Experimental investigation for field-induced interaction force of two spheres

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An apparatus is developed to study the interaction forces between two spheres under an external ac electric field. The interaction forces of a pair of spheres as the function of interspherical spacing, electric field strength, and electric field frequency are measured precisely. The results reveal that much stronger interaction can be obtained for metal spheres and high-dielectric spheres compared with the spheres with a very low dielectric constant, such as glass and polymer. The measured forces increase very steeply as the gap of the spheres decreases, and become much larger than those of available theoretical predictions when the two single-crystalline spheres of $SrTiO_3$ are closely spaced. The frequency dependence of the interaction force also shows an anomalous behavior. Our measurements indicate that a more accurate theoretical calculation should be performed to explain the experimental results. [DOI: 10.1063/1.1560556]

It is well known that the particle–particle interaction under an external electric field is an interesting issue for many fields experimentally and theoretically, for example, for the investigation of electrorheological (ER) fluids.[1–6] The electric-field-induced polarization gives rise to such an interaction force. However, it has not yet been precisely obtained experimentally even though quite a lot of theoretical work has been done in the last decades.[7–11] In 1993, Tan and Jones measured the interaction force between two 3.34-mm-diameter chrome steel balls under an applied magnetic field, and their experimental results can be explained successfully with multipolar theory.[12] However, up to now, there has been lack of systematic and accurate measurement of interparticle forces under an electric field, with the exception of Atten *et al.*, who calculated and measured the mutual force between two hemispheres fixed to the electrodes in a dc field.[13] In this letter, we report the experimental investigation regarding mutual forces between two identical spheres in an ac electric field with our recently developed apparatus.

The apparatus, shown in Fig. 1, consists mainly of two horizontally fixed copper plates (160.0 mm in diameter) separated with a gap of 49.1 mm by Teflon™ props, where the plates serve as the electrodes that are connected to a high-voltage transformer driven by a frequency-convertible power supply. The frequency and amplitude of ac output can be adjusted through a function generator and an amplifier, respectively. The plates are mounted on a stage driven by a computer-controlled elevator, which is used to adjust the spacing between two spheres with an accuracy of 0.001 mm. The spheres used in our experiments were all 6.30 ± 0.01 mm in diameter, of which the surfaces were polished carefully in order to avoid any discharge. Two spheres were glued onto the end of a thin Al_2O_3 insulating tubes, by which one sphere was fixed onto the bottom plate and another sphere was suspended to the below hook of the electronic balance through a small hole at the middle of the upper plate. Both the spheres were located in the center region of two plates by adjusting the lengths of the tubes. Two spheres were precisely arranged between two electrodes, with their vertical axes parallel to the applied electric field. In the experiments, the attractive force between the two spheres was measured with an electronic balance (sensitivity is 0.001 g). The whole apparatus was sealed in a plexiglass box into which the dry nitrogen gas was filled to protect measurements from moisture and dust.

Four kinds of bispherical coordinates, gold–gold, $SrTiO_3$–$SrTiO_3$, quartz-glass–quartz-glass, polytetrafluoroethylene (PTFE)–PTFE, were chosen for the measurements. For the couples of identical spheres separated with a gap spacing $\delta = 0.01$ mm under a 150-Hz ac field, the measured interaction force versus field strength is shown in Fig. 2. It can be seen that the interaction forces for the couples of gold and single-crystalline $SrTiO_3$ spheres are much stronger than

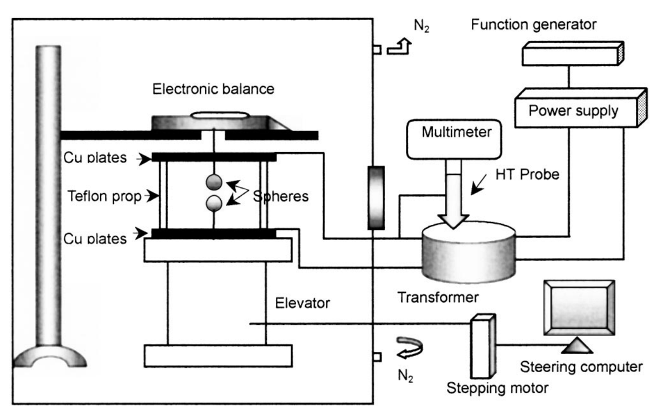

FIG. 1. Apparatus used to measure electric inter-particle forces between two spheres in an applied electric field.

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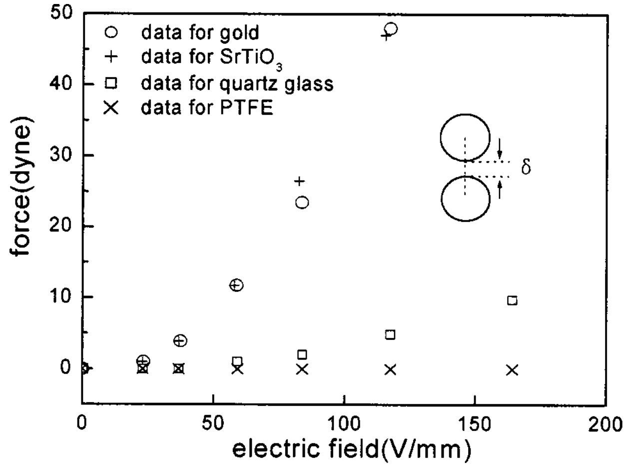

FIG. 2. Attractive force between two identical spheres at gap spacing δ (0.01 mm) for four samples: gold, $SrTiO_3$, quartz glass, and PTFE, vs the applied electric field.

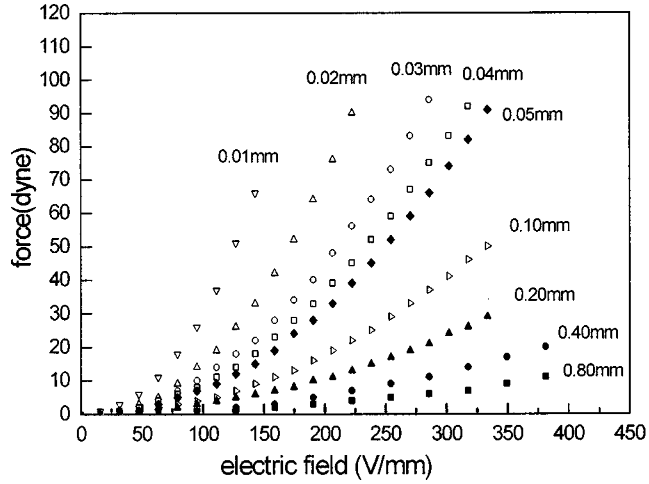

FIG. 3. Attractive forces between two identical spheres of $SrTiO_3$ at gap spacing δ (0.01–0.80 mm) vs the applied electric field.

those for the two quartz spheres and the two PTFE spheres, for which the interaction is almost zero for the sensitivity of the balance used for measuring forces. The obvious force differences between different pairs of spheres must be attributed to the polarization effect of the spheres under an ac electric field. That is, high dielectric constant of single-crystalline $SrTiO_3$ spheres ($\varepsilon = 294$) would result in much stronger induced dipole interaction than those of the quartz glass ($\varepsilon = 4.0$) and PTFE ($\varepsilon = 2.6$) spheres. We also note that the force between two gold spheres was almost no different from that between two $SrTiO_3$ spheres, despite the fact that there was a very distinctive difference in the electromechanical origin of the attractive forces between them. Here, we take the case of perfectly conducting spheres as a special case of the upper limit of particle polarizability. If two gold spheres are considered to be subjected to a uniform dc field in the range of the applied low-frequency field, the experimental fact will be accordant with the conventional dipole approximation because both perfectly conductive and high-dielectric spheres have almost the same value of the complex dismatch factor β.[5,14]

The spacing and field-strength dependence of interaction forces between the pair of identical single-crystalline $SrTiO_3$ spheres were further studied. The variation of interaction force (F) between two $SrTiO_3$ spheres with field strength (E_0) measured at different interspherical gaps is given in Fig. 3, where the electric field frequency is fixed to be 50 Hz and the gap between two spheres is from 0.01 to 0.8 mm. From the measurements, we find that the interaction force increases as the field strength increases and satisfies the relation of $F \sim E_0^2$, as predicted by the dipole approximation.[14–16] In fact, the mutual force can be normalized to E_0^2 versus gap spacing δ according to the experimental results shown in Fig. 3, which demonstrates that the normalized force has indeed nothing to do with the field strength (95.3–317.7 V/mm). This normalized curve and three kinds of calculations are plotted in Fig. 4. It is obvious that the experimental results are much different from the theoretical calculations for two models based on simple dipole and modified dipole approximation,[14,15] while the results are closer to the finite element analysis model.[16] However, it should be emphasized that all three models are no longer valid for the case of a very thin gap. The interaction increases very sharply as the gap spacing decreases and is much larger than the theoretical calculations when the normalized spacing δ/R (R are radii of the spheres) is less than about 0.03. This result indicates that these available calculations for the mutual forces should be improved when the spheres are arranged closely.

Frequency dependence of interparticle interaction under an ac electric field has not been well studied, although some experimental data were obtained in the case of ER fluids.[17,18] By employing our apparatus, we are able to measure the two-sphere interaction force as a function of frequency. As an example, the frequency dependence for a couple of single-crystalline $SrTiO_3$ spheres is shown in Fig. 5. During the measurement, the gap between the two spheres and the field strength were fixed at 0.01 mm and 25.2 V/mm, respectively. It is found that the force between two spheres increased as the frequency changed from 100 Hz to 1 kHz, where it is noted that the force measured at 1 kHz is over 40 times larger than that at 100 Hz. According to the conventional dipole–dipole model, the mutual force is just proportional to β^2, for the experiment:[17]

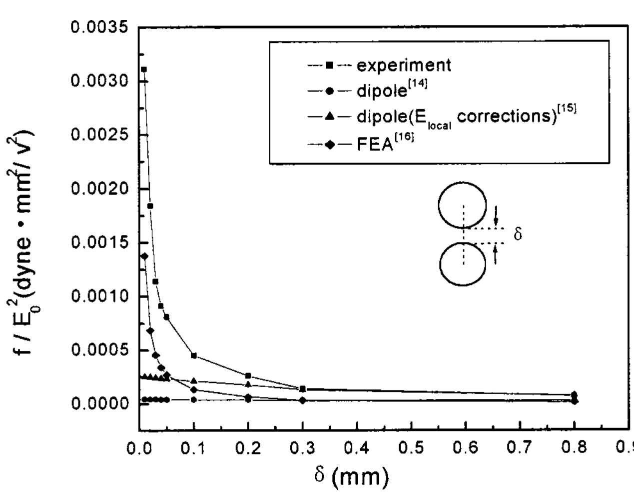

FIG. 4. Comparison between the experimental results and theoretical calculations based on the different modeling.

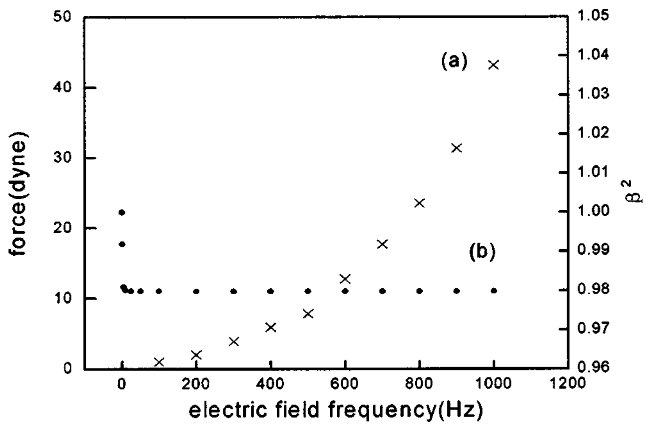

FIG. 5. Frequency dependence of the interaction force, where curve (a) shows the experimental results and (b) the theoretical prediction.

$$\beta^2=\frac{(\sigma_p-\sigma_m)^2+\omega^2\varepsilon_0^2(\varepsilon_p-\varepsilon_m)^2}{(\sigma_p+2\sigma_m)^2+\omega^2\varepsilon_0^2(\varepsilon_p+2\varepsilon_m)^2}, \tag{1}$$

where ε and σ represent the dielectric constant and conductivity, respectively. The subscript p indicates the sphere and the subscript m indicates the medium phase. ω is the radian frequency of the alternative field. Assuming $\varepsilon_m=1$ and $\sigma_m=0$ for the medium of N_2, we substituted them and the values of ε_p and σ_p of $SrTiO_3$ measured with HP 4192A LF impedance analyzer to Eq. (1). Consequently, the calculated frequency dependence was drawn as solid circles in Fig. 5. It is obvious that the predicted forces between two spheres are approximately independent of the electric frequency from 100 Hz to 1 KHz. The difference between experimental results and theoretical prediction based on the simple dipole–dipole model indicates that the present model is not valid to explain our experimental results. Therefore, we believe that there must be some other factors influencing the interaction force except for a complex dismatch factor of two phases.

In conclusion, by using our apparatus, the interaction forces between two spheres under an ac electric field are measured for different pairs of spheres. The respective dependence of the forces on interspherical spacing, field strength, and electric frequency is also given. The results demonstrate that the available theoretical calculations are not valid to fit the behaviors of the interaction forces, especially for the case of the spheres being closely spaced. A more quantitative theoretical study needs to be performed to explain the measured results.

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