# Transmission resonances of electromagnetic wave through metallic gratings: phase and field characterizations 

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#### Abstract

We investigate experimentally and numerically the transmission resonances of electromagnetic wave through a metallic grating of periodic slits as well as an array of subwavelength holes. The far-field transmission (magnitude and phase) spectra and the detection of surface electric field are used to characterize two kinds of transmission resonances (Fabry-Perot-like and structural factor resonance) occurring to the slit grating. In addition, the comparison between the slit grating and the subwavelength hole array reveals the effect of the aperture property on the phase and field of resonantly transmitted electromagnetic wave.


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## 1. Introduction

Recently, a renewed interest has been cast to resonant transmissions of electromagnetic (EM) wave through subwavelength apertures in metallic gratings, which is inspired by the observation of extraordinary optical transmission through a metallic film with an array of subwavelength holes [1-9]. EM transmissions of the metallic gratings at particular frequencies are seen much larger than the values expected from the area fractions of apertures in the limit of geometric optics. The underlying transmission mechanisms are attributed to EM resonances established on the gratings and occurring to appropriate structural dimensions. Typically, two types of transmission resonances are identified in terms of the relevant wavelength $\lambda$, and their associated peaks are found to exhibit distinguishable spectral behaviors. One type of resonances, related to the structural lattice, has $\lambda \sim a$ ( $a$ being periodicity of the grating) and is dependent strongly on the incidence angle of EM wave and the lattice pattern. Whereas, the other type, related to the waveguiding mode of individual aperture, has the wavelength comparable to the transversal/longitudinal dimension of the aperture, for example $\lambda \sim t$ ( $t$ being thickness of the grating), and is not sensitive to the incidence angle, and persists to even the random array of apertures.

In this study, we mainly investigated EM wave transmission through the grating of onedimensional (1D) periodic slits. This kind of gratings has been found to support both types of resonances mentioned above, and therefore exhibits multiple transmission peaks which reach $100 \%$ in magnitude in ideal cases and behave differently in the spectra [3, 4]. Considering that EM fields at resonance has deterministic spatial distributions in the structure/material, just like discrete resonant modes in microwave cavities [10], we are interested in characterizing the relevant field modes on the 1D gratings for these two types of resonances by measuring the far-field transmission (magnitude and phase) and scanning the near-field profile. Finally, we compared the $\lambda \sim a$ resonance in the 1 D slit gratings with the resonance responsible for extraordinary transmission of EM wave through the grating of subwavelength hole array. Although occurring in different systems, they belong commonly to the forementioned lattice-related type which has been regarded as, specially for perfect conducting gratings, structural factor $(\mathrm{SF})$ resonance due to the structural factor $G_{E}$ involved [11, 12]. We found that the $\lambda \sim a$ resonances in two systems exhibit distinct phase characteristics.

## 2. Experiments and simulations

Figure 1 illustrates two metallic grating structures: the 1D periodic slits and the twodimensional (2D) array of subwavelength holes in our investigations. The experimental samples were made of good conductors (aluminum or brass) which behave like perfect metal at the operative microwave frequencies. The 1D slit grating was constructed by stacking 60 cm long aluminum rods with spacers at both ends and the resultant sample had the transverse dimensions of $6060 \mathrm{~cm}^{2}$. The 2D hole array was fabricated through drilling in a $3030 \mathrm{~cm}^{2}$ brass plate. Two identical double ridged waveguide horn antennas (HP 11966E) were connected with an S-parameter network analyzer (Agilent 8720ES) and used as microwave emitter and receiver. The gratings were located between the two horn antennas with enough distance away from the emitter to approximate the incidence of plane wave. The EM wave of magnitude $\left(E_{0}, H_{0}\right)$ and of specific polarization illuminated the gratings at normal incidence,
as depicted in Fig. 1. The polarization direction is readily controlled through orienting the horn antennas with respect to the grating, since the double ridged waveguide horn generates a linearly polarized EM wave in the far field. We obtained the far-field transmission (magnitude $T$ and phase $\phi$ ) spectrum of the grating from parameter S 21 recorded by the analyzer. Here, the transmission phase $\phi$ refers to the phase difference of EM wave between the front and the rear surface of the sample. The phase values are plotted in the main value region $[-\pi,+\pi]$ in our paper with understanding $\phi$ is actually a continuous function of wavelength and $2 \pi$ should be deducted from the left branches upon an abrupt jump $-\pi \rightarrow+\pi$. Its negative sign represents the clockwise rotation in the complex plane with increasing frequency.


Fig. 1. (color online) Photos and schematic pictures of the metallic gratings under investigation: (a) one-dimensional periodic slits and (b) two-dimensional array of subwavelength holes, where $a$ is the periodicity, $d$ the aperture size (slit width/hole diameter), and $t$ the thickness of the grating. The gratings are illuminated by a plane EM wave of normal incidence with electric field along the $y$-direction. $E_{0}$ and $H_{0}$ denote the magnitude of the incident electric and magnetic field, respectively.

In order to characterize the field modes on the gratings, we replaced the receiving horn with small wire antennas. One was 10 mm long dipole antenna made by setting apart the inner and outer conductors of a section of coaxial, while the other was 5 mm long monopole made by stripping off the outer conductor of the coaxial. The wire antennas were mounted on a 2D translational stage, and were brought as close to the rear surface of the grating as possible to detect the local fields $E_{y}$ and $E_{z}$ with designated orientations, as illustrated in Figs. 3(a) and 4(a). The stage was controlled to move within the $x-y$ plane at a spatial resolution $1 \times 1 \mathrm{~mm}^{2}$, so that the spatial distributions of the fields could be mapped point-by-point.

Besides the measurements, we also implemented the finite-difference time-domain simulations to compare with the measured results. On one hand, the simulations are used to justify the experimental measurements, especially for the detection of local fields which are disturbed inevitably by the wire antennas of length $0.27 a$ and $0.135 a$. On the other hand, they are also good supplements to the experiments where the local fields on the front surface are difficult to detect without interfering with the incident wave. The numerical simulations were done by employing a commercial software CONCERTO 3.5 (Vector Fields Limited, England,
2003). In the simulations, the periodic boundary condition was imposed and the perfect electrical conductor approximation was assumed.

## 3. Results and discussion

### 3.1. Transmission resonances on the grating of periodic slits

Figure 2(a) shows the measured and simulated transmission (magnitude and phase) spectrum of microwave through a grating of periodic slits (see the figure caption for the geometry of the grating) at normal incidence, where the overall agreement between the experiment and the simulation is good. Using the dimensionless unit $\lambda / a$, we recognize readily the Wood's anomaly $\lambda=a$ in the spectrum where the transmission minimum appears whenever the diffraction beam of first order propagates at the grazing angle. Immediately above the Wood's anomaly, $\lambda=1.03 a$, the simulation reveals a sharp peak with the transmission phase $\phi \approx-2 \pi$, while the measured result is a lower peak that occurs to a bit larger wavelength $\lambda=1.13 a$ and has the phase larger somewhat than $-2 \pi$. At a larger wavelength $\lambda=1.75 a$, both simulation and experiment display a broad peak of magnitude $100 \%$ and phase $\sim-\pi$.


Fig. 2. (color online) (a) Transmission magnitude and phase of EM wave through a grating of periodic slits with $a=37 \mathrm{~mm}, d=12 \mathrm{~mm}$, and $t=25 \mathrm{~mm}$ at normal incidence. (b) Snapshots of the $E_{y}$ distribution across a slit unit at two transmission resonances, $\lambda=1.75 a$ and $1.03 a$, in the simulation. The snapshot moment was when the field amplitude reached its maximum. Only one periodicity along the $y$-direction was shown. The field quantity was normalized with respect to the incident field $E_{0}$.

According to the peak positions, the former peak is considered to be induced by the SF resonance of the grating which comes from the structural coherence of the grating and involves a structural factor $G_{E}$ in the collective reaction of all apertures [11, 12], and the latter is caused by the Fabry-Perot-like (FP-like) resonance of each slit where the propagation of TEM wave is allowed. The features of these two resonances can be visualized and distinguished by investigating the relevant field modes on the structure at the peak frequencies. Figure 2(b) is the snapshots from the simulations for the $E_{y}$ distribution across a slit unit at these two resonances, where $E_{y}$ has been normalized to the incident field. The $\lambda=1.75 a \mathrm{map}$ exhibits a feature of FP resonance, where a wave node is in the midst of the slit along the $z$ direction, consistent with the corresponding transmission phase. It is noted that the $E_{y}$ field is subject to an enhancement at the entrance and the exit of the slit. For the $\lambda=1.03 a \mathrm{map}$, however, the field has two nodes inside the slit and hence gives rise to the $-2 \pi$ phase and a strong enhancement in the midst of the slit. In addition, it is noted that the intensity
modulation along the $y$-direction near the grating structure is obvious, which is due to the fact that the diffraction beam of first order begins pronouncing at $\lambda \sim a$ in a form of evanescent wave.

The large discrepancy between the measurement and the simulation for the SF resonance might be caused by the finite size effect of the sample, the limited quality/quantity of the periodicity, and the imprecision of the normal incidence. The SF resonance is intrinsic from the collective diffractions of infinite apertures, and its intensity and position are very sensitive to these imperfect factors in experiments [13, 14]. On the other hand, for the FP-like resonance which becomes localized on each aperture, the agreement is noted to be excellent.


Fig. 3. (color online) (a) Schematic picture of the 10 mm dipole antenna scanning the rear surface of the grating. (b) The measured magnitude distributions, $\left|E_{y}\right|$, on the rear surface of the grating at two transmission resonances. (c) The simulated magnitude distributions for the corresponding resonances. Only one periodicity along the $y$-direction was shown. The dash lines in (a) and (b) delineate the scanning region. The field quantity in (b) and (c) was normalized with respect to the incident field $E_{0}$.

We experimentally scanned the magnitude of $E_{y}$ component on the rear surface $z_{2}$ of the grating at the two resonances by directing the dipole antenna along the $y$-axis and recording the magnitude of the received signal, i.e. $|\mathrm{S} 21|$, as illustrated in Fig. 3(a). In this case, the parameter S 21 was proportional to the quantity, $E_{0}\left(x, y, z_{1}, \lambda\right) T(\lambda) F\left(x, y, z_{2}, \lambda\right) A^{\text {dipole }}(\lambda)$, where $A^{\text {dipole }}$ was a wavelength-dependent factor describing the receiving ability of the dipole. In order to extract the distribution function $F\left(x, y, z_{2}, \lambda\right)$ which characterized the magnitude of $E_{y}$, we did the following normalization procedure. The recorded quantity was divided by the incident field whose magnitude was proportional to $E_{0}\left(x, y, z_{1}, \lambda\right) A^{\text {dipole }}(\lambda)$, i.e. the signal which was received on the dipole after only the sample was removed and the detector was brought forward to the position $z_{1}$ of the front surface of the grating. On the other hand, the scattering loss caused by the finite size and the defect of the sample has the effect of decaying the transmitted signal. Thus, the field data were corrected through divided by the corresponding transmission magnitude $T(\lambda)$ at each resonance. The normalized and corrected data were plotted in Fig. 3(b) and were compared to the simulated results in Fig. 3(c). We found that the enhanced field distributes almost uniformly across the slit width for the resonance $\lambda=1.75 a$. For the SF resonance, the field is strongly enhanced only at two edges of the slit from the simulated result. Such unequal field distributions are the
manifestation of the distinct nature of these two resonances. The corresponding measured result did not resolve this spatial characterization, which is due possibly to those reasons for the discrepancy in the transmission spectra and the perturbation of the dipole detector to the local field.


Fig. 4. (color online) (a) Schematic picture of the 5 mm monopole antenna scanning the rear surface of the grating. (b) The measured distributions of $E_{z}$ component on the rear surface of the grating at two transmission resonances. (c) The simulated distributions of $E_{z}$ component across a slit unit for the corresponding resonances, where the snapshot moment was when the field amplitude reached its maximum. Only one periodicity along the $y$-direction was shown. The dash lines in (a) and (b) delineate the scanning region. The field quantity in (b) and (c) was normalized with respect to the incident field $E_{0}$.

We also scanned the $E_{z}$ distribution on the rear surface of the grating at the two resonances by directing the monopole antenna perpendicularly to the rear surface, as illustrated in Fig. 4(a). The measured results in Fig. 4(b) were for the real part of $E_{z}$ obtained from the real part of S 21 whose phase angle represented the relative phase of $E_{z}$. In this situation, the S21 data was proportional to $E_{0}\left(x, y, z_{1}, \lambda\right) T(\lambda) F\left(x, y, z_{2}, \lambda\right) A^{\text {monopole }}(\lambda)$ where $A^{\text {monopole }}$ described the receiving ability of the monopole antenna and $F\left(x, y, z_{2}, \lambda\right)$ characterized the $E_{z}$ distribution on the rear surface. The data was corrected for the scattering loss, i.e. divided by $T(\lambda)$, and they were also normalized to the incident field $E_{0}\left(x, y, z_{1}, \lambda\right) A^{\text {dipole }}(\lambda)$. In the normalization, an extra magnitude ratio $A^{\text {monopole }}(\lambda) / A^{\text {dipole }}(\lambda)$, due to using the different detectors, need be eliminated. We tested the receiving abilities of the dipole and the monopole through locating them at the identical position in the far field of the horn antenna and adjusting the right polarization orientation and recording the received signal. The value of this ratio was determined simply by comparing the magnitude of the received signal. The experimental results indicate an alternation of positive and negative values of $E_{z}$ along the $y$-direction within one period for both resonances, and a slightly stronger magnitude of $E_{z}$ at the resonance $\lambda=1.13 a$ than at $\lambda=1.75 a$. The corresponding simulated results are shown in Fig. 4(c), where the illustrated region is the $y-z$ plane to facilitate viewing the field on both surfaces of the grating simultaneously. Besides the opposite symmetry property of the field modes which coincides with their transmission phases, the $E_{z}$ magnitude is different significantly for these two resonances. For the FP-like resonance, nearly no $E_{z}$ field enhancement is noticed on both surfaces but near the edges of the slit. In sharp contrast, the massively enhanced $E_{z}$ field for the SF resonance dominates on each surface of the grating and exhibits a feature which is the standing wave resonance on the
length of one periodicity along the $y$-direction. Although the measured results show, to some extent, the difference in the $E_{z}$ magnitude for both resonances, the difference is not as drastic as the simulation. Possible reasons are suspected to be that the SF resonance is degraded by the imperfections of the grating and the perturbation of the detector, hence the $E_{z}$ field at $\lambda=1.13 a$ not appearing intense enough in the experiment.

This drastically enhanced field on the both surfaces of the grating along the transverse direction is consistent with the sharpness of the SF resonance peak in the transmission spectrum. From the simulations, we found that the $E_{y}$ and $E_{z}$ components oscillate synchronously, which means the resonance $\lambda=1.03 a$ not only intakes a part of incident EM energy into the slits to enhance $E_{y}$ there (see Fig. 2(b)), but also stores a quite large amount of EM energy on the surfaces to produce the intense $E_{z}$ (see Fig. 4(c)). Consequently, the resonance accommodates much more energy than the FP-like one, $\lambda=1.75 a$, where the surface field effect is absent, and correspondingly a higher quality factor Q or a narrower peak for the resonance $\lambda=1.03 a$ is seen in the transmission spectrum.

### 3.2. Comparison between periodic slits and hole array

It has found that the FP-like resonance may transit smoothly to the SF one in the $a / \lambda$ vs. $a / t$ plot as the ratio alt increases for the grating structure [15, 16]. This is because the former is determined mainly by the thickness $t\left(\lambda_{\mathrm{FP}} \sim 2 t\right.$ for the lowest order) whereas the latter by the periodicity $a\left(\lambda_{\mathrm{SF}} \sim a\right)$. These two resonances may interact with each other in the case of an appropriate ratio alt, and may also become well separated in the spectrum in other cases of alt. For $a / t<2$, the FP-like resonance appears at wavelength well above the Wood's anomaly, as the above slit grating. Given the value $a$, the FP-like resonance will shift towards the Wood's anomaly with decreasing $t$. For a reasonably large value $a / t>2$, there will only be the SFinduced peak at the long wavelength region above the Wood's anomaly, except one at the zero frequency.

We made another slit grating with larger $a$ and smaller $t$, see Fig. 5 caption. We obtained the transmission spectrum and the resonant $E_{z}$ field mode for this grating, as shown in Figs. 5(a1) and 5(a2). A peak with transmission phase $\sim-\pi$ is observed near the Wood's anomaly, from both experiment and simulation. The map of simulated $E_{z}$ field indicates the features of surface field enhancement and the $\sim-\pi$ phase. Both the peak position and the $E_{z}$ field enhancement point out it is a SF resonance. However, comparing with the SF resonance $\lambda=1.03 a$ in the previous slit grating, this resonance has the different phase $\phi \approx-\pi$ which implies the midst of the slit along the $z$-direction is now a node for $E_{y}$.

It is also interesting to compare this case with the grating of hole array where only SF resonance exists above the Wood's anomaly because of the cutoff wavelength of the holes [17]. We measured and simulated a sample of hole array, as picturized in Fig. 1(b). The results for the sample of hole array are shown in Figs. 5(b1) and 5(b2). As expected, the peak occurs at $\lambda \approx a$ and exhibits a strong enhancement of $E_{z}$ field on both surfaces. However, its transmission phase is $\sim 0$, which is in sharp contrast to the slit case in Figs. 5(a1) and 5(a2) and also different from the case of SF resonance of the previous slit grating, where $\phi \approx-2 \pi$. Accordingly, the field patterns and the surface charges derived from the EM boundary condition on perfect conductor [10] present the opposite symmetry about the mirror plane $z=\left(z_{1}+z_{2}\right) / 2$ in Figs. 5(a2) and 5(b2). The $\sim 0$ phase of the hole array may be understood from the fact that the hole has the cutoff frequency for EM wave propagation. When the EM wave of $\lambda \approx a$ transmits through the hole array, each hole acts as a barrier due to the wave frequency much lower than the cutoff frequency of the small hole, and the wave has to tunnel resonantly through the hole in a form of evanescent field. So the phase change of EM wave across the grating of hole array approximates zero.


Fig. 5. (color online) (a1) and (b1) are the transmission magnitude and phase of normally incident EM wave through the grating of periodic slits with $a=56 \mathrm{~mm}, d=18 \mathrm{~mm}$, and $t=19 \mathrm{~mm}$, and the grating of hole array with $a=20 \mathrm{~mm}, d=8 \mathrm{~mm}$, and $t=0.5 \mathrm{~mm}$, respectively. (a2) and (b2) are the simulated $E_{z}$ distributions across a slit unit and a hole unit at the peak frequencies, respectively. The snapshot moment was when the field amplitude reached its maximum. Only one periodicity along the $y$-direction was shown. The field quantity was normalized with respect to the incident field $E_{0}$. The induced surface charges are schematically illustrated nearby.

## 4. Conclusion

The 1D metallic gratings of periodic slits demonstrate two types of resonances (FP-like and SF ones) responsible for the transmission peaks located at particular wavelengths in the spectrum. The resonance-induced peaks were first identified through measuring the far-field transmission including the phase. Then, we experimentally characterized the local electric field on the rear surface of the gratings at the resonances by employing the near-field scanning technique. With the support of numerical simulations, we showed the specific features of the relevant fields at these two resonances. The field at the FP-like resonance is localized highly inside the slits and shows little effect of surface field enhancement, whereas the most significant feature of the SF resonance is the strong enhancement of $E_{z}$ field on the surfaces whatever the transmission phase at the resonance was. We compared the cases of the slit grating and the hole array. Although both support the SF resonance, their field modes have different symmetry, consistent with the transmission phase measured at the far-field. The difference in symmetry/phase is related to the cutoff property of the aperture which EM wave propagates through.

Recently such metallic gratings are applied as the template of shaping and enhancing local fields for trapping/sensing biomolecules or dielectric particles [18, 19]. Recognition of the positions on these metallic structures where local fields are vastly enhanced at particular frequencies is crucial to implement these applications. The analysis on the resonant field and the transmission phase here may provide the strategic guides for the determination of optimal positions: the aperture interior, or the aperture entrance, or the aperture vicinity on the surface.

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