To search quantum number fractionalization near quantum criticality above one dimension : Holographic approach

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APCTP

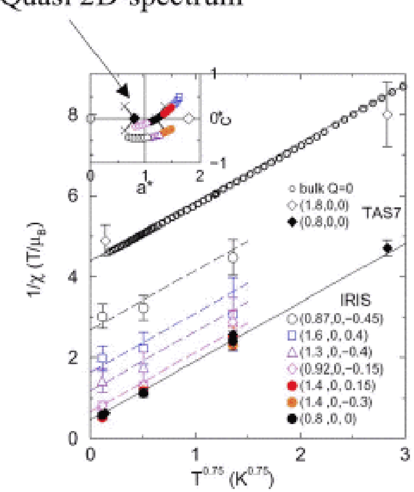
arXiv:1106.0398

MOTIVATION

Quantum criticality : Universal scaling and T-linear transport

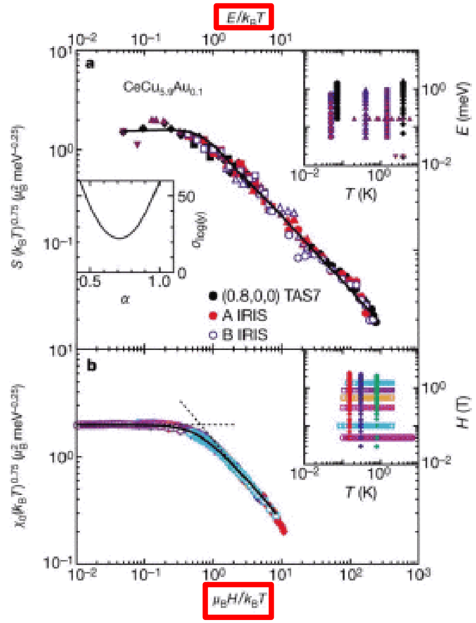
Anomalous exponent + ``local'' scaling
B/T scaling at the QCP: for any point
in the Brillouin Zone

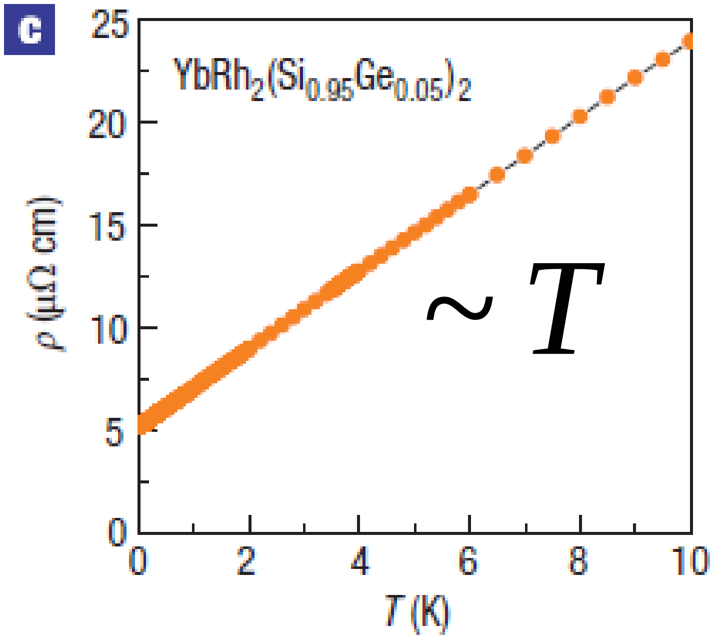
Quasi 2D-spectrum

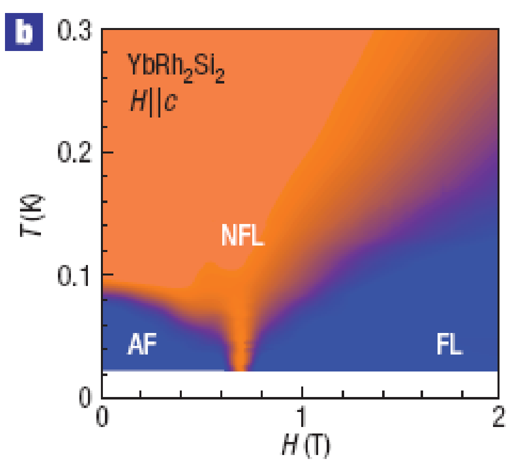


$CeCu_{6-x}Au_x$ (x=0.1)

Schroeder et al, Nature 407,351 (2000).







$$\mathrm{Im}\,\chi(\omega,T)=CT^{-\Delta_\chi}\,f\left(\frac{\omega}{T}\right)$$

Two scenarios for $Z \rightarrow 0$

- Complex textures of critical fluctuations (Weak coupling) ➔ Quantum criticality in most bosonic (scalar and vector) models and Hertz-Moriya-Millis framework in correlated fermions

- Fractionalization (Strong coupling) ➔ Gauge theory above one dimension

Mechanism of quantum number fractionalization

- Gapped

✓ Ground state degeneracy (symmetry breaking or topological order) ➔ Polyacetylene, Edge states of AKLT, Z2 spin liquid, FQHE (FQSHE), …

- Gapless

✓ Large-N Dirac or Fermi surface ➔ U(1) spin liquid (Slave boson & fermion theories)

✓ Topological term : anomaly of the Dirac theory ➔ Spin $\frac{1}{2}$ chain, FQHE (FQSHE), deconfined quantum criticality, …

SU(2) chiral anomaly and SU(2) k=1 WZW theory

$$H=\sum_{i\sigma}(t-(-1)^i\delta t)(c_{i\sigma}{}^{\dagger}c_{i+1\sigma}+h.c.)+U\sum_i n_{i\uparrow}n_{i\downarrow}$$

$\delta t>0\rightarrow$ LSLSL...
$\delta t<0\rightarrow$ SLSLS...

$$L=\overline{\psi}(i\partial\!\!\!/+mQe^{i\alpha Q\gamma^5})\psi \qquad Q\equiv\vec{n}\cdot\vec{\sigma}$$

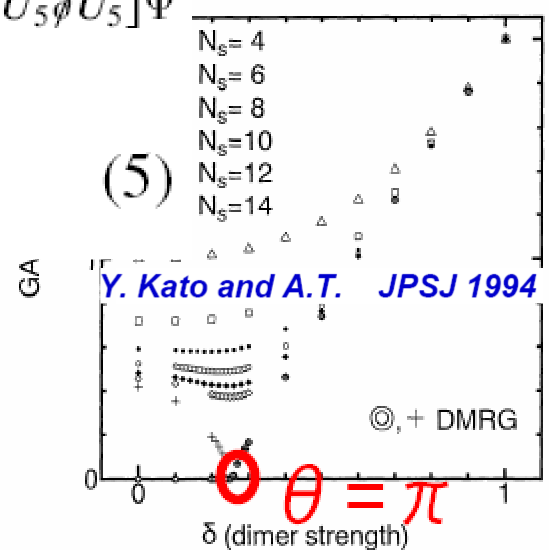
γ_5 : AF $\leftrightarrow$ VBS

$$\mathbf{j}^5_\mu\equiv\bar{\Psi}\gamma_\mu\gamma^5\frac{\vec{\sigma}}{2}\Psi$$

$$\partial_\mu \mathrm{j}^5_{\mathrm{sp},\mu}\neq 0$$

$$Z_{\rm spin}=\int\mathcal{D}\vec{n}\,\mathcal{D}\phi_+\int\mathcal{D}\bar{\Psi}\,\mathcal{D}\Psi e^{-\int d\tau\,dx\bar{\Psi}[1\otimes\partial\!\!\!/+U_5\partial\!\!\!/U_5]\Psi}$$
$$=\int\mathcal{D}\vec{n}\,\mathcal{D}\phi_+e^{-S_{\rm WZW}[g]}|_{g=e^{-i\phi+Q}}=Z_{\rm WZW}, \qquad (5)$$

$$U_5\equiv e^{-(i/2)\phi_+Q\gamma^5}$$

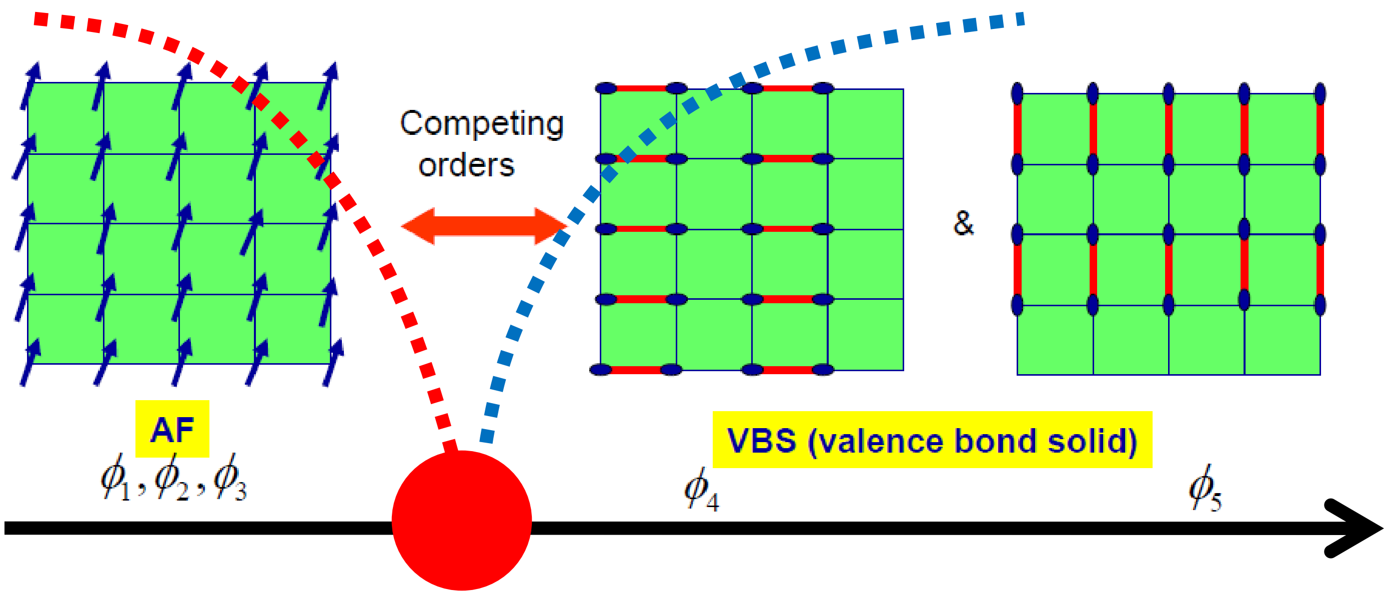


A. Tanaka & X. Hu, Phys. Rev. Lett. 88, 127004 (2002)

2D quantum antiferromagnet : O(5) WZW theory

$$\mathcal{H}_{SU(2)} = J \sum_{\langle ij \rangle} \mathbf{S}_i \cdot \mathbf{S}_j - Q \sum_{\langle ijkl \rangle} \left(\mathbf{S}_i \cdot \mathbf{S}_j - \tfrac{1}{4}\right)\left(\mathbf{S}_k \cdot \mathbf{S}_l - \tfrac{1}{4}\right)$$

Deconfined QCP
: T. Senthil, A. Vishwanath, L. Balents, S. Sachdev, and M. P. A. Fisher, Science 5, 303 (2004)



O(5) WZW theory
: A. Tanaka & X. Hu, Phys. Rev. Lett. 95, 036402 (2005)

$$S_{eff}[\phi_\alpha] = \frac{1}{g} \int d\tau d^2x \left(\partial_\mu \phi_\alpha\right)^2 + i\Gamma[\phi_\alpha]$$

$$O(3) \times Z_4 \subset O(5)$$

$$\Gamma[\phi_\alpha] = \frac{3\varepsilon^{abcde}}{4\pi} \int_0^1 ds \int d\tau d^2x \, \phi_a \partial_s \phi_b \partial_\tau \phi_c \partial_x \phi_d \partial_y \phi_e$$

Quantum MonteCarlo Simulation
: A.W. Sandvik, Phys. Rev. Lett. 98, 2272020 (2007)

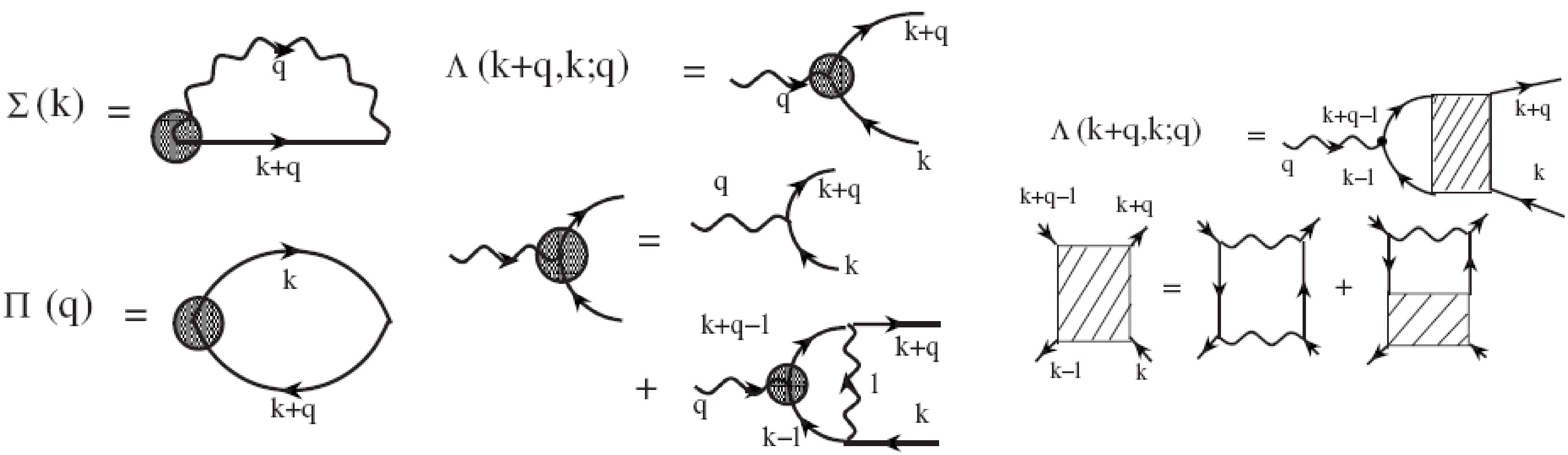
Motivation for the AdS/CFT framework : Interplay between correlations and topological terms

- To perform an exact summation for infinite number of quantum processes (all planar diagrams) in the large-N limit

- To introduce an effect of the topological (WZW) term via the classical equation of motion with the one-dimensional higher topological (Chern-Simons) term

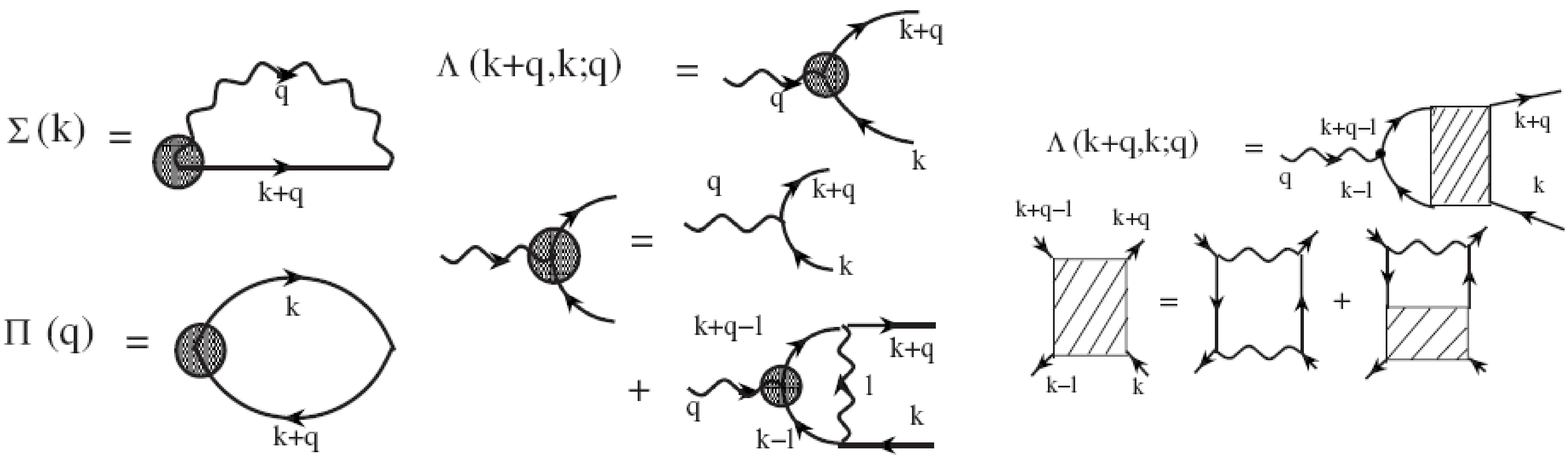
Revisit to the large N limit I

- Before 2009 : Eliashberg approximation (Only self-energy corrections) in the Fermi surface problem
- After Sung-Sik Lee, Phys. Rev. B **80**, 165102 (2009) : The Fermi surface problem ~ Matrix model (Non-abelian gauge theory) ➔ Vertex corrections !
- Boson self-energy correction (thermodynamics)
- Fermion self-energy correction (transport): Perturbative analysis based on the Eliashberg theory ➔ Maki-Thompson corrections are not important, but Aslamosov-Larkin corrections generate anomalous exponents beyond the Eliashberg theory. [Max A. Metlitski and S. Sachdev, Phys. Rev. B (2010)]



Revisit to the large N limit II

- "Vertex correction and Ward identity in U(1) gauge theory with a Fermi surface," Ki-Seok Kim, Phys. Rev. B **82**, 075129 (2010)
- "Critical particle-hole composites at twice the Fermi wave vector in U(1) spin liquid with a Fermi surface," Ki-Seok Kim, Phys. Rev. B **83**, 035123 (2011)



Mechanism of quantum number fractionalization in strongly interacting conformal field theories above one dimension : Emergent deconfined local quantum criticality in the Holographic approach

- "Derivation" of dynamical mean-field theory at d « infinity from strongly interacting field theory in the geometrical (duality) point of view ➔ Emergent local quantum criticality : E/T Scaling

- Mechanism of quantum number fractionalization ➔ Interplay between correlations and topological term : Anomalous critical exponents

INTRODUCTION OF ADS/CFT

Holography beyond the bulk-edge correspondence in QHE

Superstring Theory

Holography (AdS/CFT)

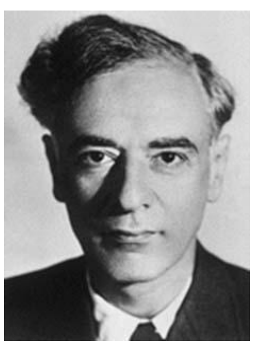
General Relativity (Supergravity)

Gauge Theory (QCD-like theory)

Duality mapping

Classical, weakly-coupled

Quantum, strongly-coupled



Condensed-matter Physics

Finite temperature,
Finite density

O. Aharony, S.S. Gubser, J. Maldacena, H. Ooguri, Y. Oz, Phys. Rept. **323**, 183 (2000).

S. Nakamura's talk in APCTP

AdS/CFT correspondence

A typical correspondence

N=4 large-Nc SU(Nc) super Yang-Mills (SYM) theory
at large ’t Hooft coupling in 3+1 dim at the quantum level

Equivalent

Type IIB supergravity on $AdS_5 \times S^5$ at the classical level

$$\text{4D generating functional}: Z_4[\phi_0(x)] = \int \mathcal{D}[\Phi] \exp\{iS_4 + i\int \phi_0(x)\mathcal{O}\},$$

$$\text{5D (classical) effective action}: \Gamma_5[\phi(x,z) = \phi_0(x)]\,;\, \phi_0(x) = \phi(x, z=0).$$

$$\text{AdS/CFT correspondence}: Z_4 = \Gamma_5.$$

Physics of the additional length scale

- An energy scale for renormalization group transformation ??
- Derivation from conformal field theories to gravity theories ??

$$S_{4D} = \int d^4x \left(i\bar{\psi}\gamma^\mu \partial_\mu \psi + \bar{\psi}\chi_\infty + \bar{\chi}_\infty \psi + \bar{\chi}_\infty G \chi_\infty \right)$$

$$S_{5D} = \frac{1}{16\pi G_5} \int dr \int d^4x \sqrt{-g} \left\{ R + \Lambda + i\bar{\chi}_r (\gamma^\mu D_\mu - m_\chi)\chi_r + \text{int.} \right\}$$

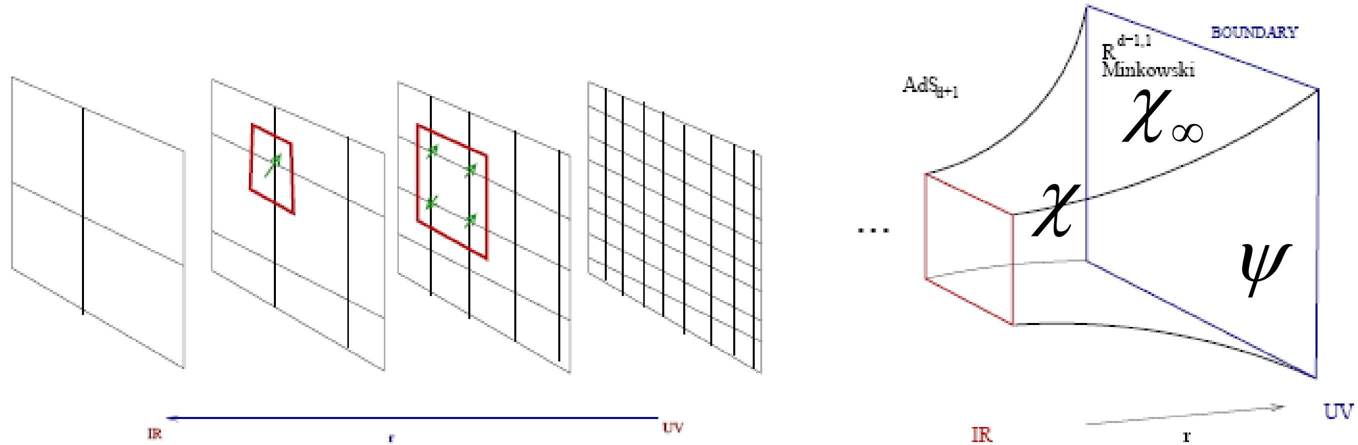


Figure 1. The left figure indicates a series of block spin transformations labelled by a parameter r. The right figure is a cartoon of AdS space, which organizes the field theory information in the same way. In this sense, the bulk picture is a hologram: excitations with different wavelengths get put in different places in the bulk image.

How to use the AdS/CFT machinery ?

- Q. How to find thermodynamics and correlation functions ?
- A. Solve classical gravity equations with boundary conditions.

$$S = -\int d^{d+1}x\sqrt{-g}\,\left[(D_M\phi)^* D^M\phi + m^2\phi^*\phi\right]$$

$$\Delta = \frac{d}{2} + \sqrt{m^2R^2 + \frac{d^2}{4}}$$

$$D_M\phi = (\partial_M - iqA_M)\phi$$

$$\phi(r, x^\mu) = \int \frac{d^dk}{(2\pi)^d}\,\phi(r, k_\mu)\,e^{ik_\mu x^\mu}, \qquad k_\mu = (-\omega, \vec{k})$$

$$-\frac{1}{\sqrt{-g}}\partial_r(\sqrt{-g}g^{rr}\partial_r\phi) + \left(g^{ii}(k^2 - u^2) + m^2\right)\phi = 0$$

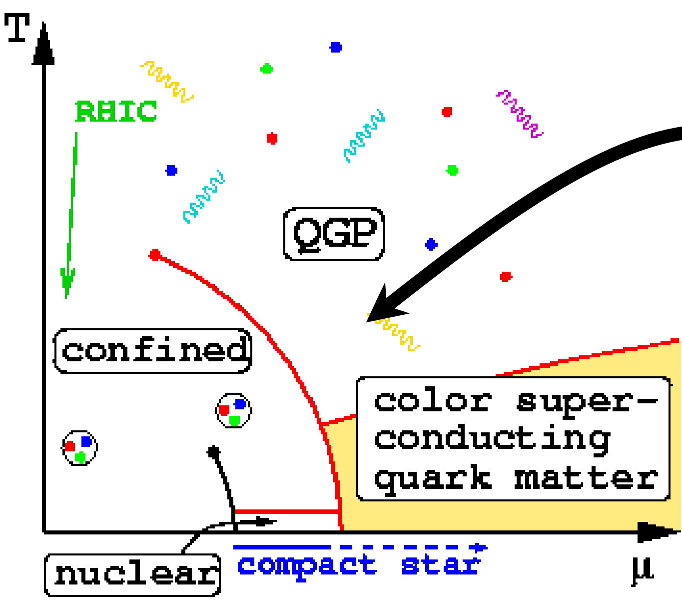
$$u(r) \equiv \sqrt{\frac{g_{ii}}{-g_{tt}}}\left(\omega + \mu_q\left(1 - \frac{r_0^{d-2}}{r^{d-2}}\right)\right)$$

$$\phi(r, k_\mu) \overset{r\to\infty}{\approx} A(k_\mu)\,r^{\Delta-d} + B(k_\mu)\,r^{-\Delta}$$

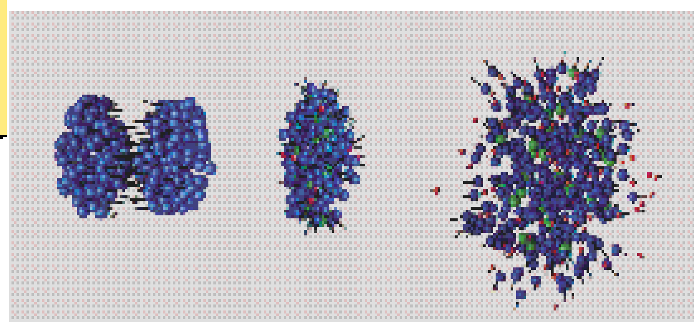
$$G_R(k_\mu) = K\frac{B(k_\mu)}{A(k_\mu)}$$

Success of the AdS/CFT correspondence

$$\eta/s = \frac{1}{4\pi} << 1 \rightarrow \text{Almost perfect fluid}$$



Strongly coupled QGP



- Look at scattering events with nonzero impact parameter (select by number of final particles)
- Distribution of particles over momentum is not axially symmetric: characterized by "elliptic flow" parameter v_2
- Naturally explained if the matter acts like a liquid: more pressure along the smaller axis of the hot region.

- Too large viscosity kills v_2
- Hydrodynamic simulations can give estimates for v_2

AdS/CMP correspondence (Hairy black hole) : Temperature and density

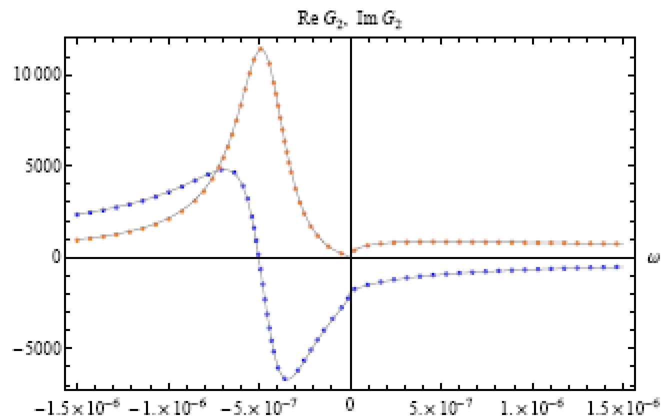
- Description of non-Fermi liquid at quantum criticality ➔ Absence of quasiparticles and T-linear resistivity
- Description of superconductivity (Holographic superconductor)

$$S_{\rm probe}[\psi] = \int d^{D+1}x\sqrt{g}\left(i\bar{\psi}\left(\not{D}-m\right)\psi + \text{interactions}\right) \qquad D_M \equiv \partial_M + \tfrac{1}{4}\omega_{M\,AB}\Gamma^{AB} - iq_\psi A_M$$

$$G_R(\omega,k) = \frac{h_1}{k_\perp + \frac{1}{v_F}\omega + c_k\omega^{2\nu_{k_F}}} \qquad \omega_\star(k) \sim k_\perp^z, \qquad z = \frac{1}{2\nu_{k_F}} > 1 \qquad \nu_{k_F} \equiv \sqrt{m^2 + k_F^2 - q^2/2}/\sqrt{6}.$$

$$\frac{\Gamma(k)}{\omega_\star(k)} \overset{k_\perp\to 0}{\to} \text{const}, \qquad Z \propto k_\perp^{\frac{1-2\nu_{k_F}}{2\nu_{k_F}}} \overset{k_\perp\to 0}{\to} 0.$$

$$G_R \approx \frac{h_1}{k_\perp + \tilde{c}_1\omega\ln\omega + c_1\omega}, \qquad \tilde{c}_1 \in \mathbb{R}, \quad c_1 \in \mathbb{C} \qquad Z \sim \frac{1}{|\ln\omega_\star|} \overset{k_\perp\to 0}{\to} 0.$$



EDC: $G_2(\omega, k = 0.9)$

Sung-Sik Lee, Phys. Rev. D **79**, 086006 (2009)

M. Cubrovic, J. Zaanen, and K. Schalm, Science **24**, 439 (2009)

T. Faulkner, N. Iqbal, H. Liu, J. McGreevy, and D. Vegh Science **27**, 1043 (2010).

S. S. Gubser, Phys. Rev. D **78**, 065034 (2008).

S. A. Hartnoll, C. P. Herzog, G. T. Horowitz, Phys. Rev. Lett. **101**, 031601(2008).

Fluctuation-driven first-order transition near quantum criticality in the holographic superconductor

$$S = \frac{1}{2\kappa_p^2}\int d^4x\sqrt{-g}\left[R + \frac{L^2}{6} - \frac{L^2}{4}F^2 - \frac{1}{2}(\mathcal{D}\eta)^2 - \frac{1}{2}\eta^2(eA_\mu - \mathcal{D}_\mu\phi)^2 - \frac{m^2}{2}\eta^2\right]$$

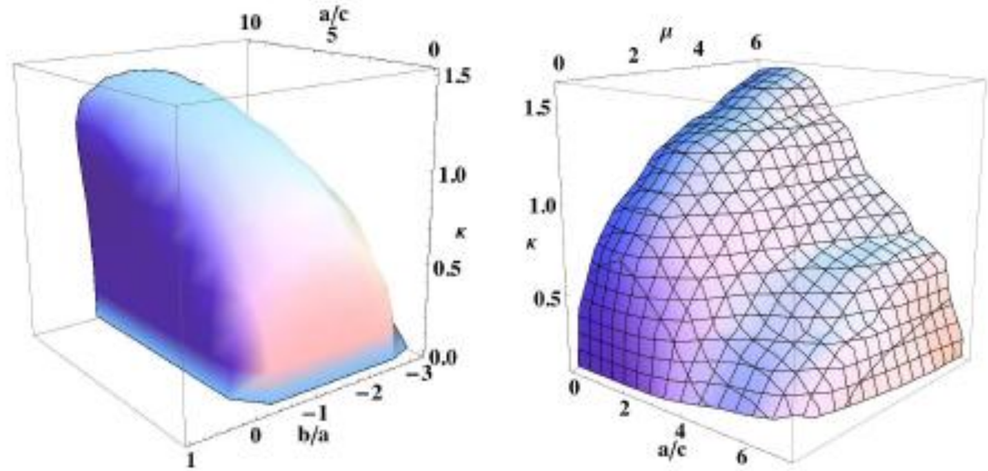


FIG. 3: Tricritical surfaces in $(b/a, a/c, \kappa)$ with a fixed μ for the left panel and $(\mu, a/c, \kappa)$ with a fixed b/a for the right panel, respectively, where vortices do not interact with each other. The interaction potential is attractive inside the ellipse while repulsive outside it.

arXiv:1104.3491v1 [hep-th]

Ki-Seok Kim[1,2], Kyung Kiu Kim[3], Youngman Kim[1,2] and Yumi Ko[1]

In this talk,

We investigate dynamics of linear fluctuations in the charged black hole of the 5-dim. Einstein (Λ<0)+ Maxwell+ Chern-Simons theory. We show that the critical exponent of the critical correlation function is enhanced due to the Chern-Simons term.

$$16\pi G_5 L = \sqrt{-g}\left(R + 12 - \frac{1}{4}F_{MN}F^{MN}\right) + \frac{\alpha}{3!}\varepsilon^{IJKLM}A_I F_{JK} F_{LM}$$

In the field-theory side:

We try to give an impression how to relate the gravity theory with the corresponding field theory, considering deconfined local quantum criticality in the problem of magnetic impurities.

Chern-Simons term

For our case, the Chern-Simons term is a supergravity realization of the R-symmetry anomaly of N=4 SYM.

Witten, Adv. Theor. Math. Phys. 2 (1998) 253.

The electric charge

The electric charge in the gravity side corresponds to the R-charge (the U(1) part) of N=4 SYM.

S. Nakamura’s talk in APCTP

Chern-Simons term

Anomaly in the SU(4) R-charge current

$$(\mathcal{D}^\mu J_\mu)^a = \frac{N^2-1}{384\pi^2} i d^{abc} \epsilon^{\mu\nu\rho\sigma} F^b_{\mu\nu} F^c_{\rho\sigma}$$

$$A^a_\mu J^a_\mu \qquad A^a_\mu \to A^a_\mu + (\mathcal{D}_\mu \Lambda)^a$$

$$\int d^4x (\mathcal{D}^\mu \Lambda)_a J^a_\mu = - \int d^4x \Lambda_a (\mathcal{D}^\mu J^a_\mu)$$

$$\frac{iN^2}{96\pi^2} \int_{AdS_5} d^5x (d^{abc} \epsilon^{\mu\nu\lambda\rho\sigma} A^a_\mu \partial_\nu A^b_\lambda \partial_\rho A^c_\sigma + \cdots)$$

$$-\frac{iN^2}{384\pi^2} \int_{\partial AdS_5} d^4x \epsilon^{\mu\nu\rho\sigma} d^{abc} \Lambda^a F^b_{\mu\nu} F^c_{\rho\sigma}$$

O. Aharony, S.S. Gubser, J. Maldacena, H. Ooguri, Y. Oz, Phys.Rept. 323, 183-386 (2000)

DECONFINED LOCAL QUANTUM CRITICALITY IN ADS/CFT

Einstein-Maxwell-Chern-Simons theory

$$S = \frac{1}{16\pi G_5}\int d^5x\sqrt{-g}\left(R + \frac{12}{l^2}\right) + S_M + S_{CS},$$
$$S_M = -\frac{1}{4e^2}\int d^5x\sqrt{-g}F_{mn}F^{mn},$$
$$S_{CS} = \frac{\kappa}{3}\int d^5x\epsilon^{lmnpq}A_l F_{mn}F_{pq}, \qquad (1)$$

$$R_{mn} - \frac{1}{2}g_{mn}R - \frac{6}{l^2}g_{mn} = 8\pi G_5 T_{mn},$$
$$-\frac{1}{e^2}\nabla_n F^{mn} + \frac{\kappa}{\sqrt{-g}}\epsilon^{mlnpq}F_{ln}F_{pq} = 0, \qquad (2)$$

$$T_{mn} = \frac{1}{e^2}\left(F_{mk}F_{nl}g^{kl} - \frac{1}{4}g_{mn}F_{kl}F^{kl}\right)$$

Reissner-Nordström (RN) AdS5 black hole

$$ds^2 = \frac{r^2}{l^2}\left(-f(r)dt^2 + dx^2 + dy^2 + dz^2\right) + \frac{l^2}{r^2 f(r)}dr^2$$

$$f(r) = 1 - \frac{M}{r^4} + \frac{Q^2}{r^6} \qquad A_t = -\frac{Q}{r^2} + \mu \qquad \mu = \frac{\sqrt{3}eQ}{4\sqrt{\pi G_5}\, l r_0^2}$$

$$T = \frac{1}{2\pi b}\left(1 - \frac{a}{2}\right)$$

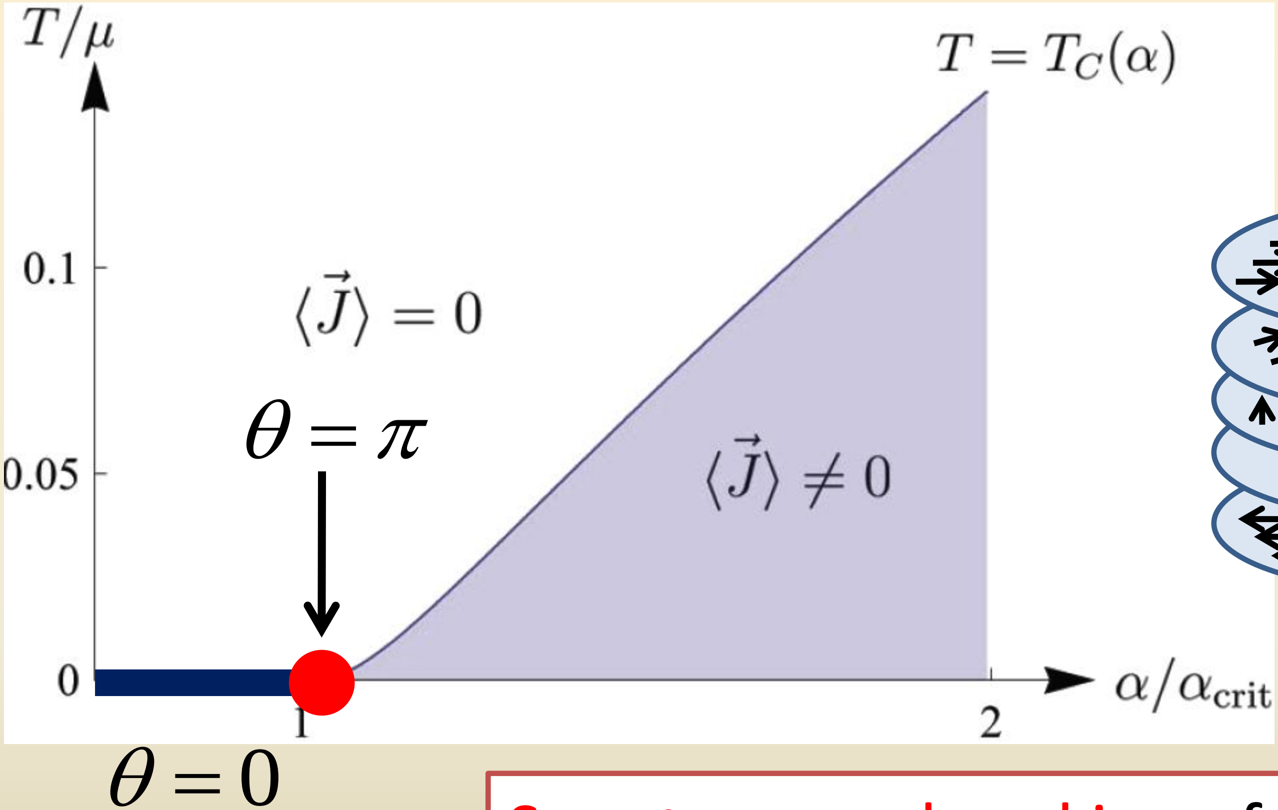
$$a = 2 - \frac{4}{1+\sqrt{1+4(\tilde{\mu}/T)^2}} \text{ and } b = \frac{1}{\pi T[1+\sqrt{1+4(\tilde{\mu}/T)^2}]}$$

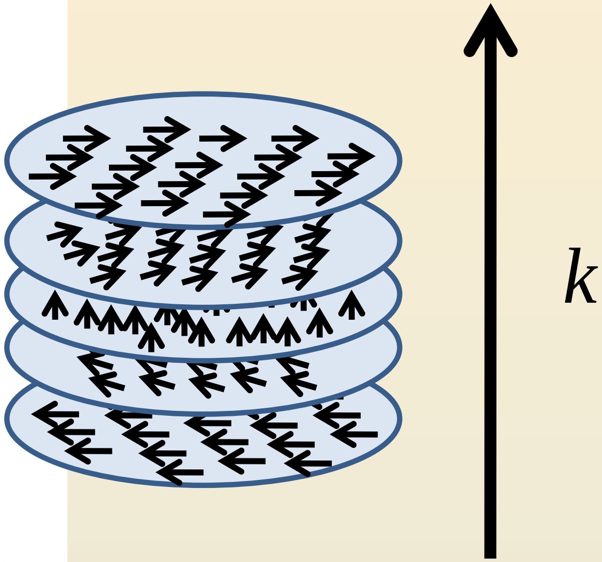
$$\tilde{\mu} = \mu\sqrt{\frac{16\pi G_5}{3(\pi e l)^2}}$$

Equations of linear fluctuations

$$g_{mn} = g^{(0)}_{mn} + h_{mn}, \qquad A_m = A^{(0)}_m + a_m$$

"Phase diagram"





Spontaneous breaking of translational and rotational symmetry.

S. Nakamura, H. Ooguri, and C.-S. Park, Phys. Rev. D **81**, 044018 (2010).

Extremal black hole : Emergence of locality

$T(r_*) = 0,$ $f(u) = (u-1)^2(2u+1)$

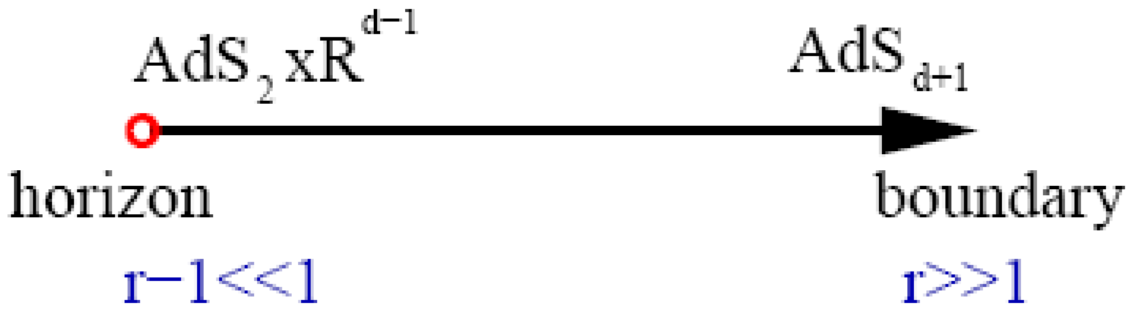


Figure 4. The geometry of the extremal AdS_{d+1} charged black hole.

Inner: $r - r_* = \omega \frac{R_2^2}{\zeta}$ for $\epsilon < \zeta < \infty$

Outer: $\frac{\omega R_2^2}{\epsilon} < r - r_*$

$\omega \to 0, \quad \zeta = \text{finite}, \quad \epsilon \to 0, \quad \frac{\omega R_2^2}{\epsilon} \to 0$

inner : $\phi_I(\zeta) = \phi_I^{(0)}(\zeta) + \omega \phi_I^{(1)}(\zeta) + \cdots$

outer : $\phi_O(r) = \phi_O^{(0)}(r) + \omega \phi_O^{(1)}(r) + \cdots$

matching ϕ_I and ϕ_O

$\zeta \to 0$ $\frac{\omega R_2^2}{\zeta} \to 0.$

Critical exponents

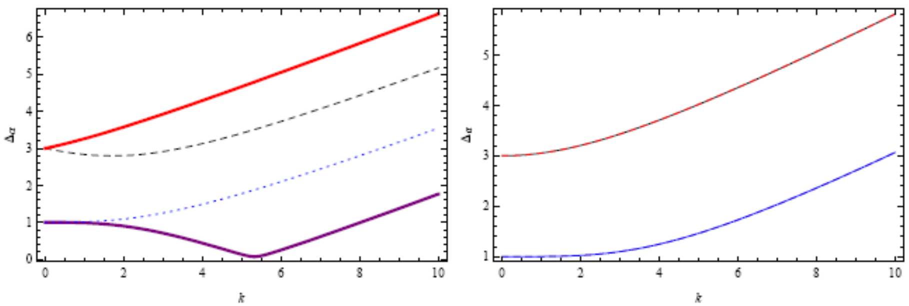


FIG. 1: Left : Critical exponents in the presence of the CS term, showing $\Delta^2_{\tilde{\kappa}}(k) < \Delta^4_{\tilde{\kappa}}(k) < \Delta^1_{\tilde{\kappa}}(k) < \Delta^3_{\tilde{\kappa}}(k)$. In particular, we see $\Delta^2_{\tilde{\kappa}}(k_c) = 0$, associated with the instability to the non-uniform current ordering. Right : Critical exponents in the absence of the CS term, showing $\Delta^2_0(k) = \Delta^4_0(k) < \Delta^1_0(k) = \Delta^3_0(k)$.

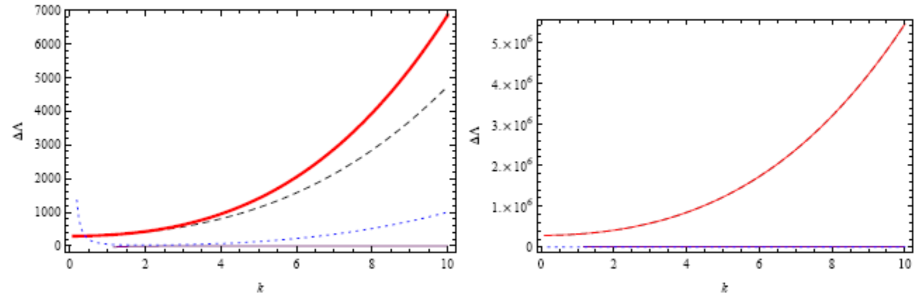


FIG. 3: $i[\Lambda_{11}(k) - \Lambda_{21}(k)] = i[\Lambda_{13}(k) - \Lambda_{23}(k)] \gg i[\Lambda_{12}(k) - \Lambda_{22}(k)] = i[\Lambda_{14}(k) - \Lambda_{24}(k)]$ in the absence of the CS term (Right) turn into $i[\Lambda_{13}(k) - \Lambda_{23}(k)] > i[\Lambda_{11}(k) - \Lambda_{21}(k)] \gg i[\Lambda_{14}(k) - \Lambda_{24}(k)] > i[\Lambda_{12}(k) - \Lambda_{22}(k)]$ in the presence of the CS term (Left).

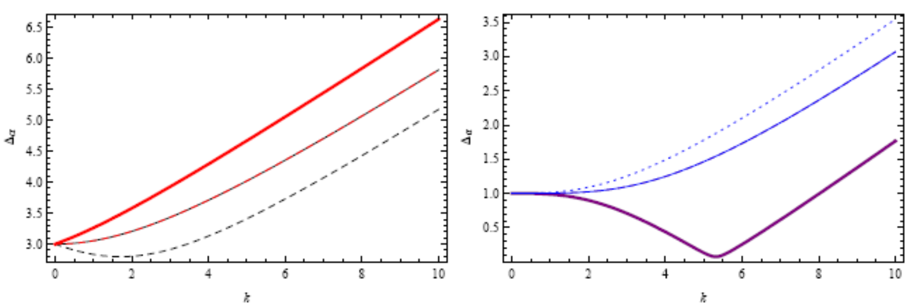
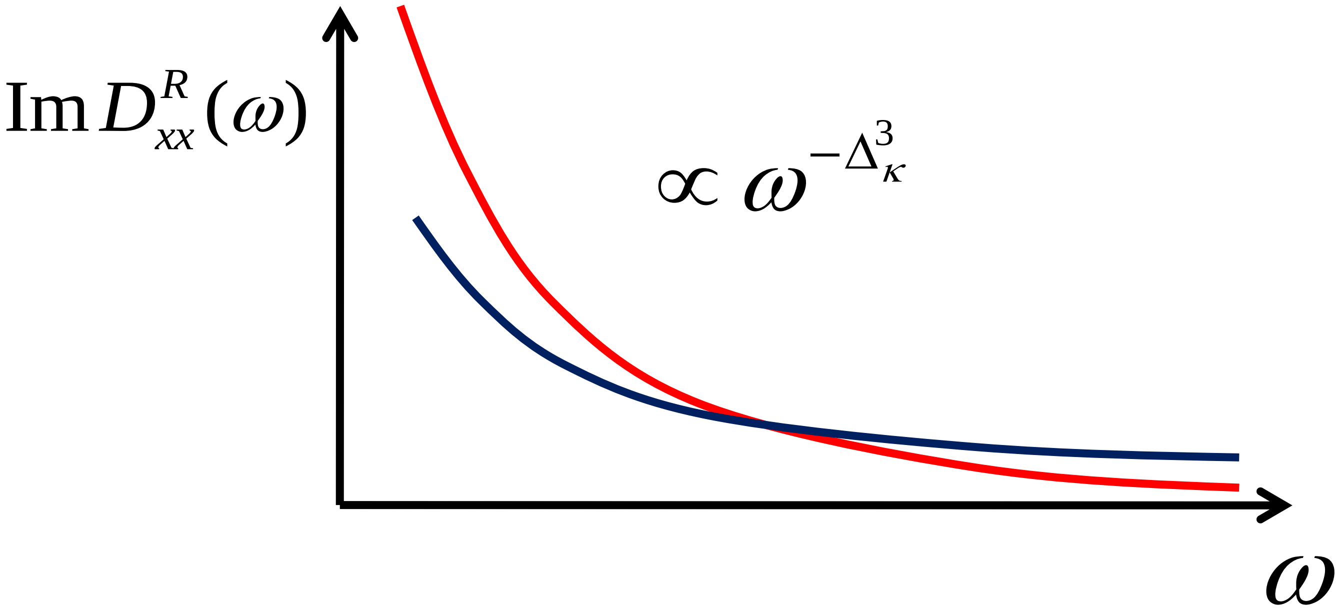


FIG. 2: Left : $\Delta^1_0(k) = \Delta^3_0(k)$ are split to $\Delta^1_{\tilde{\kappa}}(k) < \Delta^1_0(k) = \Delta^3_0(k) < \Delta^3_{\tilde{\kappa}}(k)$. Right : $\Delta^2_0(k) = \Delta^4_0(k)$ are separated into $\Delta^2_{\tilde{\kappa}}(k) < \Delta^2_0(k) = \Delta^4_0(k) < \Delta^4_{\tilde{\kappa}}(k)$.

An evidence of deconfined quantum criticality

$$\mathcal{D}^R_{xx}(\omega) = [d^{-1}_{xx}(\omega) - G^R_{xx}(\omega)]^{-1}$$

$$\Im \mathcal{D}^R_{xx}(\omega) \approx [\Im G^R_{xx}(\omega)]^{-1} \propto \omega^{-\Delta^3_{\tilde{\kappa}}(k_c)}$$



Discussion : Local conformal field theory ?

$$H = H_0 + H_1[\mathfrak{d}, c] + H_{\rm AdS}$$

$$H_1[\mathfrak{d}, c] = \sum_\alpha \int \frac{d^2k}{4\pi^2}[V_{\mathbf{k}}\mathfrak{d}^\dagger_{\mathbf{k}\alpha}c_{\mathbf{k}\alpha} + V^*_{\mathbf{k}}c^\dagger_{\mathbf{k}\alpha}\mathfrak{d}_{\mathbf{k}\alpha}]$$

$$H_0 = \sum_\alpha \int \frac{d^2k}{4\pi^2}(\varepsilon_{\mathbf{k}} - \mu)c^\dagger_{\mathbf{k}\alpha}c_{\mathbf{k}\alpha}$$

$$\langle \mathfrak{d}_{\mathbf{k}\alpha}(\tau)\mathfrak{d}^\dagger_{\mathbf{k}\beta}(0)\rangle_{H_{\rm AdS}} \sim \left[\frac{\pi T}{\sin(\pi T\tau)}\right]^{2\Delta_k}$$

$$H = -\sum_{ij,\sigma} t_{ij}c^\dagger_{i\sigma}c_{j\sigma} + E_d\sum_{i\sigma}d^\dagger_{i\sigma}d_{i\sigma} + \sum_{ii} J_{ij}\mathbf{S}_i\cdot\mathbf{S}_j + \sum_{i\sigma}(V_i c^\dagger_{i\sigma}d_{i\sigma} + \text{H.c.})$$

$$\chi(\tau) = A_f^2\beta^{-2\Delta_f}\left(\frac{\pi}{\sin(\pi\tau/\beta)}\right)^{2\Delta_f}$$

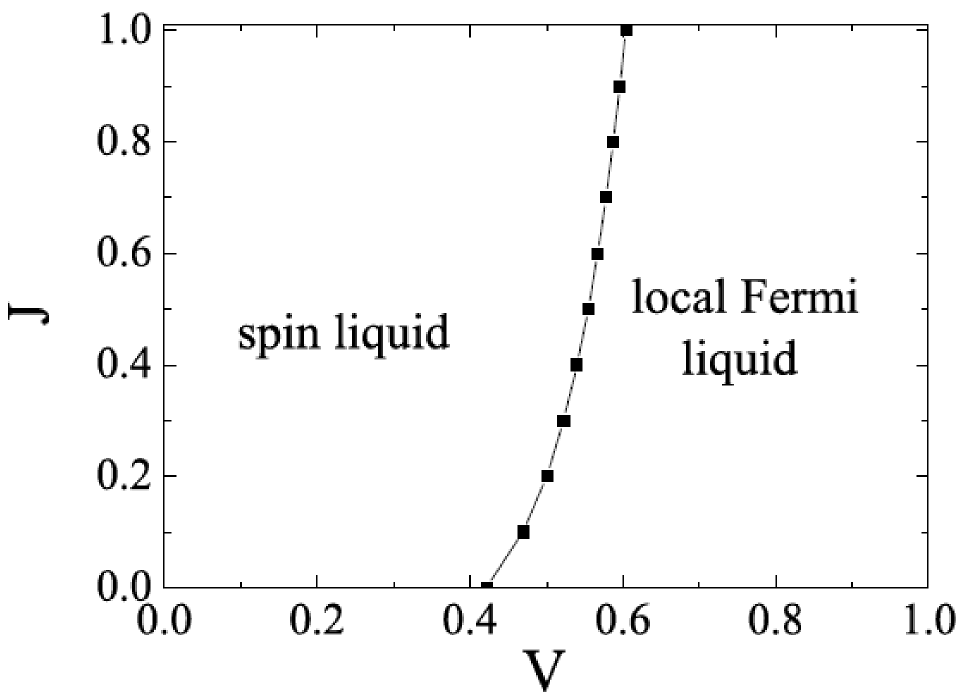
$$G_c(\tau) = A_c\beta^{-\Delta_c}g_c\left(\frac{\tau}{\beta}\right)$$

$$\mathfrak{d}_{i\alpha} = \frac{\sigma^a_{\alpha\beta}}{2}\int\frac{d^2k}{4\pi^2}\left[\frac{UV_{\mathbf{k}}e^{-i\mathbf{k}\cdot\mathbf{r}_i}}{U^2/4 - (\varepsilon_d - \varepsilon_{\mathbf{k}})^2}\right]c_{\mathbf{k}\beta}\hat{S}^a_i$$

$$g_\alpha(x) = \left(\frac{\pi}{\sin(\pi x)}\right)^{\Delta_\alpha}$$

Subir Sachdev PRL **105,** 151602 (2010)

Deconfined local quantum criticality



$$\Delta_f = \Delta_b \Rightarrow \dim S_f = \dim \rho_b$$

$$Z_{eff} = \int D\Psi^a(\tau)\delta\Big(|\Psi^a(\tau)|^2 - 1\Big)e^{-S_{eff}},$$

$$S_{eff} = -\frac{g^2}{2M}\int_0^\beta d\tau \int_0^\beta d\tau' \Psi^{aT}(\tau)\Upsilon^{ab}(\tau-\tau')\Psi^b(\tau') + S_{top},$$

$$\Psi^a(\tau) = \begin{pmatrix} S^a(\tau) \\ \rho^a(\tau) \end{pmatrix}$$

Minh-Tien Tran and Ki-Seok Kim, Phys. Rev. Lett. 105, 116403 (2010)

Conclusion

$$QCD-like\ theory\ with\ \theta=\pi\ vs.\ \theta=0$$

In conclusion, we proposed deconfined local quantum criticality above one dimension based on the holographic approach, where the Chern-Simons term on AdS_5 plays an important role in quantum number fractionalization, enhancing the critical exponent in the correlation function of critical fluctuations.

The gauge-theory side: N=4 Super Yang-Mills (SYM) theory

$$\mathcal{L} = \mathrm{tr}\Big\{ -\frac{1}{2}F_{\mu\nu}F^{\mu\nu} + \bar{\psi}_i \not{D} \psi_i + D^\mu X_p D_\mu X_p + D^\mu Y_q D_\mu Y_q$$
$$-ig\bar{\psi}_i \alpha^p_{ij}[X_p, \psi_j] + g\bar{\psi}_i \gamma_5 \beta^q_{ij}[Y_q, \psi_j]$$
$$+\frac{g^2}{2}\Big([X_l, X_k][X_l, X_k] + [Y_l, Y_k][Y_l, Y_k] + 2[X_l, Y_k][X_l, Y_k] \Big) \Big\}$$

a gauge field A_μ, four Majorana fermions ψ_i, three real scalars X_p, and three real pseudo-scalars Y_q in the adjoint representation of a compact gauge group

Supersymmetry: the symmetry of interchanging bosons and fermions

N=4 (4 suprsymmetries): 4 different ways of interchanging the bosons and the fermions in the theory

R-symmetry: the symmetry of re-arrangement of the 4 super-charges
➔ SU(4)≃SO(6) global symmetry

$$g^2_{YM} N \sim g_s N \sim \frac{R^4}{l_s^4} \ll 1$$

The gravity side: 5d SUGRA

The U(1) truncation of the S^5 reduction of 10d type IIB supergravity :

Günaydin, Sierra and Townsend, NPB242(1984)244;
Cvetic et. al., NPB668(1999)96.

$$16\pi G_5 L = \sqrt{-g}\left(R + 12 - \frac{1}{4}F_{MN}F^{MN}\right) + \frac{\alpha}{3!}\varepsilon^{IJKLM}A_I F_{JK} F_{LM}$$

$$\alpha = 1/\left(2\sqrt{3}\right)$$

The bosonic part of the N=2 5d minimal gauged super-gravity with a negative cosmological constant.

5-dim. Einstein (Λ<0)+ Maxwell theory with a CS term.

- Any solution to this theory can be consistently embedded into the 10d type IIB supergravity (namely, type IIB superstring).
- The CS coupling α is at some fixed value for the closure of SUSY.

$$\frac{R^4}{l_s^4} \sim g_s N \sim g_{YM}^2 N \gg 1$$

S. Nakamura's talk in APCTP

Type IIB super gravity on $AdS_5 \times S^5$ at the classical level

conformal symmetry of N=4 SYM

SO(6) R-symmetry (symmetry under the exchange of the 4 super charges)

If we consider only a particular sector of the global SO(6) R-symmetry in N=4 SYM, we can ``forget'' the S^5 part and the 5d analyses on AdS_5 are enough for investigation.

In such cases, we can start with 5-dimensional super gravity obtained by the S^5 reduction of the original 10d type IIB SUGRA.

S. Nakamura's talk in APCTP

Equations of linear fluctuations I

$$g_{mn} = g^{(0)}_{mn} + h_{mn}, \qquad A_m = A^{(0)}_m + a_m$$

$$0 = h_t^{x(y)\prime\prime} - \frac{1}{u} h_t^{x(y)\prime} - \frac{b^2}{uf(u)}(\omega k h_z^{x(y)} + k^2 k_t^{x(y)})$$

$$-3auB'_{x(y)}, \qquad 0 = kf(u)h_z^{x(y)\prime} + \omega h_t^{x(y)\prime} - 3a\omega u B_{x(y)},$$

$$0 = h_z^{x(y)\prime\prime} + \frac{[u^{-1}f(u)]'}{u^{-1}f(u)} h_z^{x(y)\prime} + \frac{b^2}{uf^2(u)}(\omega^2 h_z^{x(y)}$$

$$+\omega k h_t^{x(y)}), \qquad 0 = B''_{x(y)} + \frac{f'(u)}{f(u)} B'_{x(y)} + \frac{b^2}{uf^2(u)}[\omega^2$$

$$-k^2 f(u)]B_{x(y)} - \frac{1}{f(u)} h_t^{x(y)\prime} - \tilde{\kappa} \frac{ik}{f(u)} B_{y(x)}, \qquad (4)$$

Equations of linear fluctuations II

$$u = \frac{r_0^2}{r^2} \text{ with } B_{x(y)} = \frac{a_{x(y)}}{\mu} \text{ and } \tilde{\kappa} = \frac{64e^2Qb^4}{l^5}\kappa$$

Y. Matsuo, S.-J. Sin, S. Takeuchi, and T. Tsukioka, JHEP **04**, 071 (2010).

$$\Theta_{x(y)\pm} = \frac{1}{u}h_t^{x(y)\prime} - \left(3a - \frac{C_\pm}{u}\right)B_{x(y)} \qquad C_\pm = 1 + a \pm \sqrt{(1+a)^2 + 3ab^2k^2}$$

$$0 = \Theta''_{x\pm} + \frac{[u^2f(u)]'}{u^2f(u)}\Theta'_{x\pm} + \frac{b^2(\omega^2 - k^2f(u)) - uf(u)C_\pm}{uf^2(u)}\Theta_{x\pm} - \frac{iC_\pm k\tilde{\kappa}}{C_0f(u)}(\Theta_{y+} - \Theta_{y-}),$$

$$0 = \Theta''_{y\pm} + \frac{[u^2f(u)]'}{u^2f(u)}\Theta'_{y\pm} + \frac{b^2(\omega^2 - k^2f(u)) - uf(u)C_\pm}{uf^2(u)}\Theta_{y\pm} + \frac{iC_\pm k\tilde{\kappa}}{C_0f(u)}(\Theta_{x+} - \Theta_{x-}).$$

$$0 = \tilde{\Theta}'' + \frac{[u^2f(u)]'}{u^2f(u)}\tilde{\Theta}' + \tilde{\Omega}(u)\tilde{\Theta}, \qquad (5)$$

$$\Theta = \Lambda\tilde{\Theta} \text{ with } \Theta = \begin{pmatrix} \Theta_{x+} \\ \Theta_{x-} \\ \Theta_{y+} \\ \Theta_{y-} \end{pmatrix}$$

$$\tilde{\Omega}(u) = \frac{b^2}{uf^2(u)}(\omega^2 - f(u)k^2) + \frac{\mathcal{D}_{\tilde{\kappa}}(k) - 2(1+a)}{2f(u)} \qquad (6)$$

$$\mathcal{D}_{\tilde{\kappa}}(k) = diag\left(-D_- + \tilde{\kappa}k, D_- + \tilde{\kappa}k, -D_+ - \tilde{\kappa}k, D_+ - \tilde{\kappa}k\right)$$

$$D_\pm = \sqrt{(C_0 \pm \tilde{\kappa}k)^2 \pm 4\tilde{\kappa}kC_-} \qquad C_0 = C_+ - C_-$$

Analysis for the inner region I

$$f(r) = 12\frac{(r-r_*)^2}{r_*^2} \qquad r - r_* = \omega\frac{R^2}{\zeta}$$

$$0 = 3\frac{[(R^2/r_*)\omega+\zeta]^5}{\zeta^3}\left\{\frac{[(R^2/r_*)\omega+\zeta]}{\zeta}\tilde{\Theta}'' + \frac{[(R^2/r_*)\omega]}{\zeta^2}\tilde{\Theta}'\right\}$$
$$+\left[\frac{b^2[(R^2/r_*)\omega+\zeta]^2}{12\frac{R^4}{r_*^2}}\left(1-12\frac{R^4}{r_*^2}\frac{k^2}{\zeta^2}\right)-(1+a)+\frac{\mathcal{D}_{\tilde{\kappa}}(k)}{2}\right]\tilde{\Theta}.$$

$$\tilde{\Theta}_I(\zeta) = \tilde{\Theta}_I^{(0)}(\zeta) + \omega\tilde{\Theta}_I^{(1)}(\zeta) + \omega^2\tilde{\Theta}_I^{(2)}(\zeta) + \ldots$$

$$0 = \tilde{\Theta}_{I\alpha}^{(0)\prime\prime}(\zeta) + \left\{\frac{b^2}{36\frac{R^4}{r_*^2}} + \frac{\mathcal{D}_{\tilde{\kappa}}^{\alpha}(k) - 2[b^2k^2+(1+a)]}{6\zeta^2}\right\}\tilde{\Theta}_{I\alpha}^{(0)}(\zeta)$$

Analysis for the inner region II

$$\tilde{\Theta}^{(0)}_{I\alpha}(\zeta) = a^{(0)}_{I\alpha}\sqrt{\zeta}\Big[-BesselJ\Big\{\frac{\Delta^{\alpha}_{\tilde{\kappa}}(k)}{2},\Big(\frac{b}{6\frac{R^2}{r_*}}\Big)\zeta\Big\} - i\sqrt{\zeta}BesselY\Big\{\frac{\Delta^{\alpha}_{\tilde{\kappa}}(k)}{2},\Big(\frac{b}{6\frac{R^2}{r_*}}\Big)\zeta\Big\}\Big]$$

$$+b^{(0)}_{I\alpha}\sqrt{\zeta}\Big[-BesselJ\Big\{\frac{\Delta^{\alpha}_{\tilde{\kappa}}(k)}{2},\Big(\frac{b}{6\frac{R^2}{r_*}}\Big)\zeta\Big\} + i\sqrt{\zeta}BesselY\Big\{\frac{\Delta^{\alpha}_{\tilde{\kappa}}(k)}{2},\Big(\frac{b}{6\frac{R^2}{r_*}}\Big)\zeta\Big\}\Big].$$

$$\tilde{\Theta}^{(0)}_{I\alpha}(\zeta) = a^{(0)}_{I\alpha}\sqrt{\zeta}\Big[-BesselJ\Big\{\frac{\Delta^{\alpha}_{\tilde{\kappa}}(k)}{2},\Big(\frac{b}{6\frac{R^2}{r_*}}\Big)\zeta\Big\}$$

$$-i\sqrt{\zeta}BesselY\Big\{\frac{\Delta^{\alpha}_{\tilde{\kappa}}(k)}{2},\Big(\frac{b}{6\frac{R^2}{r_*}}\Big)\zeta\Big\}\Big],$$

$$\Delta^{\alpha}_{\tilde{\kappa}}(k) = \sqrt{1 - \frac{2}{3}\Big(\mathcal{D}^{\alpha}_{\tilde{\kappa}}(k) - 2(1+a) - 2b^2k^2\Big)}$$

Analysis for the outer region I

$$f(u) = (u-1)^2(2u+1)$$

$$0 = [u^2(u-1)^2(2u+1)\tilde{\Theta}']' + \left\{\frac{b^2[\omega^2 - (u-1)^2(2u+1)k^2]u}{(u-1)^2(2u+1)} + \frac{[\mathcal{D}_{\tilde{\kappa}}(k) - 2(1+a)]u^2}{2}\right\}\tilde{\Theta}.$$

$$\tilde{\Theta}_O(u) = \tilde{\Theta}_O^{(0)}(u) + \omega\tilde{\Theta}_O^{(1)}(u) + \omega^2\tilde{\Theta}_O^{(2)}(u) + ..$$

$$\begin{aligned} 0 &= [u^2(u-1)^2(2u+1)\tilde{\Theta}_{O\alpha}^{(0)}{}']' \\ &+ \frac{[\mathcal{D}^{\alpha}_{\tilde{\kappa}}(k) - 2(1+a)]u^2 - 2b^2k^2u}{2}\tilde{\Theta}_{O\alpha}^{(0)} \end{aligned}$$

$$\tilde{\Theta}_{O\alpha}^{(0)}(u) = a_{O\alpha}^{(0)}(u-1)^{\frac{-1+\Delta^{\alpha}_{\tilde{\kappa}}(k)}{2}}\mathcal{H}\Big(-\frac{1}{2}, q'_{+\alpha}; \alpha_{+\alpha}, \beta_{+\alpha}, 2, \Delta^{\alpha}_{\tilde{\kappa}}(k); u\Big) + b_{O\alpha}^{(0)}(u-1)^{\frac{-1-\Delta^{\alpha}_{\tilde{\kappa}}(k)}{2}}\mathcal{H}\Big(-\frac{1}{2}, q'_{-\alpha}; \alpha_{-\alpha}, \beta_{-\alpha}, 2, -\Delta^{\alpha}_{\tilde{\kappa}}(k); u\Big),$$

$$q'_{\pm} = \frac{1 - b^2k^2 \mp \Delta^{\alpha}_{\tilde{\kappa}}(k)}{2},$$

$$\alpha_{\pm} = \frac{\frac{3}{2} \pm \Delta^{\alpha}_{\tilde{\kappa}}(k) + \sqrt{\frac{33}{4} \mp 2\Delta^{\alpha}_{\tilde{\kappa}}(k)}}{2},$$

$$\beta_{\pm} = \frac{\frac{3}{2} \pm \Delta^{\alpha}_{\tilde{\kappa}}(k) - \sqrt{\frac{33}{4} \mp 2\Delta^{\alpha}_{\tilde{\kappa}}(k)}}{2}.$$

Analysis for the outer region II

$$\tilde{\Theta}^{(0)}_{I\alpha}(u) = a^{(0)}_{I\alpha}\left[(1-u)^{\frac{-1+\Delta^{\alpha}_{\tilde{\kappa}}(k)}{2}} + \mathcal{G}^{\alpha}_{R}(\omega)\frac{1}{(1-u)^{\frac{1+\Delta^{\alpha}_{\tilde{\kappa}}(k)}{2}}}\right]$$

$$\begin{aligned}\tilde{\Theta}^{(0)}_{O\alpha}(u\to 1) \;=\; & a^{(0)}_{O\alpha}(-1)^{\frac{-1+\Delta^{\alpha}_{\tilde{\kappa}}(k)}{2}}\mathcal{H}\Big(-\frac{1}{2}, q'_{+\alpha}; \alpha_{+\alpha}, \beta_{+\alpha}, \gamma_{+\alpha}, \delta_{+\alpha}; 1\Big)(1-u)^{\frac{-1+\Delta^{\alpha}_{\tilde{\kappa}}(k)}{2}} \\ + \; & b^{(0)}_{O\alpha}(-1)^{\frac{-1-\Delta^{\alpha}_{\tilde{\kappa}}(k)}{2}}\mathcal{H}\Big(-\frac{1}{2}, q'_{-\alpha}; \alpha_{-\alpha}, \beta_{-\alpha}, \gamma_{-\alpha}, \delta_{-\alpha}; 1\Big)(1-u)^{\frac{-1-\Delta^{\alpha}_{\tilde{\kappa}}(k)}{2}}\end{aligned}$$

$$a^{(0)}_{O\alpha} = \frac{(-1)^{\frac{1-\Delta^{\alpha}_{\tilde{\kappa}}(k)}{2}}}{\mathcal{H}\Big(-\frac{1}{2}, q'_{+\alpha}; \alpha_{+\alpha}, \beta_{+\alpha}, \gamma_{+\alpha}, \delta_{+\alpha}; 1\Big)} a^{(0)}_{I\alpha},$$

$$b^{(0)}_{O\alpha} = \frac{(-1)^{\frac{1+\Delta^{\alpha}_{\tilde{\kappa}}(k)}{2}}}{\mathcal{H}\Big(-\frac{1}{2}, q'_{-\alpha}; \alpha_{-\alpha}, \beta_{-\alpha}, \gamma_{-\alpha}, \delta_{-\alpha}; 1\Big)} \mathcal{G}^{\alpha}_{R}(\omega) a^{(0)}_{I\alpha}.$$

Retarded Green's function I

$$a_{I\alpha}^{(0)^{-1}}\tilde{\Theta}_{O\alpha}^{(0)}(u)$$

$$= \frac{\mathcal{H}\left(-\frac{1}{2}, q'_{+\alpha}; \alpha_{+\alpha}, \beta_{+\alpha}, \gamma_{+\alpha}, \delta_{+\alpha}; u\right)}{\mathcal{H}\left(-\frac{1}{2}, q'_{+\alpha}; \alpha_{+\alpha}, \beta_{+\alpha}, \gamma_{+\alpha}, \delta_{+\alpha}; 1\right)}(1-u)^{\frac{-1+\Delta_{\tilde{\kappa}}^{\alpha}(k)}{2}}$$

$$+\frac{\mathcal{H}\left(-\frac{1}{2}, q'_{-\alpha}; \alpha_{-\alpha}, \beta_{-\alpha}, \gamma_{-\alpha}, \delta_{-\alpha}; u\right)}{\mathcal{H}\left(-\frac{1}{2}, q'_{-\alpha}; \alpha_{-\alpha}, \beta_{-\alpha}, \gamma_{-\alpha}, \delta_{-\alpha}; 1\right)}\frac{\mathcal{G}_R^{\alpha}(\omega)}{(1-u)^{\frac{1+\Delta_{\tilde{\kappa}}^{\alpha}(k)}{2}}},$$

$$\mathcal{G}_R^{\alpha}(\omega) = i\pi\frac{2^{-\Delta_{\tilde{\kappa}}^{\alpha}(k)}\left(\frac{b}{6\frac{R^2}{r_*}}\right)^{\Delta_{\tilde{\kappa}}^{\alpha}(k)}}{\Gamma\left(\frac{\Delta_{\tilde{\kappa}}^{\alpha}(k)}{2}\right)\Gamma\left(1+\frac{\Delta_{\tilde{\kappa}}^{\alpha}(k)}{2}\right)}\left(2\frac{R^2}{r_*}\right)^{\Delta_{\tilde{\kappa}}^{\alpha}(k)}\omega^{\Delta_{\tilde{\kappa}}^{\alpha}(k)}$$

Retarded Green's function II

$$\tilde{\Theta}^{(0)}_{O\alpha}(u \to 0) \;=\; a^{(0)}_{I\alpha}\Big[\frac{\mathcal{H}\Big(-\frac{1}{2}, q'_{+\alpha}; \alpha_{+\alpha}, \beta_{+\alpha}, \gamma_{+\alpha}, \delta_{+\alpha}; u\Big)}{\mathcal{H}\Big(-\frac{1}{2}, q'_{+\alpha}; \alpha_{+\alpha}, \beta_{+\alpha}, \gamma_{+\alpha}, \delta_{+\alpha}; 1\Big)} + \mathcal{G}^{\alpha}_{R}(\omega)\frac{\mathcal{H}\Big(-\frac{1}{2}, q'_{-\alpha}; \alpha_{-\alpha}, \beta_{-\alpha}, \gamma_{-\alpha}, \delta_{-\alpha}; u\Big)}{\mathcal{H}\Big(-\frac{1}{2}, q'_{-\alpha}; \alpha_{-\alpha}, \beta_{-\alpha}, \gamma_{-\alpha}, \delta_{-\alpha}; 1\Big)}\Big]$$

$$\tilde{\Theta}^{(0)}_{O\alpha}(r \to \infty) \approx A_{\infty} r^{\Delta_A - d} + B_{\infty} r^{-\Delta_A}$$

$$\Theta^{(0)}_{O\alpha}(u \to 0) = \sum_{\beta=1}^{4} \Lambda_{\alpha\beta}(k)\tilde{\Theta}^{(0)}_{O\beta}(u \to 0)$$

$$= \sum_{\beta=1}^{4} \Lambda_{\alpha\beta}(k) a^{(0)}_{I\beta}\Big[\frac{\mathcal{H}\Big(-\frac{1}{2}, q'_{+\beta}; \alpha_{+\beta}, \beta_{+\beta}, \gamma_{+\beta}, \delta_{+\beta}; u\Big)}{\mathcal{H}\Big(-\frac{1}{2}, q'_{+\beta}; \alpha_{+\beta}, \beta_{+\beta}, \gamma_{+\beta}, \delta_{+\beta}; 1\Big)} + \mathcal{G}^{\beta}_{R}(\omega)\frac{\mathcal{H}\Big(-\frac{1}{2}, q'_{-\beta}; \alpha_{-\beta}, \beta_{-\beta}, \gamma_{-\beta}, \delta_{-\beta}; u\Big)}{\mathcal{H}\Big(-\frac{1}{2}, q'_{-\beta}; \alpha_{-\beta}, \beta_{-\beta}, \gamma_{-\beta}, \delta_{-\beta}; 1\Big)}\Big]$$

$$B_x = \frac{u}{C_+ - C_-}\Big(\Theta_{x+} - \Theta_{x-}\Big), \qquad B_y = \frac{u}{C_+ - C_-}\Big(\Theta_{y+} - \Theta_{y-}\Big)$$

Retarded Green's function III

$$G_{xx}^R(\omega) = \frac{\mathcal{C}}{16\pi G_5 e^2} \frac{\sum_{\beta=1}^4 [\Lambda_{1\beta}(k) - \Lambda_{2\beta}(k)] a_{I\beta}^{(0)} \frac{\lim_{u\to 0}\left\{\mathcal{H}\left(-\frac{1}{2}, q'_{-\beta}; \alpha_{-\beta}, \beta_{-\beta}, \gamma_{-\beta}, \delta_{-\beta}; u\right) u^{\frac{\Delta_A}{2}-2}\right\}}{\mathcal{H}\left(-\frac{1}{2}, q'_{-\beta}; \alpha_{-\beta}, \beta_{-\beta}, \gamma_{-\beta}, \delta_{-\beta}; 1\right)} \mathcal{G}_R^\beta(\omega)}{\sum_{\beta=1}^4 [\Lambda_{1\beta}(k) - \Lambda_{2\beta}(k)] a_{I\beta}^{(0)} \frac{\lim_{u\to 0}\left\{\mathcal{H}\left(-\frac{1}{2}, q'_{+\beta}; \alpha_{+\beta}, \beta_{+\beta}, \gamma_{+\beta}, \delta_{+\beta}; u\right) u^{-\frac{\Delta_A}{2}}\right\}}{\mathcal{H}\left(-\frac{1}{2}, q'_{+\beta}; \alpha_{+\beta}, \beta_{+\beta}, \gamma_{+\beta}, \delta_{+\beta}; 1\right)}}$$

$$\left(\frac{\mathcal{C}}{16\pi G_5 e^2}\right)^{-1} G_{xx}^R(\omega)$$

$$\approx \frac{\lim_{u\to 0}\left\{\mathcal{H}\left(-\frac{1}{2}, q'_{-3}; \alpha_{-3}, \beta_{-3}, \gamma_{-3}, \delta_{-3}; u\right) u^{\frac{\Delta_A}{2}-2}\right\}}{\lim_{u\to 0}\left\{\mathcal{H}\left(-\frac{1}{2}, q'_{+3}; \alpha_{+3}, \beta_{+3}, \gamma_{+3}, \delta_{+3}; u\right) u^{-\frac{\Delta_A}{2}}\right\}} \mathcal{G}_R^3(\omega)$$