Dissipatively Driven Many-Body Pairing State for Cold Fermions in an Optical Lattice

Wei Yi

University of Science and Technology of China

中国科学技术大学

Innsbruck collaborators:
Sebastian Diehl
Andrew Daley
Peter Zoller

Outline

- General idea of reservoir engieering
- Some examples
- Dissipatively driven pairing states of Fermions
  - Simple case: Anti-ferromagnetic Néel state
  - d-wave pairing state
  - General strategy
  - p-wave pairing state. Topological order?
- Mean field theory
- Physical implementation
- Application: effective cooling scheme via adiabatic connection
- Summary

Conventional ground state preparation

Start from a given many body Hamiltonian H, cool the system into the ground state

$$\rho \xrightarrow{T\to 0} |\psi_g\rangle\langle\psi_g|$$

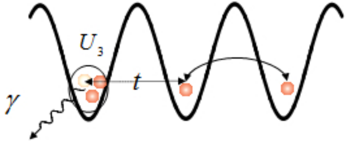
Open system with drive and dissipation

$$\frac{d\rho}{dt} = -i\,[H,\rho] + \mathcal{L}\rho$$

$$\rho(t) \xrightarrow{t\to\infty} \rho_{\mathrm{steady}}$$

Engineer system-reservoir coupling, so that ρ_{steady} is localized in the Hilbert space.

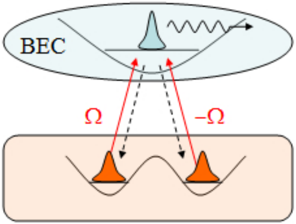
Example I: Stabilization of p-wave superfluid



- System cannot thermalize due to large three-body losses
- In an optical lattice, three-body losses suppressed by quantum Zeno effect, i.e.loss blockade

Y.-J, Han *et al.*, Phys. Rev. Lett. 103, 070404 (2009)

Example II: Driven dissipative BEC



$$J_{ij} = (a_i^\dagger + a_j^\dagger)(a_i - a_j)$$

- Local jump operators lock phases between adjacent sites
- Long range order established by sequence of local jump operators
- The coherence of BEC eventually comes from the laser
- Final steady state not dependent upon initial density matrix

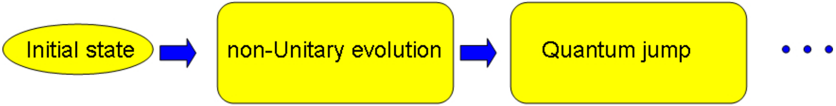
S. Diehl *et al.*, Nature Physics 4, 878 (2008)

Master equation

$$i\frac{\partial\rho}{\partial t} = -iH_{eff}\rho + i\rho H_{eff}^{\dagger} + \kappa\sum_{l} J_l\rho J_l^{\dagger}$$

$$H_{eff} = H - \frac{i}{2}\kappa\sum_{l} J_l^{\dagger}J_l$$

Quantum trajectory picture



$|\psi\rangle$ $\quad$ $e^{-iH_{eff}\Delta t}|\psi\rangle$ $\quad$ $J_l|\psi(\Delta t)\rangle$ with rate $\kappa\| J_l|\psi\rangle\|^2$

State preparation based on dissipative processes

Look for appropriate set of quantum jump operators $\{J_l\}$, so that they have a unique dark state

$$J_l|\varphi\rangle = 0 \qquad \forall l$$

Requirement

- Non-Hermitian
- Particle number conserving
- Possible to connect any state to the dark state
- Quasi-local
- Single particle operator

More examples:

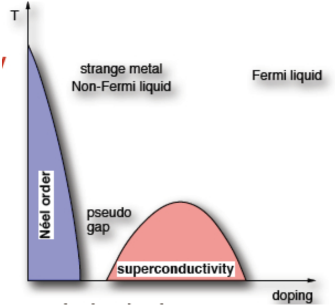
- η-condensate of fermion pairs

  S. Diehl *et al.*, Nature Physics 4, 878 (2008)
- Reservoir engineering for general quantum simulation of spin-models

  H. Weimer *et al.*, Nature Phys. 6, 382-388 (2010)
- Dissipation-induced squeezing

  G. Watanabe and H. Mäkelä, arXiv:1101.4845 (2011)
- Fermion pairing states with various symmetries, e.g. d-wave symmetry

High Tc superconductors



- Repulsive interaction
- Experimental evidence for fermion pairing with d-wave symmetry
- Simplified model: 2-d Fermi-Hubbard Model
- Difficult to solve

Quantum simulation of Fermi-Hubbard Model

- Fermi-Hubbard Hamiltonian can be implemented with an optical lattice potential
- How to cool the system to its ground state?

$$T_c/T_F \sim 0.02 \qquad T_{\rm exp}/T_F \sim 0.2 \quad \text{(ETH)}$$

A possible solution

- Prepare pure state with appropriate symmetry
- Connect to the ground state of the Fermi-Hubbard model via adiabatic passage

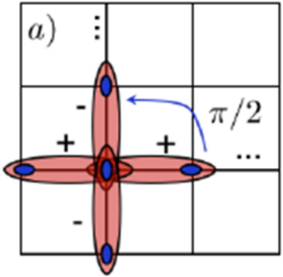
BCS-type d-wave pairing state

$$|d\rangle = \left(d^\dagger\right)^N |vac\rangle$$

$$d^\dagger = \sum_i \left[\left(a^\dagger_{i+e_x,\uparrow}a^\dagger_{i,\downarrow} - a^\dagger_{i+e_x,\downarrow}a^\dagger_{i,\uparrow}\right) - \left(a^\dagger_{i+e_y,\uparrow}a^\dagger_{i,\downarrow} - a^\dagger_{i+e_y,\downarrow}a^\dagger_{i,\uparrow}\right)\right]$$

Under translational symmetry

$$d^\dagger = \left[\left(a^\dagger_{i+e_x,\uparrow} + a^\dagger_{i-e_x,\uparrow}\right) a^\dagger_{i,\downarrow} - \left(a^\dagger_{i+e_y,\uparrow} + a^\dagger_{i-e_y,\uparrow}\right) a^\dagger_{i,\downarrow}\right]$$



Simplified version

$$d^{\dagger} = \sum_{i,\lambda} f(\lambda) S^{\pm}_{i+\lambda}$$

$$f(\lambda) = \begin{cases} 1 & \lambda = e_x \\ -1 & \lambda = e_y \end{cases}$$

With

$$S^{+}_{i+\lambda} = a^{\dagger}_{i+\lambda} \sigma^{+} a^{\dagger}_{i} = a^{\dagger}_{i+\lambda,\uparrow} a^{\dagger}_{i,\downarrow}$$

$$S^{-}_{i+\lambda} = a^{\dagger}_{i+\lambda} \sigma^{-} a^{\dagger}_{i} = a^{\dagger}_{i+\lambda,\downarrow} a^{\dagger}_{i,\uparrow}$$

Anti-ferromagnetic Néel state

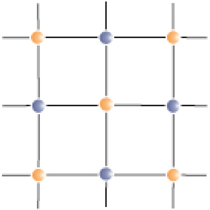
$$|AF\pm\rangle = \prod_{i\in A,\lambda} a^{\dagger}_{i+\lambda} \sigma^{\pm} a^{\dagger}_{i} |vac\rangle = \prod_{i\in A,\lambda} S^{\pm}_{i+\lambda} |vac\rangle$$

How to prepare anti-ferromagnetic Néel state?

$$|AF+\rangle = \prod_{i\in A,\lambda} a^{\dagger}_{i+\lambda,\uparrow} a^{\dagger}_{i,\downarrow}|vac\rangle$$

Consider jump operators on the two-site unit cell

$$a^{\dagger}_{j,\uparrow} a_{i,\downarrow}|\uparrow,\downarrow\rangle = 0$$
$$a^{\dagger}_{j,\downarrow} a_{i,\uparrow}|\uparrow,\downarrow\rangle = 0$$
$$a^{\dagger}_{j,\uparrow} a_{i,\downarrow}|\downarrow,\uparrow\rangle = 0$$
$$a^{\dagger}_{j,\downarrow} a_{i,\uparrow}|\downarrow,\uparrow\rangle = 0$$

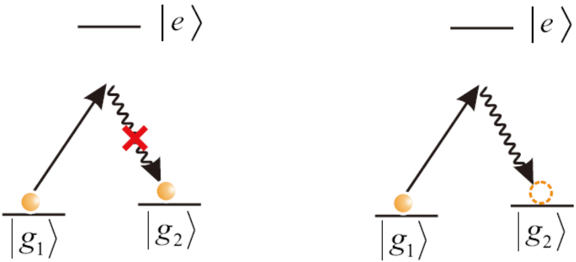


The set of jump operators $\left\{a^{\dagger}_{j,\uparrow} a_{i,\downarrow}, a^{\dagger}_{j,\downarrow} a_{i,\uparrow}\right\}$ dictates that any given site should have opposite spin with its neighbouring sites.

Additional jump operator to get rid of degeneracy of 'dark' state:

$$J_i = a_{i,\sigma}^\dagger a_{i,\sigma}$$

Physical origin: Pauli blocking



Building d-wave jump operators

- Local singlet pairing
- Spatial d-wave symmetry

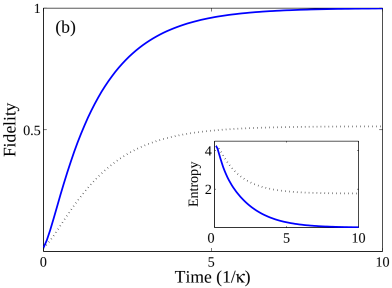
- Singlet pairing state in $1d$

$$|d\rangle_{1d} = \left[\sum_i (a^\dagger_{i+1,\uparrow} + a^\dagger_{i-1,\uparrow}) a^\dagger_{i,\downarrow}\right]^N |vac\rangle$$

- Jump operator

$$J_i^\pm = (a^\dagger_{i+1} + a^\dagger_{i-1})\sigma^\pm a_i$$
$$J_i^\pm |d\rangle_{1d} = 0$$

Uniqueness

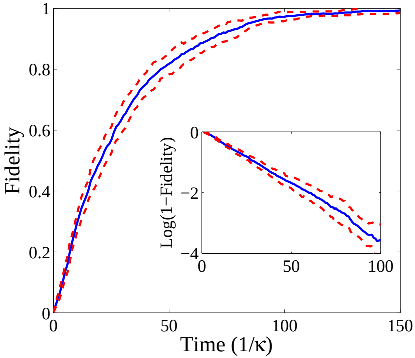


- Dark state has two-fold degeneracy with $\{J_i^{\pm}\}$
- The degeneracy can be removed by

$$J_i^z = (a_{i+1}^\dagger + a_{i-1}^\dagger)\sigma^z a_i$$

- Both dark state and steady state unique under $\{J_i^{\pm}, J_i^z\}$

What about $2d$? (2x6 ladder with 4 atoms)



With jump operators

$$J_i^{\pm} = \left(a_{i+e_x}^{\dagger} + a_{i-e_x}^{\dagger}\right)\sigma^{\pm}a_i - \left(a_{i+e_y}^{\dagger} + a_{i-e_y}^{\dagger}\right)\sigma^{\pm}a_i$$

$$J_i^{z} = \left(a_{i+e_x}^{\dagger} + a_{i-e_x}^{\dagger}\right)\sigma^{z}a_i - \left(a_{i+e_y}^{\dagger} + a_{i-e_y}^{\dagger}\right)\sigma^{z}a_i$$

States that can be constructed from Néel state unit cell operators

$$\beta_i^\dagger = \sum_\nu \rho_\nu a^\dagger_{i+e_\nu,\sigma_1} a^\dagger_{i,\sigma_2}$$

Jump operator

$$J_i = \sum_\nu \rho_\nu a^\dagger_{i+e_\nu,\sigma_1} a_{i,\sigma_2}$$

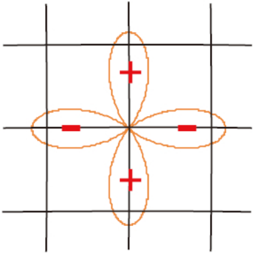
With

$$\sum_{\mu,\nu} \rho_\mu \rho_\nu a^\dagger_{j+\mathbf{e}_\nu,\sigma_1} a^\dagger_{j+\mathbf{e}_\mu,\sigma_1} = 0, \qquad \text{for } \sigma_1 \neq \sigma_2$$

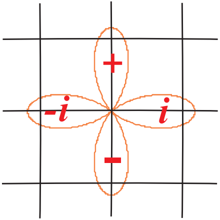
$$\sum_{\mu,\nu} \rho_\mu \rho_\nu (a^\dagger_{j+\mathbf{e}_\nu} - a^\dagger_{j-\mathbf{e}_\nu}) a^\dagger_{j+\mathbf{e}_\mu} = 0, \qquad \text{for } \sigma_1 = \sigma_2$$

Examples

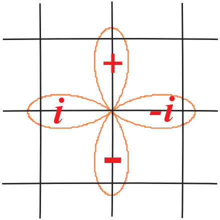
$$\rho_{\pm x} = -\rho_{\pm y} = 1$$



$$\rho_x = -\rho_{-x} = \pm i$$
$$\rho_y = -\rho_{-y} = 1$$

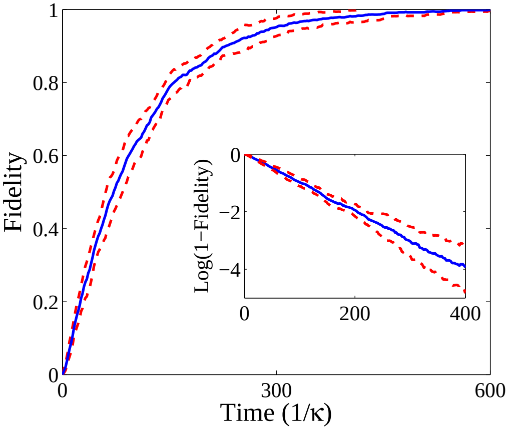


$p_x + ip_y$



$p_x - ip_y$

p-wave pairing state (4x4 plaquette with 4 atoms)



- Final steady state is approached exponentially fast, as in the d-wave case

Discussion on uniqueness condition

Consider the ‘parent’ Hamiltonian

$$H_p = \sum_{i,\sigma=\pm,z} \left(J_i^\sigma\right)^\dagger J_i^\sigma$$

- H_p semi-positive definite
- Dark state is unique if the ground state of H_p is non-degenerate
- Check if symmetries operations of H_p leaves the dark state unaltered
- For example, the reduced ‘parent’ Hamiltonian $H_p^r = \sum_{i,\sigma=\pm} \left(J_i^\sigma\right)^\dagger J_i^\sigma$ has a discrete symmetry

$$\begin{aligned} T_d: \quad & a_{i,\uparrow} \to -a_{i,\uparrow}; \quad a_{i,\downarrow} \to a_{i,\downarrow} \text{ for } i \in A, \\ & a_{i,\uparrow} \to a_{i,\uparrow}; \quad a_{i,\downarrow} \to a_{i,\downarrow} \text{ for } i \in B \end{aligned}$$

Mean field description

- The BCS pairing state in coherent state form

$$|D(\theta)\rangle = \prod_{\mathbf{q}\in BZ}\left[\frac{1}{\sqrt{1+|\varphi_{\mathbf{q}}|^2}} + \frac{e^{i\theta}\varphi_{\mathbf{q}}}{\sqrt{1+|\varphi_{\mathbf{q}}|^2}} a^{\dagger}_{\mathbf{q},\uparrow} a^{\dagger}_{-\mathbf{q},\downarrow}\right]|\mathsf{vac}\rangle,$$

- Gutzwiller ansatz in momentum space

$$\rho = \prod_{\mathbf{q}} \rho_{\mathbf{q}}, \quad \rho_{\mathbf{q}} = \mathrm{tr}_{\neq \mathbf{q}}\rho, \quad \mathrm{tr}\rho_{\mathbf{q}} = 1\,\forall\ \mathbf{q}$$

- Trace over the master equation

$$\mathrm{tr}_{\neq \mathbf{q}}\left\{\frac{\mathrm{d}\rho}{\mathrm{dt}}\right\} = \mathrm{tr}_{\neq \mathbf{q}}\left\{-\mathrm{i}\left[\mathrm{H},\rho\right] + \mathcal{L}\rho\right\}$$

- The Louisvillian part

$$\mathrm{tr}_{\neq \mathbf{q}} \mathcal{L}\rho = 2\kappa_{\mathbf{q}} \times \left\{ \gamma_{\mathbf{q},\uparrow} \rho_{\mathbf{q}} \gamma^{\dagger}_{\mathbf{q},\uparrow} + \gamma_{\mathbf{q},\downarrow} \rho_{\mathbf{q}} \gamma^{\dagger}_{\mathbf{q},\downarrow} - \frac{1}{2} \left[\gamma^{\dagger}_{\mathbf{q},\uparrow} \gamma_{\mathbf{q},\uparrow} + \gamma^{\dagger}_{\mathbf{q},\downarrow} \gamma_{\mathbf{q},\downarrow}, \rho_{\mathbf{q}} \right] \right\}$$

- The new jump operators

$$\gamma_{\mathbf{q},\uparrow} = \frac{1}{\sqrt{1 + |\varphi_{\mathbf{q}}|^2}} (c_{\mathbf{q},\uparrow} - \varphi_{\mathbf{q}} c^{\dagger}_{-\mathbf{q},\downarrow}),$$
$$\gamma_{\mathbf{q},\downarrow} = \frac{1}{\sqrt{1 + |\varphi_{\mathbf{q}}|^2}} (c_{-\mathbf{q},\downarrow} + \varphi_{\mathbf{q}} c^{\dagger}_{\mathbf{q},\uparrow})$$

- The damping spectrum is gapped

$$\kappa_{\mathbf{q}} = 2A\kappa(1 + |\varphi_{\mathbf{q}}|^2), \qquad A \equiv \frac{1}{(2\pi)^d} \int d\mathbf{q} \frac{|\varphi_{\mathbf{q}}|^2}{1 + |2\varphi_{\mathbf{q}}|^2}$$

- Approach to the steady state exponentially fast at late times

Examples

- 1d singlet pairing $\varphi_q = \cos q, A = (1 - 1/\sqrt{2})$
- 2d d-wave pairing $\varphi_{\mathbf{q}} = \cos q_x - \cos q_y, A \sim 0.36$
- Generalize to spinless fermions: p-wave pairing

$$|P\rangle = \mathcal{N} \prod_{\mathbf{q}} \left(\frac{1}{\sqrt{1 + |2\varphi_{\mathbf{q}}|^2}} + \frac{2\varphi_{\mathbf{q}}}{\sqrt{1 + |2\varphi_{\mathbf{q}}|^2}} c^\dagger_{\mathbf{q}} c^\dagger_{-\mathbf{q}} \right) |\mathrm{vac}\rangle,$$

$$\varphi_{\mathbf{q}} = 2i(\sin q_x \pm i \sin q_y), A \sim 0.12$$

- Jump operator for p-wave pairing

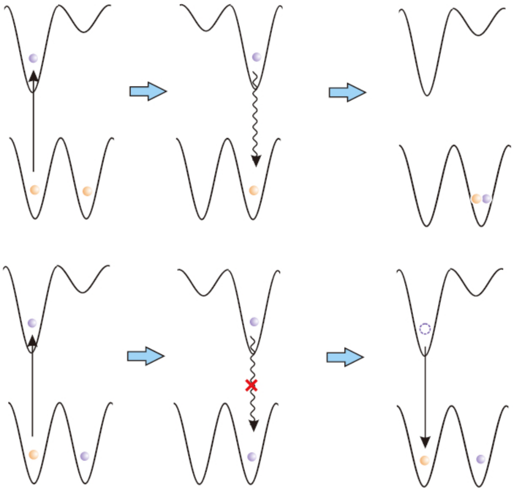
$$\gamma_{\mathbf{q}} = \frac{1}{\sqrt{1 + |2\varphi_{\mathbf{q}}|^2}} (c_{\mathbf{q}} - 2\varphi_{\mathbf{q}} c^\dagger_{-\mathbf{q}})$$

- However, this p-wave state is in the strong pairing phase and topologically trivial

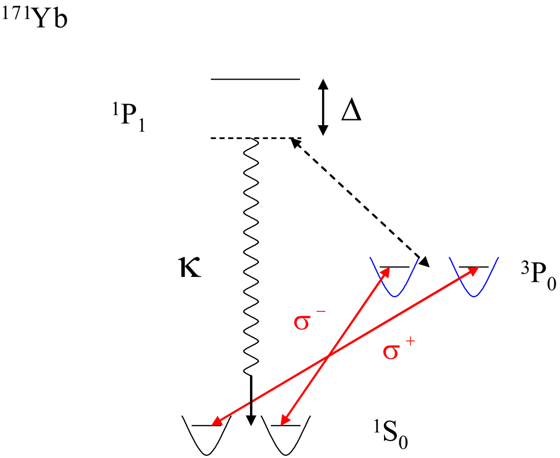
Jump operators for AF Néel state

$a^{\dagger}_{j,\downarrow} a_{i,\uparrow} |\uparrow,\uparrow\rangle \Rightarrow |0,\uparrow\downarrow\rangle$

$a^{\dagger}_{j,\downarrow} a_{i,\uparrow} |\uparrow,\downarrow\rangle \Rightarrow |\uparrow,\downarrow\rangle$



Alkaline earth like atoms

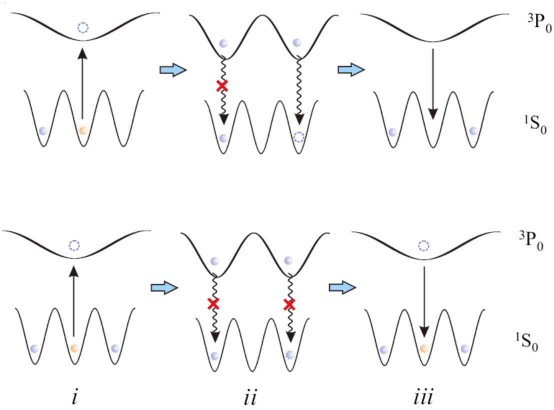


- Long lived metastable state $^3P_0$, $\Gamma \sim 2\pi \times 10\text{mHz}$
- Electronic spin and nuclear spin decoupled in the ground state

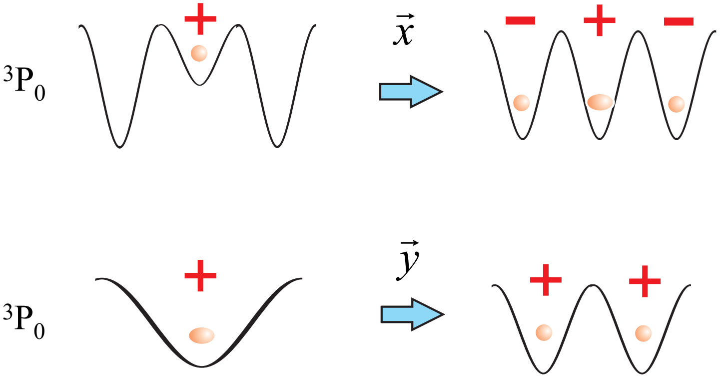
Singlet pairing in $1d$

$$(a^\dagger_{i+1,\downarrow} + a^\dagger_{i-1,\downarrow}) a_{i,\uparrow} |\downarrow, \uparrow, 0\rangle$$

$$(a^\dagger_{i+1,\downarrow} + a^\dagger_{i-1,\downarrow}) a_{i,\uparrow} |\downarrow, \uparrow, \downarrow\rangle$$



d-wave pairing



Effective cooling scheme

- Dissipatively prepare steady state with the desired symmetry properties
- Adiabatically connect to the ground state of the Hamiltonian with the same symmetry

Problem

Competition between H, e.g. FHM, and dissipative process leads to mixed state

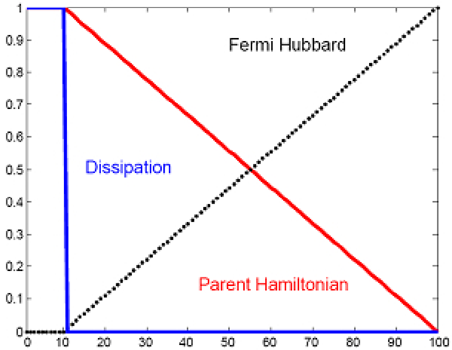
Solution

Recall the 'parent' Hamiltonian

$$H_p = \sum_{i,\sigma=\pm,z} \left(J_i^\sigma\right)^\dagger J_i^\sigma$$

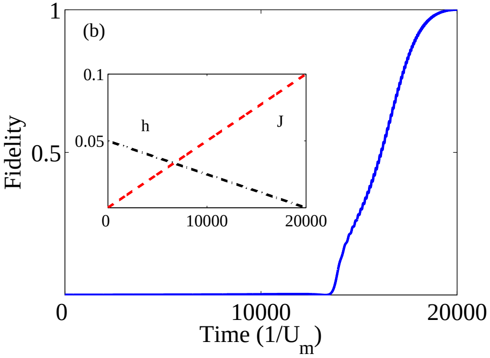
$$H_p|d\rangle = 0$$

Adiabatic passage



$$H(t) = h(t)H_p + U(t)\sum_i n_{i,\uparrow}n_{i,\downarrow} - J(t)\sum_{\langle i,j\rangle,\sigma} a^\dagger_{i,\sigma}a_{j,\sigma}$$

Adiabatic passage for small plaquette (4 atoms on 2x4 ladder)

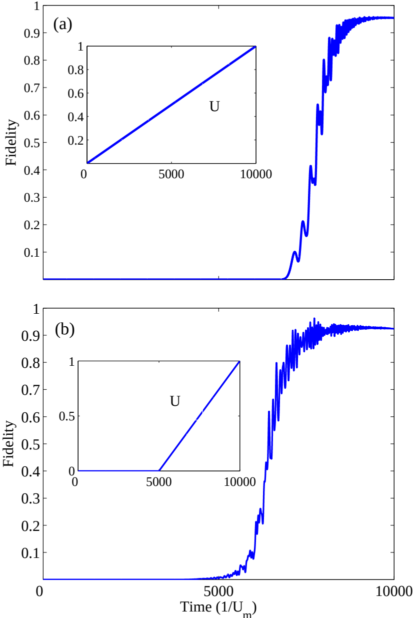


- The dissipative gap becomes an excitation gap of H_p
- With adiabatic process would be gapped so long as all the symmetry properties of the Hamiltonian are captured by the steady state

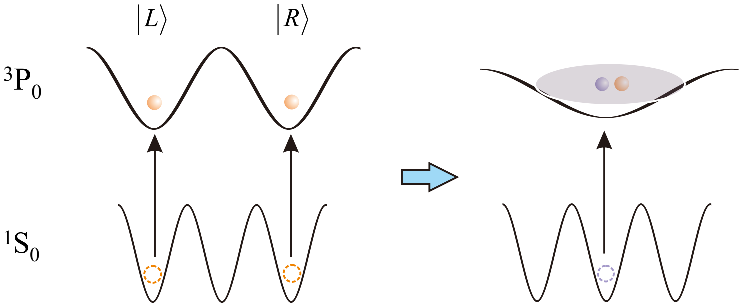
Complete vs. reduced H_p

$$H_p = \sum_{i,\sigma=\pm,z} \left(J_i^\sigma\right)^\dagger J_i^\sigma$$

$$H_p = \sum_{i,\sigma=\pm} \left(J_i^\sigma\right)^\dagger J_i^\sigma$$



Implementation of reduced H_p



$$\left(J_i^-\right)^\dagger J_i^- = n_{i,\uparrow}(a_{i+1,\downarrow}^\dagger + a_{i-1,\downarrow}^\dagger)\left(a_{i+1,\downarrow} + a_{i-1,\downarrow}\right) + 2n_{i,\uparrow}$$

- Time evolution in a digital fashion

Summary

- Designing jump operators for fermions
- Pairing states with different symmetries
- Implementation and application
- More exotic stuff:
  topologically non-trivial state?
  Majorana fermions?
  S. Diehl, E. Rico, M. A. Baranov, P. Zoller, arXiv:1105.5947 (2011)

S. Diehl, W. Yi, A. J. Daley, P. Zoller, Phys. Rev. Lett. 105, 227001 (2010)
W. Yi, S. Diehl, A. J. Daley, P. Zoller (in preparation)