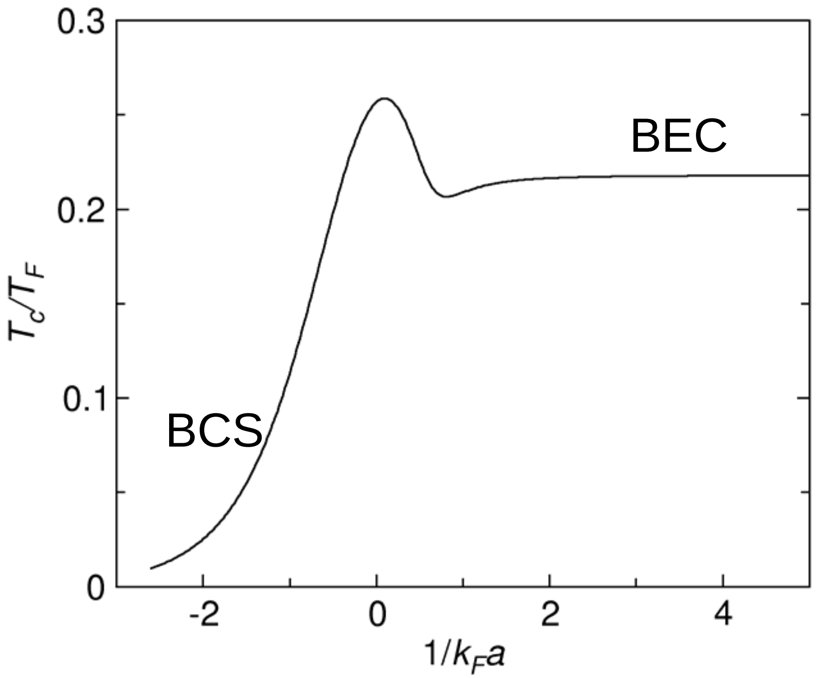
Behavior of gaps vs *T*



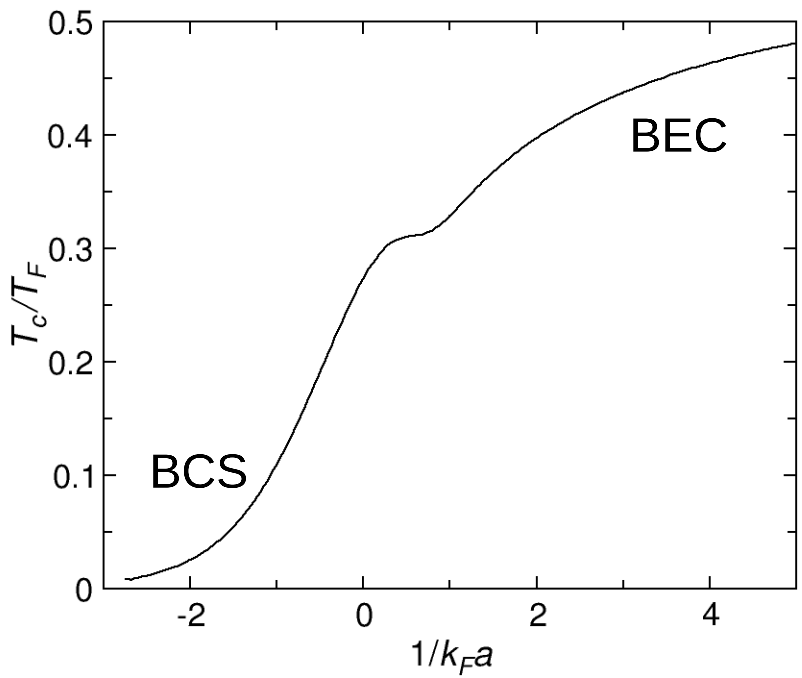
How to determine Tc?

$$\frac{\Delta_{sc}^2}{\Delta^2}$$ --- Fraction of condensed pairs.

Typical behavior of Tc vs interaction



Homogeneous

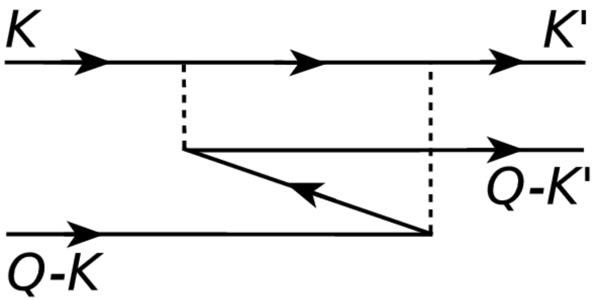


In a trap

Particle-hole channel

- Lowest order induced interaction U^0_{ind}

$$\chi^0_{ph}(P) = \chi^0_{ph}(K+K'-Q) =$$



$$= \sum_K G_0(K)G_0(K-P) = \sum_{\mathbf{k}} \frac{f(\xi_{\mathbf{k}}) - f(\xi_{\mathbf{k}-\mathbf{p}})}{\xi_{\mathbf{p}} - \xi_{\mathbf{k}-\mathbf{p}} - i\Omega_n}$$

$$U^0_{ind}(P) = -U^2\chi^0_{ph}(P)$$

$$G_0^{-1}(K) = i\omega_l - \xi_{\mathbf{k}}$$

Real and imaginary parts

- After analytical continuation, $i\Omega_n \rightarrow \Omega + i0^+$

$$\chi^0_{ph}(\Omega + i0^+, \mathbf{p}) \equiv \mathrm{Re}\,\chi^0_{ph} + i\mathrm{Im}\,\chi^0_{ph}$$

$$\mathrm{Re}\,\chi^0_{ph}(\Omega, p) = \sum_{\mathbf{k}} \frac{f(\xi_{\mathbf{k}}) - f(\xi_{\mathbf{k}-\mathbf{p}})}{\xi_{\mathbf{k}} - \xi_{\mathbf{k}-\mathbf{p}} - \Omega}$$

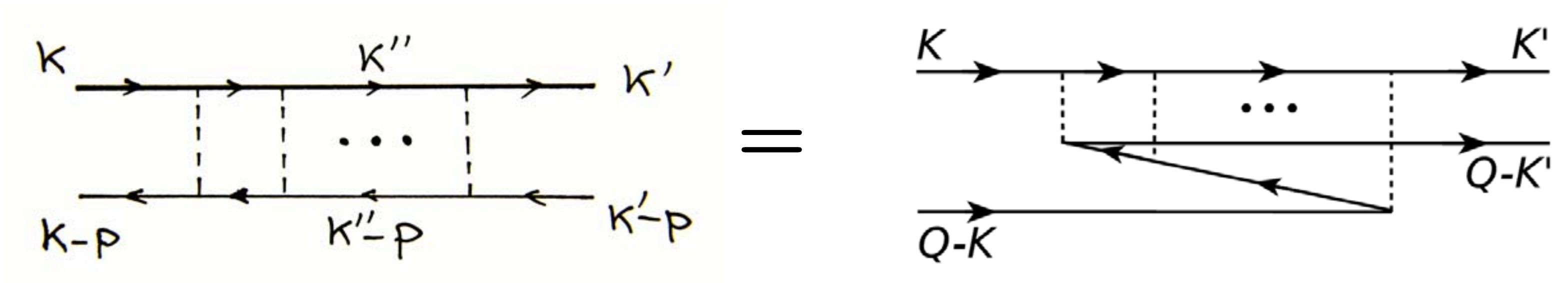
$$\mathrm{Im}\,\chi^0_{ph}(\Omega, p) = \pi \sum_{\mathbf{k}} [f(\xi_{\mathbf{k}}) - f(\xi_{\mathbf{k}-\mathbf{p}})]\delta(\xi_{\mathbf{k}} - \xi_{\mathbf{k}-\mathbf{p}} - \Omega)$$

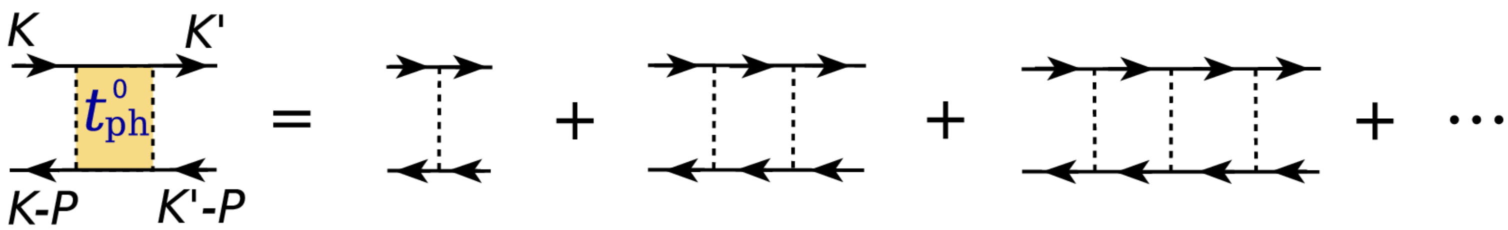
$$\mathrm{Re}\,\chi^0_{ph}(0, p) = \int_0^\infty \frac{k\mathrm{d}k}{2\pi^2} \frac{m}{p} f(\xi_{\mathbf{k}}) \ln \left| \frac{2k - p}{2k + p} \right|$$

$$\mathrm{Im}\,\chi^0_{ph}(\Omega, 0) = 0$$

Sum of particle-hole ladders

- One may sum over the infinite ladders of particle-hole scattering, i.e., the particle-hole t-matrix = total interaction





$$t^0_{ph}(P) = \frac{U}{1 + U\chi^0_{ph}(P)} = U^0_{eff}(P)$$

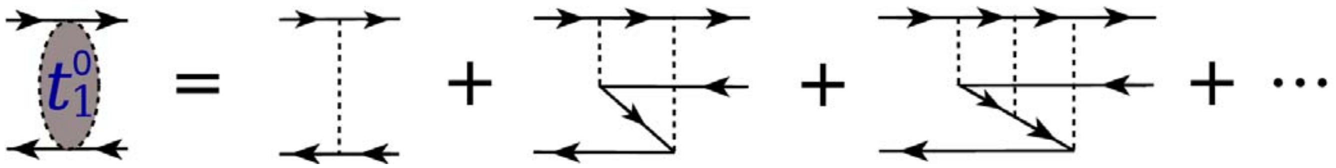
Induced interaction:

Leading term *U* belongs to particle-particle channel

$$U^0_{ind}(P) = t^0_{ph}(P) - U = -\frac{U^2\chi^0_{ph}(P)}{1+U\chi^0_{ph}(P)}$$

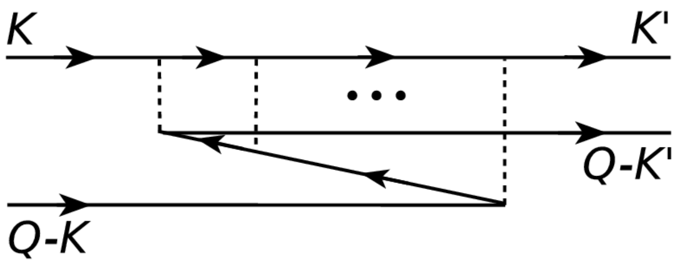
- To proceed, either replace U with t^0_{ph} in particle-particle T-matrix $t^0_1(Q)$
- or
- Replace U with t^0_1 in the particle-hole T-matrix t^0_{ph}

- We take the 2nd route

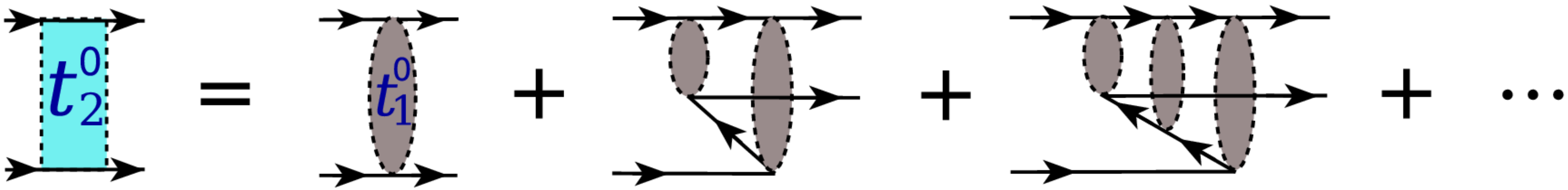


$$= \frac{1}{U^{-1}+\chi^0(Q)}$$

Effective T-matrix t_2^0: Diagrammatic resummation



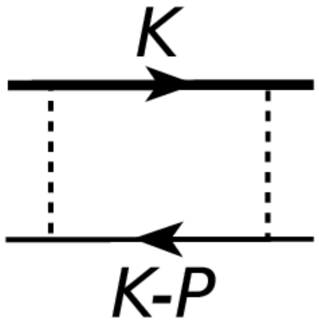
(Replace U with t_1^0)



$$t_2^0 = \frac{t_1^0}{1 + t_1^0 \chi_{ph}^0} = \frac{1}{{t_1^0}^{-1} + \chi_{ph}^0} = \frac{1}{U^{-1} + \chi^0 + \chi_{ph}^0}$$

Particle-hole susceptibility χ_{ph} in the presence of feedback effects

χ_{ph} =



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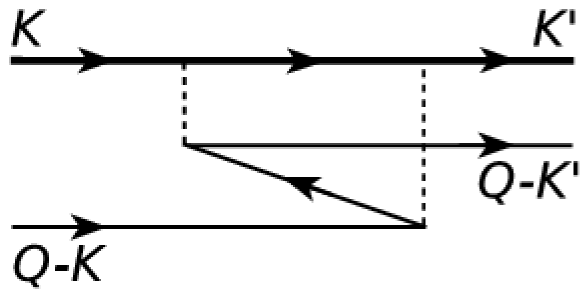
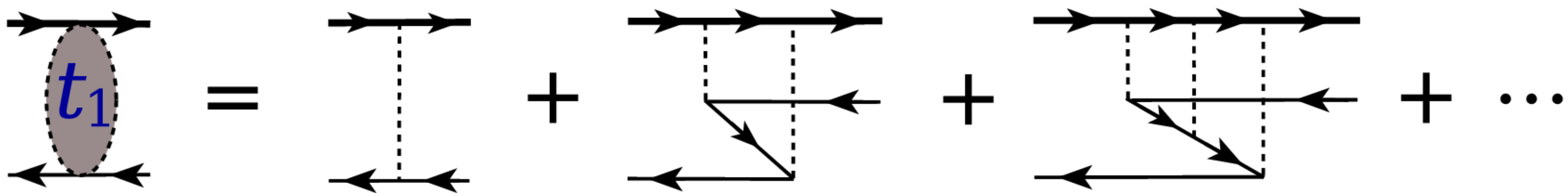


$$\chi_{ph}(P) = \chi_{ph}(K+K'-Q) = \sum_{K'} G(K')G_0(K'-P)$$

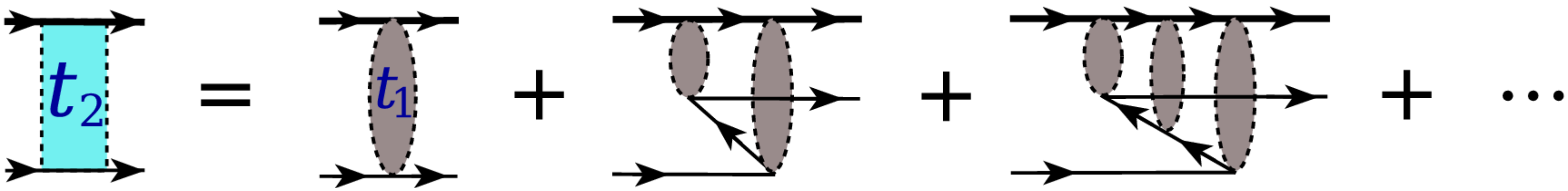
$$= \sum_{\mathbf{k}} \left[\frac{f(E_{\mathbf{k}}) - f(\xi_{\mathbf{k-p}})}{E_{\mathbf{k}} - \xi_{\mathbf{k-p}} - i\Omega_n} u_{\mathbf{k}}^2 - \frac{1 - f(E_{\mathbf{k}}) - f(\xi_{\mathbf{k-p}})}{E_{\mathbf{k}} + \xi_{\mathbf{k-p}} + i\Omega_n} v_{\mathbf{k}}^2 \right]$$

U_{eff} contains both Ω and momentum dependence, in general is no longer a separable potential.

Effective T-matrix t_2 : Diagrammatic resummation



$$= \frac{1}{U^{-1} + \chi(Q)}$$

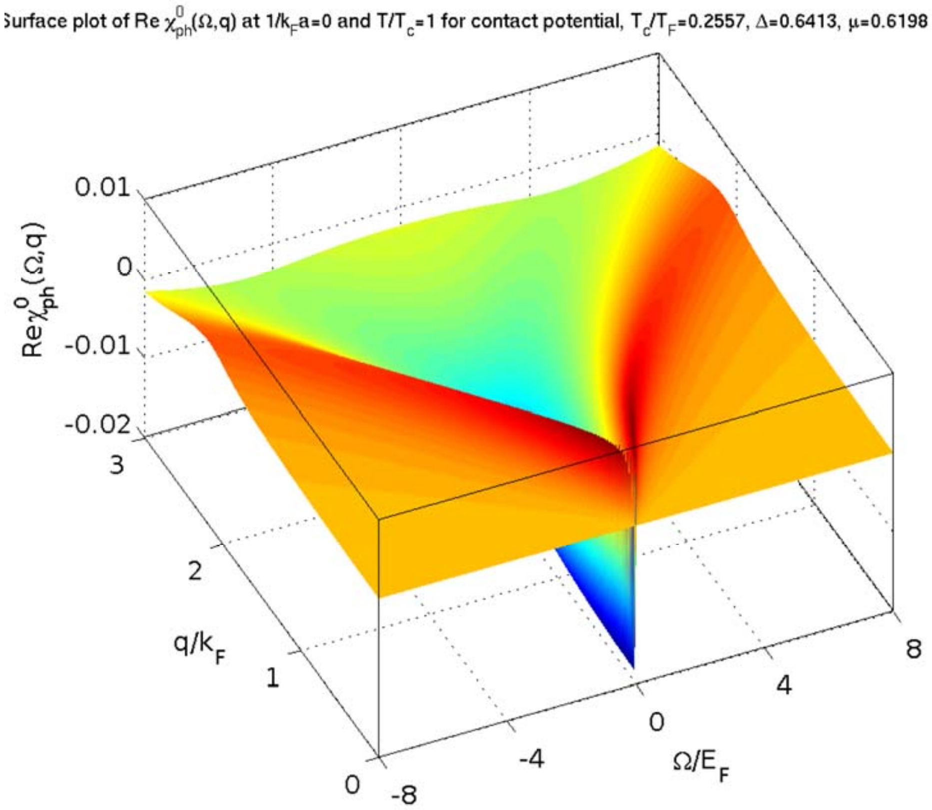


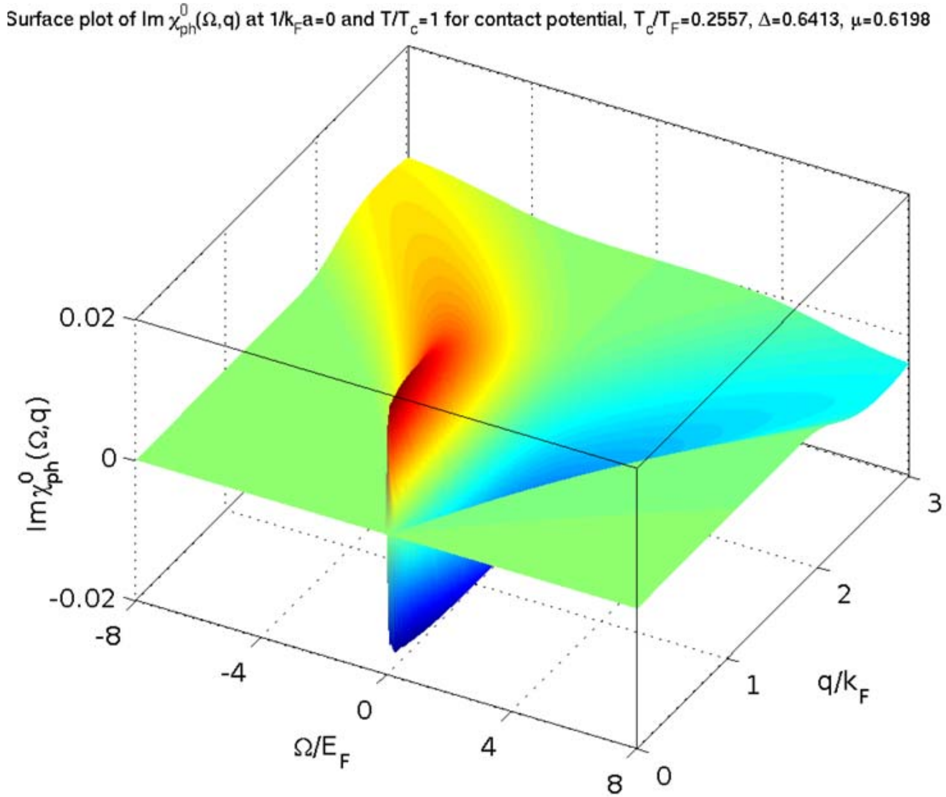
$$t_2 = \frac{t_1}{1 + t_1 \chi_{ph}} = \frac{1}{t_1^{-1} + \chi_{ph}} = \frac{1}{U^{-1} + \chi + \chi_{ph}}$$

Structure of $\chi_{ph}(Q)$

Here we use *Q* for *P*

At unitarity and *T*=*Tc*

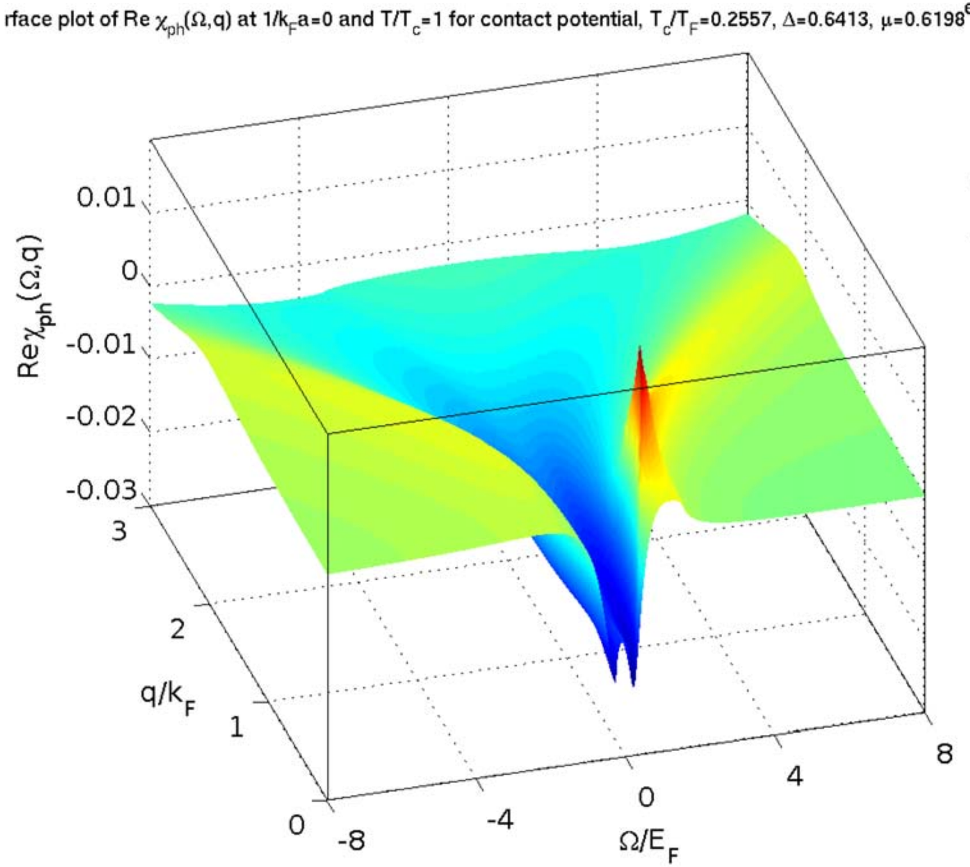




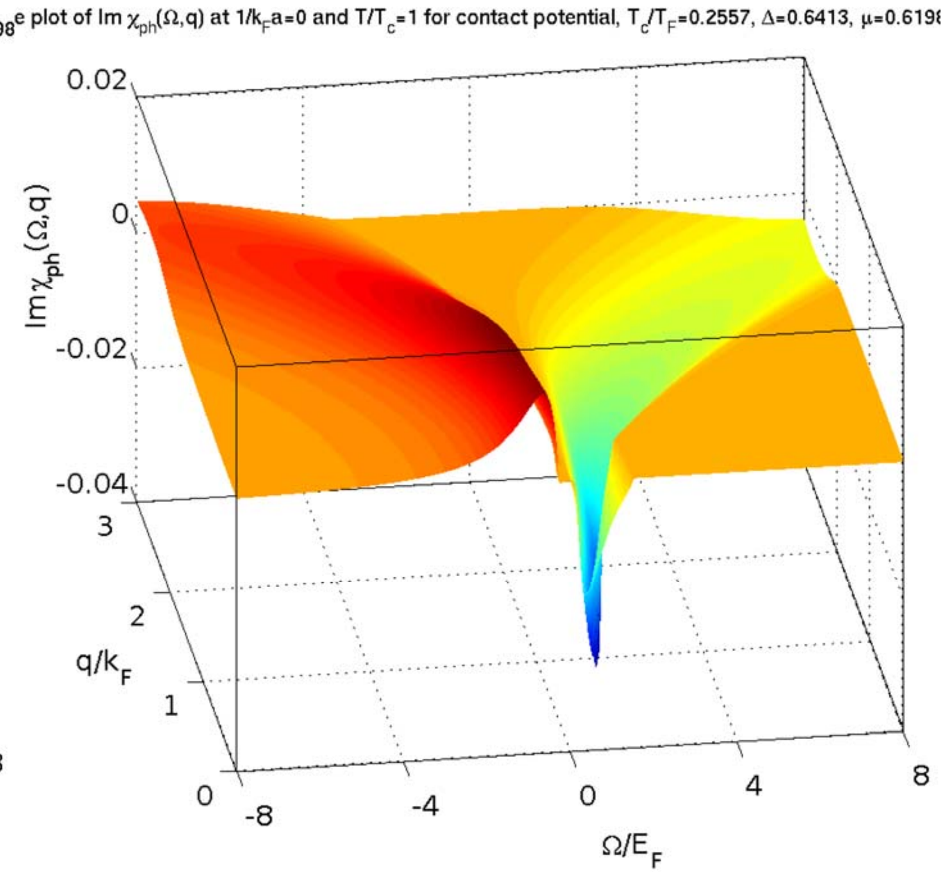
Re $\chi_{ph}{}^0(\Omega, q)$

Im $\chi_{ph}{}^0(\Omega, q)$

Existence of a gap makes $\chi_{ph}(Q)$ more complex

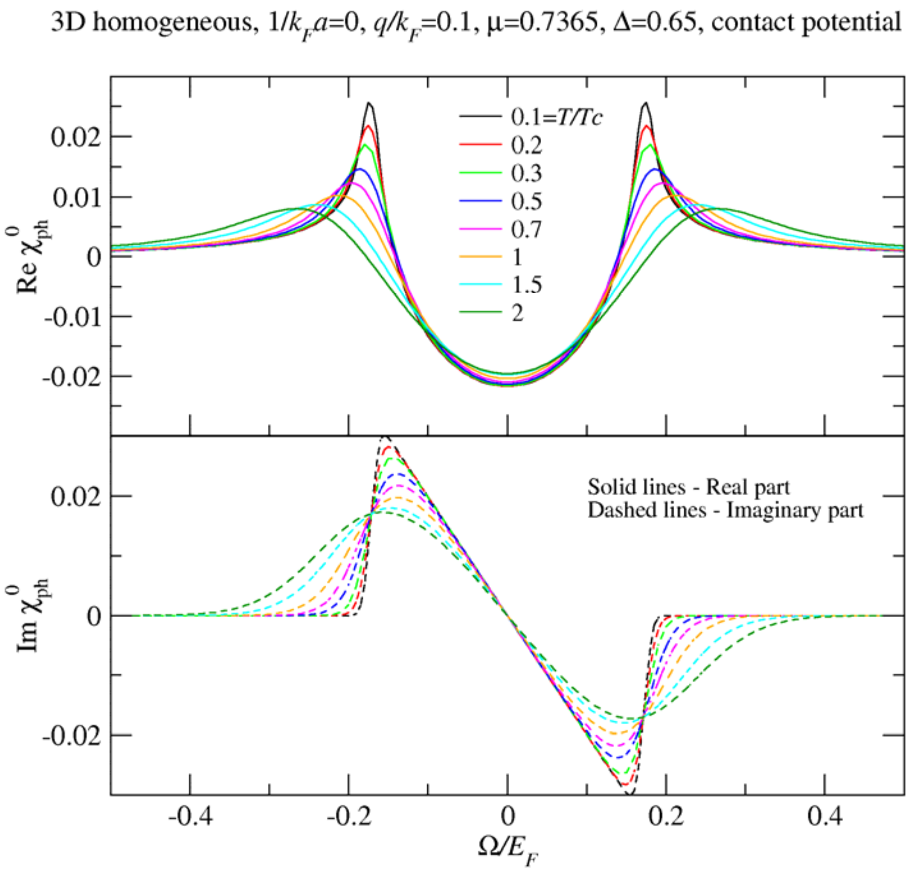


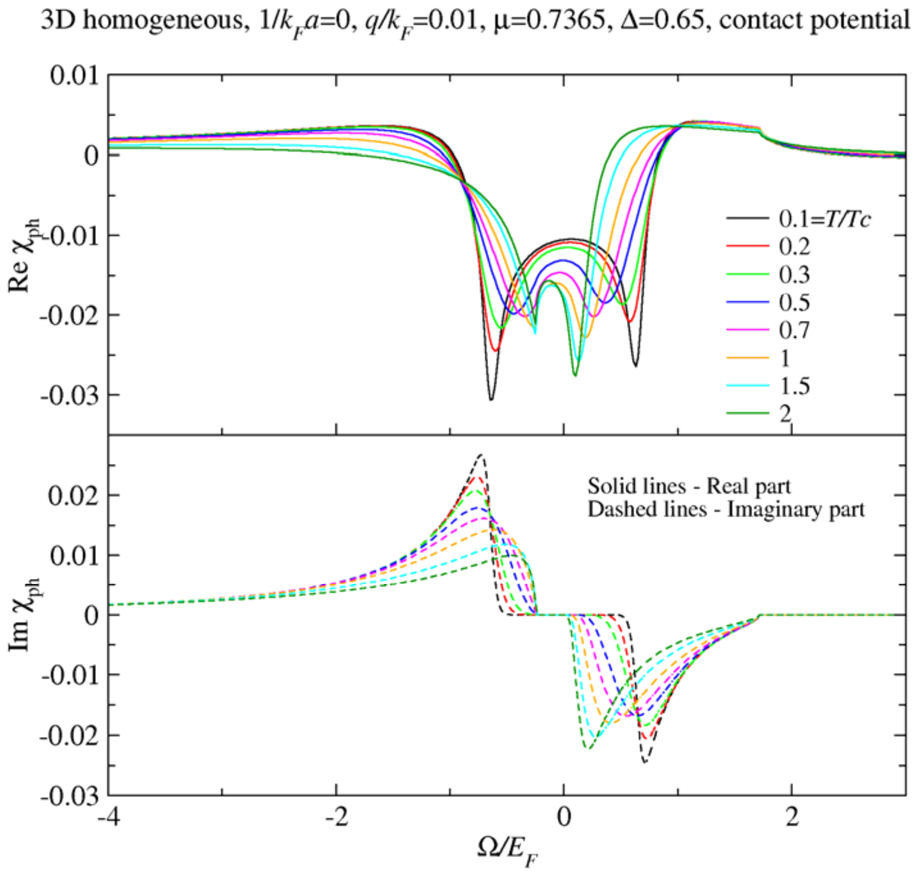
Re $\chi_{ph}(\Omega, q)$



Im $\chi_{ph}(\Omega, q)$

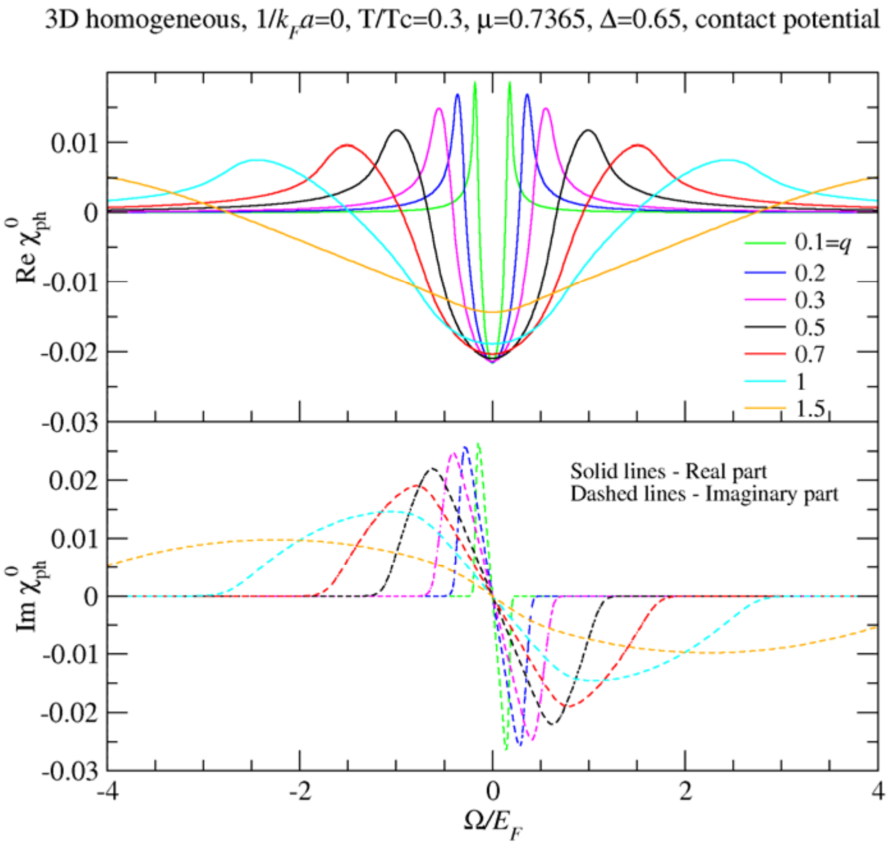
Temperature dependence



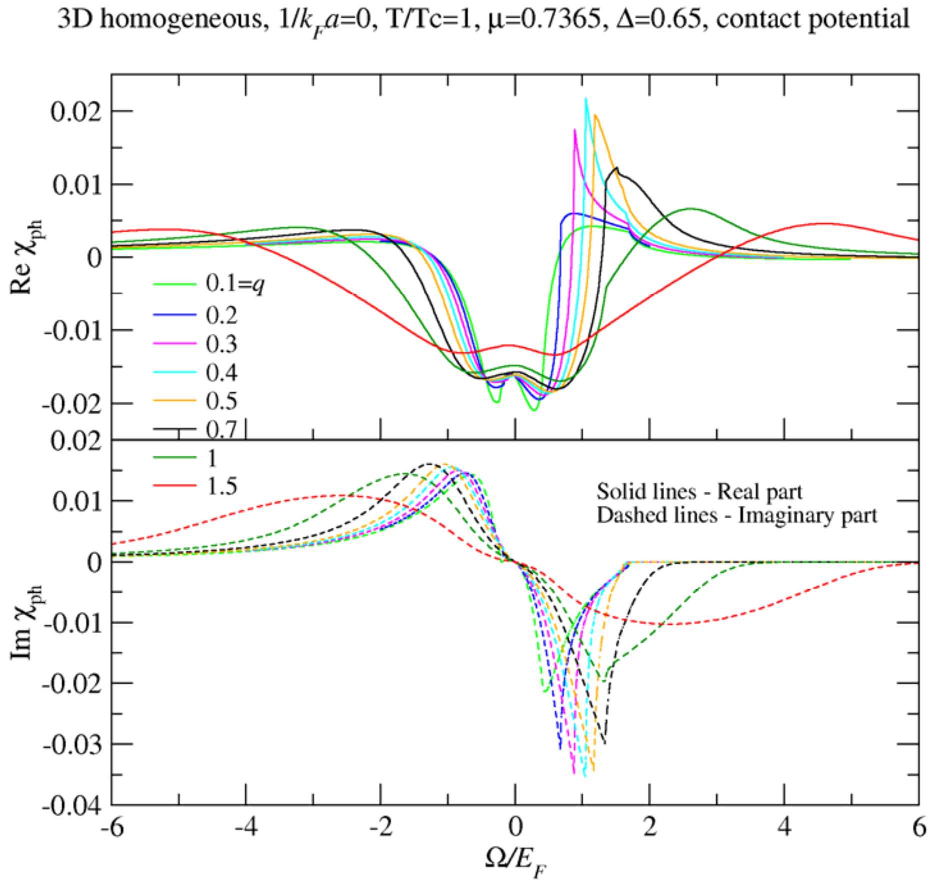


$\chi_{ph}^{0}(\Omega, q)$ $\chi_{ph}(\Omega, q)$

Momentum dependence

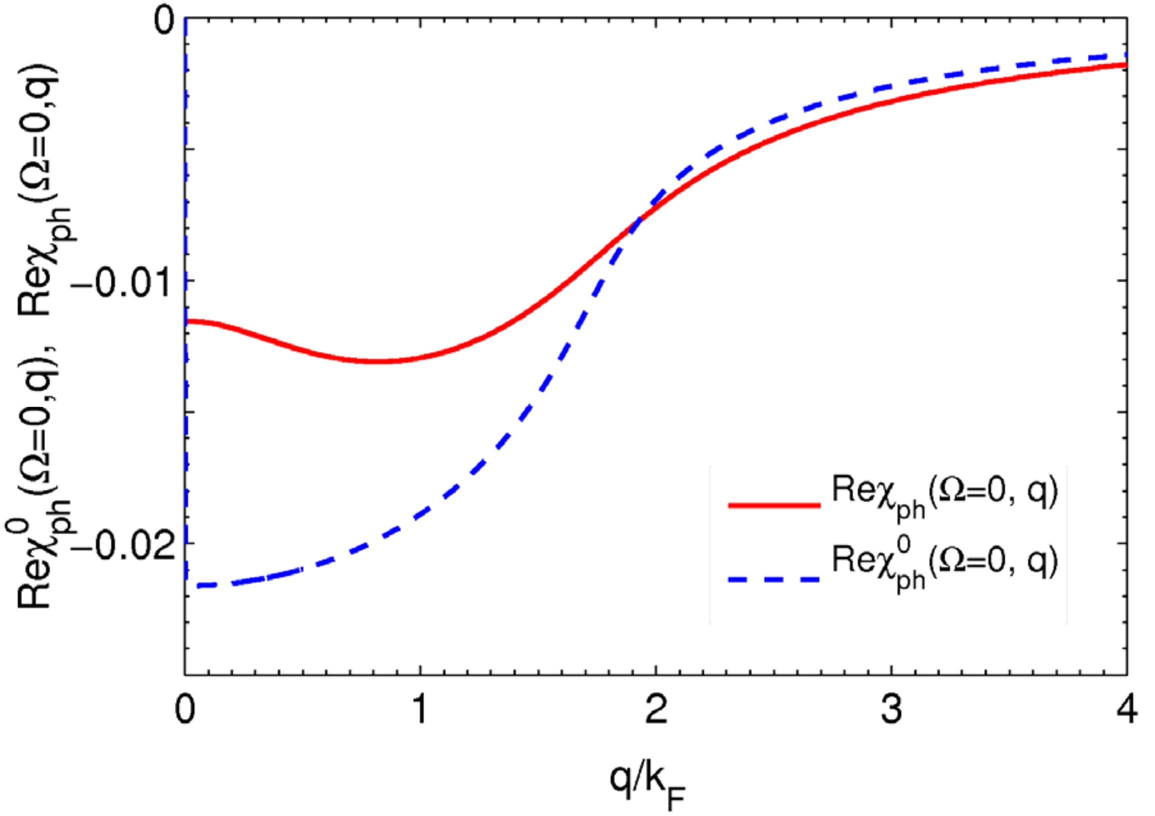


$\chi_{ph}{}^{0}(\Omega, q)$



$\chi_{ph}(\Omega, q)$

Momentum dependence of $\chi_{ph}(\Omega=0)$



Angular average of $\chi_{ph}(K+K')$

$\rightarrow <\chi_{ph}>$

- Zero momentum Q=0 Cooper pairs:

  $P = K+K' = (i\Omega, \mathbf{k}+\mathbf{k}')$.

- Choose $\Omega = 0$, and $k = k'$ --- On-shell scattering, then average over angles

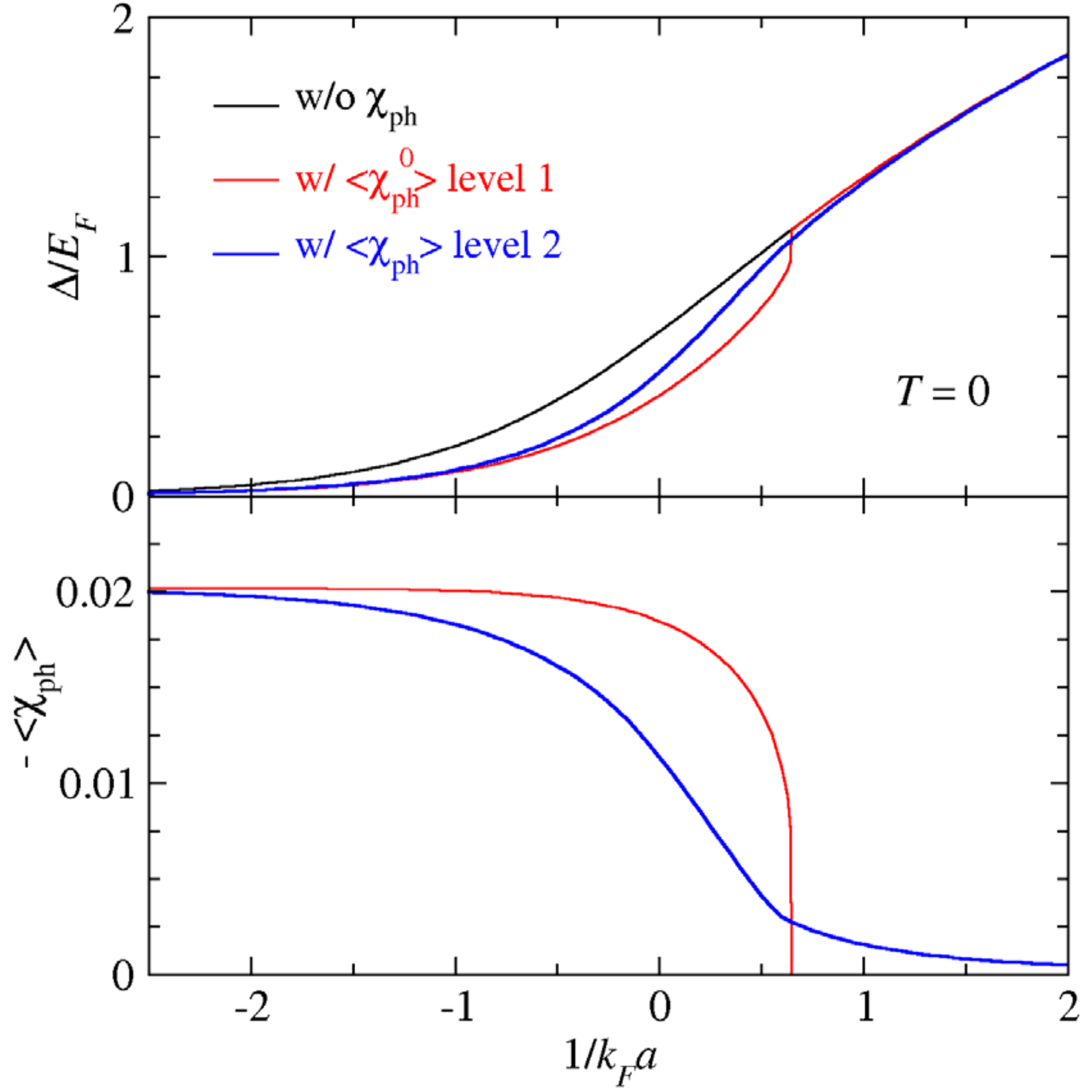
$$p = |\mathbf{k} + \mathbf{k}'| = 2k\sqrt{1 + \cos\theta}$$

- Off-shell scattering leads to imaginary part of $\chi_{ph} \rightarrow$ imaginary part and frequency dependence in order parameter.

- Further (level 2) average over a range of k such that $E_k < (E_k)_{min} + \Delta$

$$(E_{\mathbf{k}})_{min} = \Delta \qquad \text{if } \mu > 0.$$
$$(E_{\mathbf{k}})_{min} = \sqrt{\mu^2 + \Delta^2} \qquad \text{if } \mu < 0.$$

Effect on zero T gap

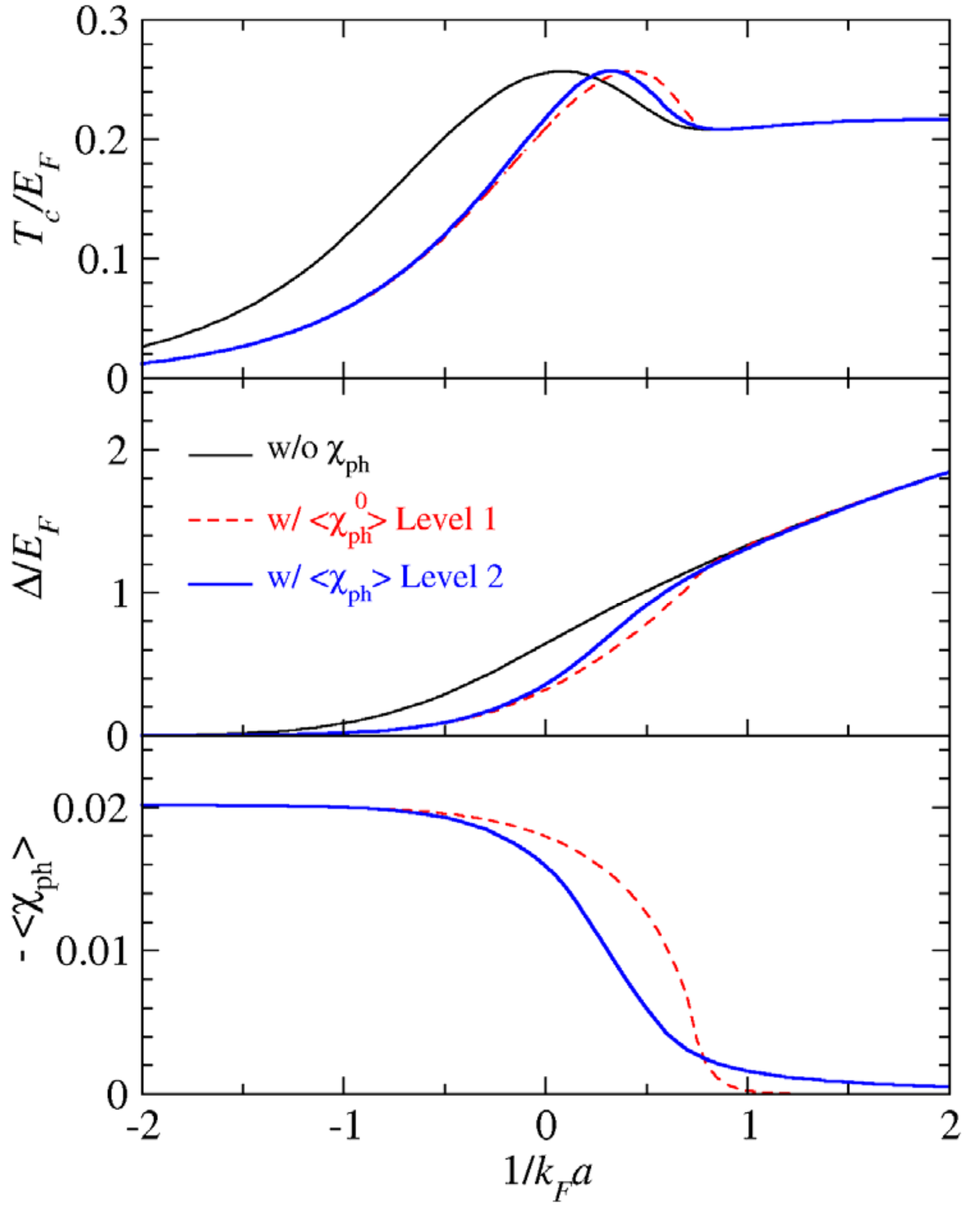


Level 1: Average at Fermi level only

Level 2: Average over a range of k near Fermi level

χ_{ph} effects diminish quickly in the BEC regime where $\mu < 0$.

Effects on *Tc*



T_c/T_F is suppressed from 0.255 to 0.215 at unitarity

Hartree self-energy not included in this calculation.

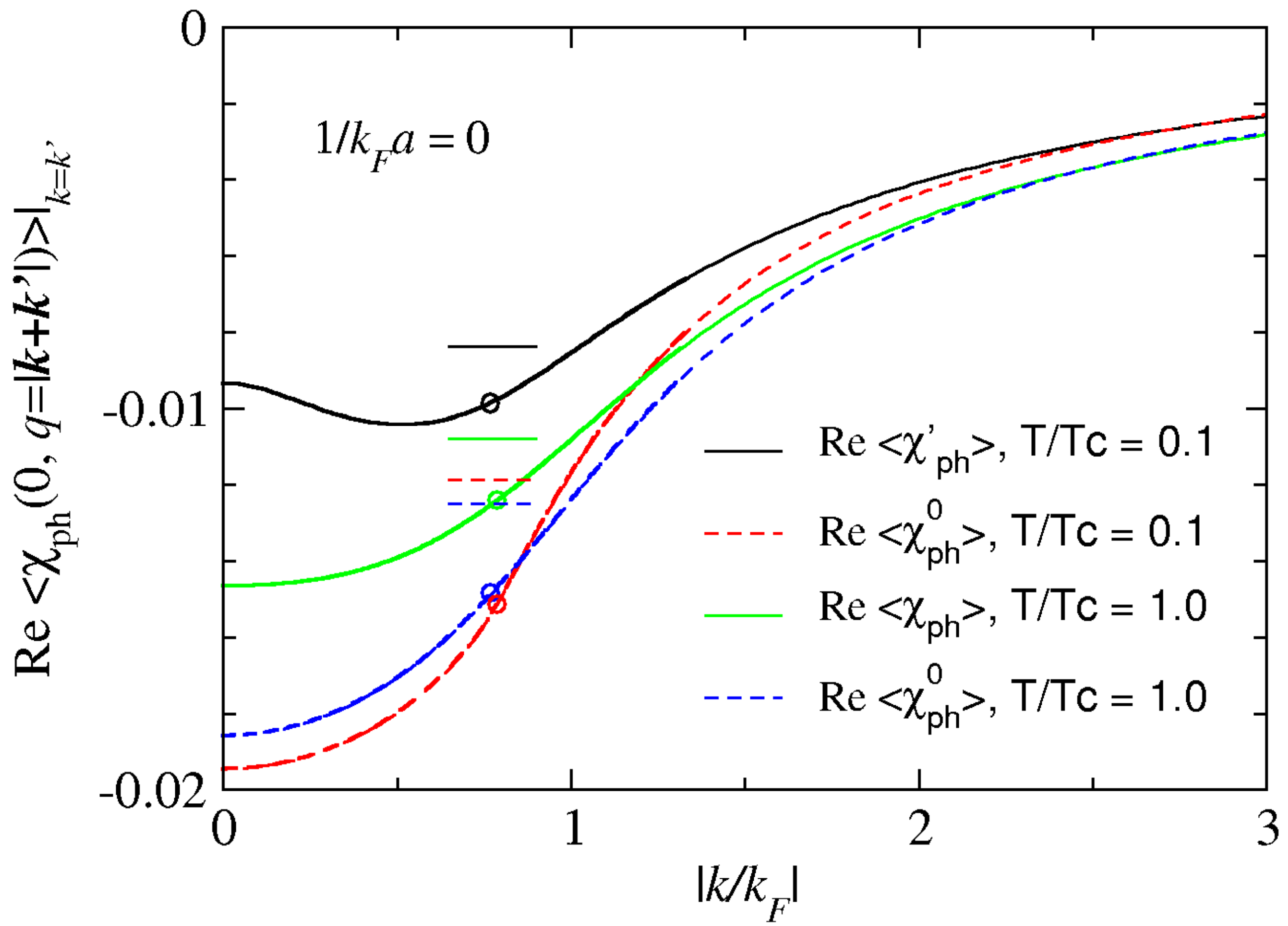
χ_{ph} effects diminish quickly in the BEC regime where $\mu < 0$.

Results agree in the BCS limit.

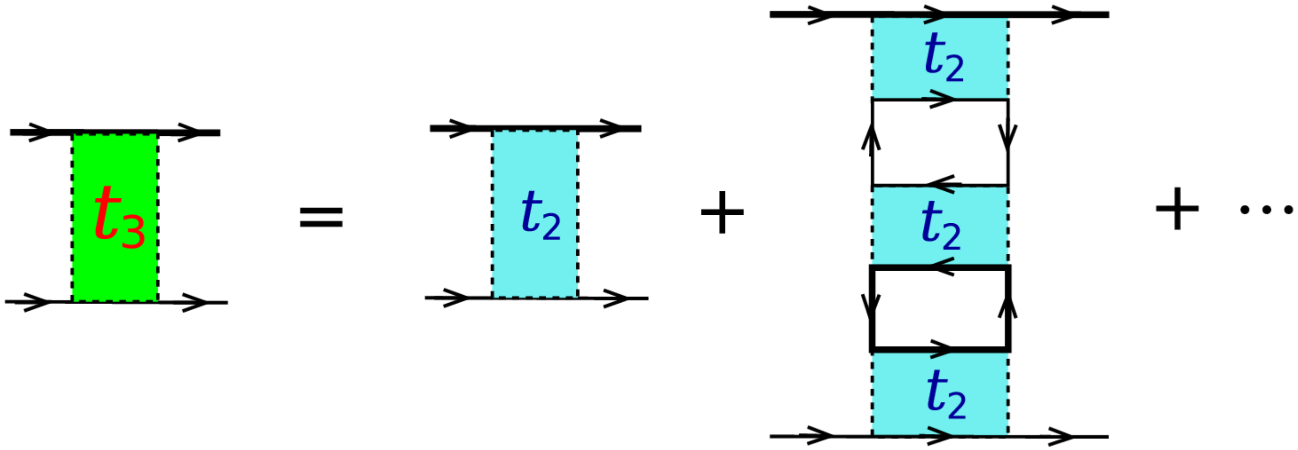
Effects on $2\Delta(0)/T_c$

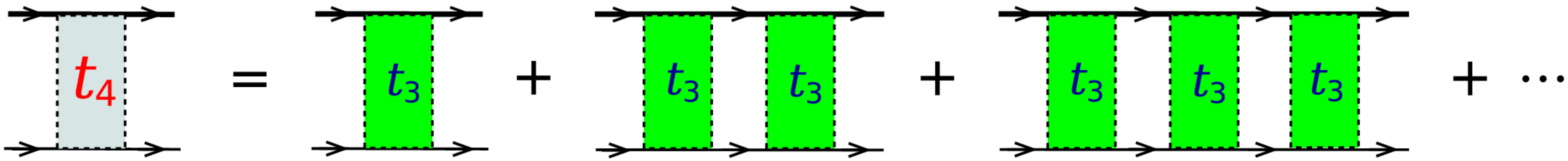
- Strong T dependence → ratio change.

3D homo, contact potential, unitary, Tc=0.2557, angular average of $\chi_{ph}(0,q)$



Where to stop?





Summary

- We have studied the effects of particle-hole channel on BCS-BEC crossover and compared with lower level approximations.
- We included the mode-coupling effects on the particle-hole susceptibility χ_{ph}, which leads to substantial differences.
- Strong temperature and momentum/frequency dependences of χ_{ph} are discovered. Away from the BCS limit, $\Delta(0)$ and T_c are suppressed differently.
- The particle-hole channel effects diminish quickly once the system enters BEC regime.
- Full-fledged calculations without taking simple angular average and setting $\Omega=0$ are needed.
- It is unclear whether higher order T-matrices will make a difference or not.

Thank you !