

Subwavelength Photonic Band Gaps from Planar Fractals

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(Received 26 March 2002; published 12 November 2002)

We show by both experiment and theory that a specific class of planar conducting fractals possesses a series of self-similar resonances, leading to multiple gaps and pass bands for electromagnetic waves over an ultrawide frequency range. A double stack of these fractal patterns exhibits polarization-independent absolute gaps over a wide range of incidence angles. These characteristics are retained even when the fractal patterns are significantly subwavelength in all dimensions. In addition, the transmittance can be modulated by an external current source.

DOI: 10.1103/PhysRevLett.89.223901

PACS numbers: 42.70.Qs, 47.53.+n

Conventional photonic band gap (PBG) materials [1] employ Bragg scattering to create forbidden band gaps. The thickness and lateral dimensions of a photonic crystal must be a few times that of the wavelength — a natural consequence of the Bragg mechanism. Frequency-selective surfaces (FSS) [2] can selectively reflect an electromagnetic (EM) wave of a prespecified frequency. It operates on the principle of resonance intrinsic to a number of interacting metallic elements arranged periodically. PBG and FSS systems typically operate at one single frequency range, with the relevant wavelength dictated by the size of the periodically arranged basic building blocks. Here we show that a particular kind of planar conducting fractals [3] can exhibit multiple stop and pass bands over a broad frequency range, with the desirable property that the fractal patterns can be significantly subwavelength in lateral dimensions and can simulate three-dimensional (3D) photonic crystals in terms of total reflection that is independent of both the polarization and the incident angle. Finite difference time domain (FDTD) simulations [4] are in excellent agreement with the experiments. The multiband functionality, covering an ultrawide spectral range, and with the low frequency gaps corresponding to wavelengths that can be significantly larger than the size of the sample makes fractal plates an interesting and potentially very useful frequency-selective material. Such unique property is the manifestation of a series of self-similar resonances intrinsic to the fractal structures.

There is an extensive literature on optical properties of fractals [5], many of which deal with random fractals. In contrast, our work employs deterministic geometric patterns, which can be fabricated easily in a controllable manner, with precisely predictable properties. Deterministic patterns such as Sierpinski gaskets or Koch curves have been studied as diffraction plates or antennas [6], but we are here interested in fractal geometries that can function as an ultrathin PBG slab or as a multiband subwavelength FSS. As such, we found that a special form of a space-filling curve is especially suitable. We are primarily interested in low frequency regimes, and

scaling to optical frequencies requires careful consideration of dispersion and absorption.

Our fractal [3] pattern (see Fig. 1) was generated from a line of length $a = 144.78$ mm, defined as the first level of the structure, placed parallel to the y axis in the xy plane. The $(k + 1)$ th level structure contains 2^k lines, with the midpoint of each perpendicularly connected to the ends of the k th level lines. The length of the $(k + 1)$ th level

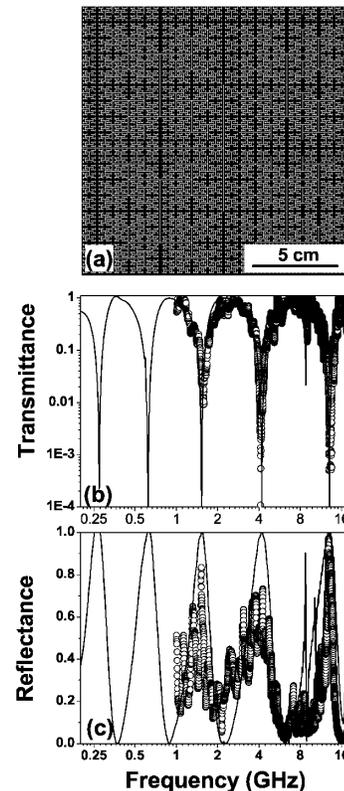


FIG. 1. (a) A computer-generated image of a part of the fractal structure. The white lines corresponds to copper in the real sample. (b) Measured (circles) and calculated (lines) transmittance; and (c) reflectance of a 15-level fractal pattern for electromagnetic waves polarized with the E vector along the y axis.

lines was scaled from that of the k th level line by a factor of $2(1)$ if k is an even (odd) number. The number of levels (N), the initial line length (a), together with the line width and thickness, collectively define the fractal structure. Simply described, the fractal has an H-shaped generator, a geometrical scaling factor of 2, with no self-intersections. As N tends to infinity, the structure eventually becomes a space-filling curve that tiles a $2a \times 2a$ square. Figure 1(a) shows a computer-generated image of part of a 15-level fractal. The real sample is made by the shadow-masking/etching technique on a $30 \text{ cm} \times 30 \text{ cm} \times 1 \text{ mm}$ dielectric plate. The linewidth and thickness of the copper lines are both 0.1 mm. Figures 1(b) and 1(c) show the measured normal transmission and reflection of the fractal plate from 1 to 18 GHz. A HP83711B generator was connected to a double-ridged waveguide horn antenna (HP11966E) to generate the microwave. Another identical receiver horn was placed at a distance of 100 cm from the source, connected to a power meter (Agilent E4418B). The fractal plate was placed on a stage, 15 cm from the receiving horn, which can be rotated about the z axis (defined as $\theta = \phi = 0^\circ$) by an angle θ (\mathbf{E} perpendicular to the plane of incidence) and tilted by an angle ϕ (\mathbf{E} parallel to the plane of incidence). Reflection measurements were carried out by putting both the source and receiver horns on one side, separated by a small angle of 8° . The open circles in Figs. 1(b) and 1(c) denote the measured transmission and reflection, respectively, when the normally incident E field was polarized along the y axis (for off-normal incidence, see below). The measured transmission [Fig. 1(b)] shows three stop bands within the 1–18 GHz range at which the EM wave transmission was strongly attenuated, and Fig. 1(c) shows strong reflection at these same three frequency ranges. Between any stop bands there is a pass band where the maximum transmission can be close to 100%. This is intriguing at first sight, since the metallic lines are connected and cover the entire plate. The relevant wavelength is also much larger than the spaces between the metallic lines.

Understanding of the observed phenomena was obtained using the FDTD simulations [4,7]. Perfect conductor boundary conditions, excellent for microwave frequencies, were applied to the metal/air interfaces. We consider an infinite xy plane tiled by a periodic replica of the fractal plate, with an incident plane wave. The substrate dielectric constant was taken to be $\epsilon_r = 5.3$, obtained by comparing with the data. Calculations revealed that certain frequencies would excite particular subdomains of the fractal, with the higher frequency characteristics governed by the localized currents in higher level substructures. The calculated results are shown as solid lines in Fig. 1. The simulations reproduce all the salient features of the experiments [8]. In particular, the calculated and measured spectra are both log periodic, an inherent consequence of a fractal pattern's self-similarity

[9]. In addition to manifesting the mathematical beauty of self-similarity, the log periodicity means that the resonances (hence the stop bands) of the fractal cover an ultrabroad frequency range. This unique feature is hard to match with periodic frequency-selective systems such as conventional PBG slabs or FSS, which typically have one single dominant response.

An analysis of the simulation results shows that the EM field excites currents in the metallic lines of the fractal, with the current amplitude reaching maxima at those strongly reflecting frequencies. The relative phase of the current with respect to the incident wave undergoes a π jump whenever the frequency was varied across each maximum reflection point, indicating a resonance behavior. The FDTD simulations showed that an N -level fractal contains N resonances. Each resonance has currents excited mainly along the metallic lines of one specific level, flowing towards the higher (finer) level structures. For example, the resonance at 13.1 GHz excites surface currents [10] with peak amplitude localized at the 13th level lines, flowing to higher level lines on both sides. The fractal response at normal incidence can be accurately modeled by a thin homogeneous plate (of the same thickness) with an effective dielectric constant of the form:

$$\epsilon_{\text{eff}}(f) = \epsilon_0 + \sum_l \frac{\beta_l}{f_l^2 - f^2}, \quad (1)$$

where f denotes frequency in GHz, the index l runs over all resonances, and ϵ_0 , f_l , and β_l are parameters obtainable from FDTD calculated spectra [11]. Equation (1) makes clear that in varying from one resonance frequency f_l to the next f_{l+1} , there is necessarily a point at which $\epsilon_{\text{eff}} = 1$, hence the existence of the pass band.

It is generally desirable if the reflection/transmission can be modulated by an external “knob.” Every line segment in our fractal is connected to each other. This property makes it possible to use an external source to drive a surface current on the fractal structure, so as to interfere with the current induced by the primary incident wave, leading to changes in the far field radiation pattern. The secondary source can be an external electric current fed into the center of the first level line [12], with a definite phase relationship to the incident beam. Figure 2 shows the modulation effect achieved by feeding a current into the fractal. We placed the 15-level fractal plate at 20 cm in front of a receiver horn connected to a HP 8563 Spectrum Analyzer. The fractal plate has the same structure as that shown in the inset of Fig. 1, except that the first level (the longest metal line) was broken in the middle to allow for external wiring to feed in an ac signal with variable frequency. A 2.13 GHz, x -polarized microwave was normally incident from the other side of the plate, in line with the receiver horn (see the schematic in the inset). Trace 2(a) shows the measured transmission due to the primary source when the current source was

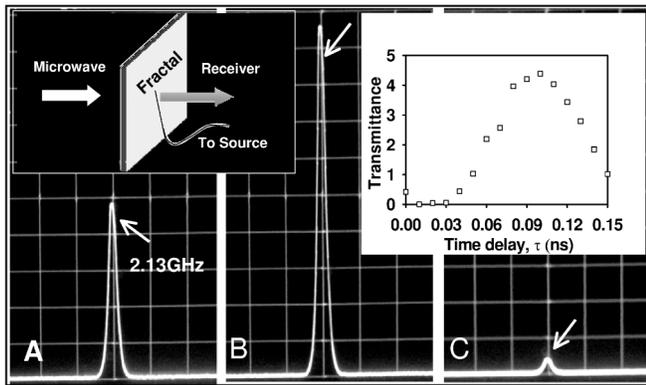


FIG. 2. Signals as a function of frequency (horizontal axis) photographed from the scope of the Spectral Analyzer. Panel (a) shows the amplitude of the transmitted microwave through the fractal due to the primary source. When the secondary current source was turned on and adjusted to be the same frequency of 2.13 GHz, the measured transmission was amplified (b) when they were in phase and diminished (c) when out of phase. The inset on the left is a schematic of the experimental setup. The inset on the right shows FDTD simulations of the transmittance modulation of a y -polarized 6.8 GHz microwave through the 7-level fractal plate, by a central-fed ac current of the same frequency. The magnitude of the modulation is governed by the phase difference (time delay) between the incident microwave and the central-fed current.

off. Traces 2(b) and 2(c) show the measured transmission when the current source (with the same frequency of 2.13 GHz) was turned on. The phase of the current source can be varied to enhance [Trace 2(b)] or reduce [Trace 2(c)] the forward transmission.

The modulation effect can be verified by FDTD simulations. We worked with a 7-level fractal, instead of the 15-level fractal used in experiment, to make the simulations more tractable. The right inset in Fig. 2 shows the calculated transmittances for a 7-level fractal pattern under a central-fed modulation current with different phase delays. In the simulations, the primary incident plane wave has a frequency of 6.8 GHz, and a modulation current of the same frequency was injected into the fractal structure from a point source placed in the middle of the first level line (with an amplitude such that the radiation power of the fractal, which now also serves as an active element, is equal to that of the primary beam). The right inset of Fig. 2 shows the numerically calculated transmittance to be strongly modulated, controlled by the phase difference (depending on the input signal time delay τ relative to the primary incident beam in the calculation) between the incident microwave and the injected current.

By stacking two identical fractal patterns, one rotated 90° relative to the other to form a rotationally invariant structure, we demonstrate that our double stack structure can simulate the incident angle and polarization-

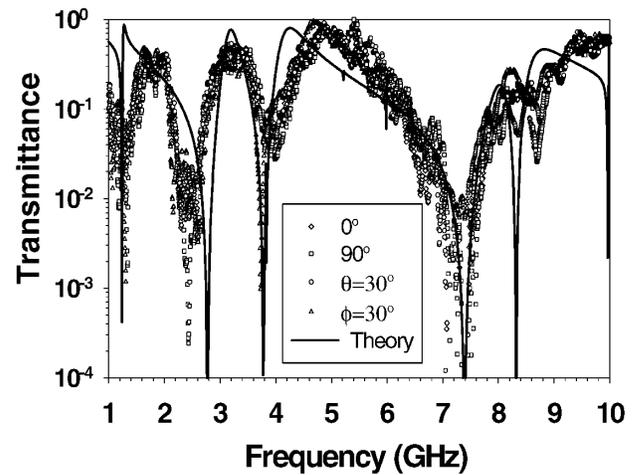


FIG. 3. Transmission through two stacked identical fractal plates with a 90° rotation with respect to each other. The diamonds and squares denote measured normal transmission through the double stack, with \vec{E} polarized along the x and y -axes, respectively, showing the gaps to be polarization independent. The lines are calculated results for normal incidence (identical for both polarizations). The circles and triangles are transmissions measured with the plates rotated (θ) and tilted (ϕ) by 30° , respectively, from the normal.

independent total reflection that are usually characteristics of a 3D photonic crystal. Figure 3 shows the transmission of such a double stack when it was rotated and tilted, so as to change the angle of incidence. We found the transmissions to be nearly identical for different angles of incidence [13], hence the spectral gap frequencies are stable with respect to a large range of incidence angles. In addition, the transmission is independent of polarization since the double stack is rotationally invariant. We note that conventional 3D photonic crystals must be at least a few times the wavelength before fully exhibiting its PBG property; thus for 1 GHz microwave the thickness would be on the order of 1 m.

Another point about our planar structure is that the size of the stop bands, measured by the dimensionless parameter $\Delta f/f_0$ (gap/midgap ratio, $\sim 5\%$ for a single fractal) can be significantly enhanced by stacking identical fractals together, in contrast to conventional 3D photonic crystals, where increasing the slab thickness sharpens the band edges. The effect of stacking up to five fractals is shown in Fig. 4 for three resonance frequencies (1.6, 4.4, and 13 GHz). The $\Delta f/f_0$ ratio [14] is seen to increase rather significantly according to the FDTD simulation. A stack of several fractals, plus the corresponding 90° rotated counterparts, is still negligible in thickness compared to the relevant wavelengths. The stop bands can also be expanded by increasing the width of metal lines in the fractal.

An empirical formula for the resonance wavelengths of an N -level fractal structure is given by

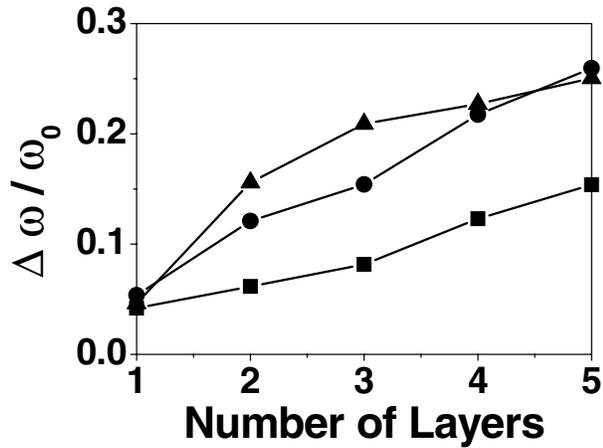


FIG. 4. The calculated gap/midgap ratio as a function of the number of stacking layers for three spectral gaps at 1.6 (square), 4.4 (circle), and 13 (triangle) GHz. The dielectric substrates are 1 mm thick, separated by an air gap of 1 mm.

$$\lambda_i = \sqrt{\bar{\epsilon}_r} \sum_{j=1}^{N+1-i} \alpha(j)b(i+j-1), \quad (2)$$

where $\{\lambda_i\}$ is for $y(x)$ polarization if i is an odd (even) number, and $\{\alpha(1) = \alpha(2) = 2.1, \alpha(3) = \alpha(4) = 3.5, \alpha(2i+1) = \alpha(2i+2) = 2\alpha(2i-1) = 2\alpha(2i), i \geq 2\}$ are a set of empirical parameters, $b(i)$ is the length of the i th level, and $\bar{\epsilon}_r = (\epsilon_r + 1)/2$. Equation (2) enabled us to extrapolate to low and high frequencies where direct simulation is too costly. We also note that Eq. (2) predicts $\lambda_i/\lambda_{i+2} \rightarrow 2$ as $N \rightarrow \infty$ [remembering that $b(i) = 2b(i+2)$], i.e., approaching exact self-similarity in the limit of large N .

A direct consequence of Eq. (2) is that the resonance wavelength can be much larger than the sample size. This is because the low frequency resonances are essentially determined by the longest continuous metallic line in the fractal, and such a line is easily much longer than the linear dimension of the fractal. This gives the fractal its “subwavelength” property, i.e., it can effectively reflect EM waves with wavelength much larger than its lateral dimensions. For example, we found from experiment and simulation that a 28×29 mm plate containing a 6-level fractal can almost completely reflect a 3.85 GHz microwave (coinciding with one of the resonances of the 6-level fractal pattern) coming from a monopole antenna placed at the near field, while a copper plate of the same dimension failed to block the EM wave since it is far less than the wavelength of approximately 78 mm.

This subwavelength property means that our fractal plates can act as compact reflectors. We note that a single low frequency resonance can also be achieved in periodic resonating systems such as the “high-impedance sur-

face” of Sievenpiper *et al.* [15], while the fractal can have a multiple of very low frequency resonances.

In short, we found that a particular fractal pattern (space-filling curves) has a self-similar series of resonances, leading to a log-periodic sequence of gaps. In addition, the lower frequency stop bands correspond to wavelengths that are significantly longer than the lateral dimension of the fractal plate, making the fractal plates subwavelength reflectors. The connected topological structure makes it possible to modulate the transmission property via a second source.

This work was supported by RGC Hong Kong through Grants No. N_HKUST025/00 and No. HKUST6145/99P.

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- [7] Simulations were performed using the software package CONCERTO, developed by Vector Fields Limited, England, 2001.
- [8] The smaller amplitude of the measured reflection is due to the diffraction effect of our finite-sized sample.
- [9] The self-similarity and log periodicity is approximate for finite-sized plates with a finite number of levels.
- [10] Calculated from the curl of the tangential magnetic field.
- [11] For example, simulation results shown in Fig. 1 can be reproduced accurately using the following model parameter values (β_l in units of GHz², f_l in units of GHz): $\epsilon_0 = 11$, $\beta_1 = 10$, $\beta_2 = 23$, $\beta_3 = 60$, $\beta_4 = 190$, $\beta_5 = 500$, $f_1 = 0.274$, $f_2 = 0.623$, $f_3 = 1.529$, $f_4 = 4.19$, and $f_5 = 13.056$.
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