In the present study, we show that in two-dimensional (2D) photonic crystals the periodic structure can be utilized to tune the birefringence as a function of frequency, in the regime where scattering is significant. Thus both the form factor and the structure factor can affect the birefringence and the interplay between the two (e.g., for systems consisting of 2D photonic crystals composed of anisotropic materials) is envisioned to exhibit rich possibilities.

Exactly analogous to electronic systems, photonic crystals, and metamaterials are artificial structures relying on periodicity and local resonances for their manifest characteristics. When a photonic crystal is constructed according to different periodic patterns, the effective dielectric constant of the structure, evaluated at the long wavelength limit, can provide us with a means to relate the PC band structure and the strength of the birefringence. It is also shown that the microwave birefringence can be tuned as a function of frequency by utilizing the band structures of a two-dimensional photonic crystal consisting of metallic cylinders arranged in a two-dimensional square lattice. By measuring the transmission and mapping the field inside of the sample, the birefringence was directly determined. An agreement between band structure calculations and experiment measurements was achieved, with the frequency at the center of transmission band showing the least birefringence and the frequency at the band edge exhibiting the most.

For a homogeneous sample with an anisotropic dielectric constant, the equation for the phase velocity may be expressed as

$$\frac{s_1}{v_1^2} + \frac{s_2}{v_2^2} + \frac{s_3}{v_3^2} = 0,$$

where $k = k^2$, $|\beta| = 1$, $v_1^2 = \omega^2/k^2$, $v_2 = 1/\mu\varepsilon_\perp$, and $s_j$ is the Cartesian component of the unit vector $\hat{s}$ along the direction of the wave vector $k$. From symmetry of our sample, we have $\varepsilon_\perp = \varepsilon_\parallel = \varepsilon_1 = \varepsilon_\|$, and $\varepsilon_\perp = \varepsilon_\parallel = \varepsilon_2$, where the subscripts $\perp$ and $\parallel$ means perpendicular and parallel to the cylindrical axis, respectively.

By solving Eq. (1), we obtain two solutions, denoted by superscripts (1) and (2)

$$v_j^{(1,2)} = v^2 \quad \text{and} \quad v_j^{(2,2)} = \sin \theta v^2 + \cos \theta v^2,$$

where $\theta$ is the angle between the incident wave’s propagating direction and the $z$-axis, $v_1^2 = v_2^2 = v_\parallel = 1/\varepsilon_\parallel \varepsilon_\|$, and $v_3^2 = v_\perp^2 = 1/\mu\varepsilon_\perp$. The optical axis is defined as the direction along which the phase velocities of both the ordinary beam (denoted as $O$, with velocity $v^{(1)}$) and the extraordinary beam (denoted as $E$, with velocity $v^{(2)}$) are the same. Accordingly, the $z$-axis is the optical axis of our sample.

The 2D photonic crystal examined in our experiment consists of $9 \times 9$ metallic cylinders each 8 mm in diameter arranged with a lattice constant of 30 mm. The structure is illustrated in Fig. 1. Two horn antennas were used as the microwave source and receiver. The sample was placed in the middle of the two horns. For mapping the local phase distribution, the horn receiver was replaced by a small dipole antenna (a dipolelike receiver), affixed so as to move along the $x$-$y$ plane, controlled by a computer. The local field’s phase pattern inside the photonic crystal was scanned for the plane indicated in Fig. 1.

FIG. 1. (Color online) Schematic drawing showing the sample geometry with the measurement configuration.
We employ the Finite-Difference Time-Domain (FDTD) approach, which is particularly good at handling dispersive media such as metal.

To obtain the band structure, we use random initial data at discrete nodal points in the unit cell, and propagate the fields forward in time by using the Maxwell equation with the perfect metallic boundary condition at the rod surfaces (which is an excellent approximation for waves in the microwave regime) and the Bloch boundary condition at the cell boundaries

$$\tilde{E}(\vec{r} + \vec{R}_n) = e^{i\vec{k} \cdot \vec{R}_n} \tilde{E}(\vec{r}),$$

where $\tilde{E}$ denotes the electric field, $\vec{k}$ is the Bloch wave vector in the first Brillouin zone, and $\vec{R}_n$ is the lattice vector of the unit cell. After running the FDTD code for a sufficient long time, the time series $\tilde{E}(t)$, summed over all the points in the unit cell, is Fourier-transformed into the frequency domain. The peaks in the frequency domain denote the frequency values corresponding to the particular chosen $k$. In our experiment, the transmission spectra were first measured by setting the receiver’s $E$-polarization perpendicular to that of the source [see the top illustration in Fig. 2(a)]. In this case, free space gives very low signal as expected, shown by the black line. When the sample was placed between the two horns (red line), a weak transmission peak appeared at 8.3 GHz. By maintaining the orthogonality of the source and receiver polarizations but rotating the source horn’s $E$-polarization to 45° from the optical axis (the $z$-axis) of the sample, the amplitude of transmittance increased rapidly to 100% at 8.25 GHz. We conclude that for the case illustrated at the top panel of Fig. 2(a), the birefringence is much weaker than it is in the case illustrated at the bottom of Fig. 2(a). In theory, for the top case, the birefringence should not occur. Hence the appearance of a peak is probably due to the misalignment of the cylinders in the crystal as well as sample’s finite size. We also note from Fig. 2(a) that the optical axis of our sample is indeed parallel to the $z$-axis.

To observe the differences between the $O$-component and the $E$-component for polarized incident wave, we set the polarization of the source at 45° angle to the $z$-axis, whereas the polarization of the receiver was first set parallel (to detect the $E$-component) and then perpendicular to the $z$-axis (to detect the $O$-component). The measured transmission spectra are given by the four curves in Fig. 2(b). The birefringence phenomenon is indeed confirmed by the divergence of the green and black curves at a number of frequencies. Moreover, it is clear that the birefringence is highly frequency dependent.

We have calculated the band structure of our sample, shown in Fig. 3(d). It is seen that there is a stop band from 7 to 8 GHz for the TE mode.

In order to have a visible image of the birefringence, we designed and constructed a set of scanning devices for local field mapping inside of the sample. This could easily be realized by means of moving a dipole antenna throughout the plane indicated in Fig. 1. LabVIEW software was used to
ranging from 35° to 45° relative to the Fig. 1. During the measurement, the polarization of the different points in intervals between the rows parallel to the panel is for the TM mode (E-component) and the TE mode (O-component) at the same frequency—larger the separation, larger the magnitude of the birefringence. Thus in Figs. 3(a)–3(c) we see that as the frequency increases from 8.06 to 8.50 to 8.94 GHz, the angle of deviation between the two modes varies from 6° to 4° and then back to 6°. This variation is noted to correspond very accurately with the band structure predictions, based on the separation between the TE and TM modes, as shown in Fig. 3(d).

The field mapping shown in Fig. 3(a) is also noted to display disorder. This is due to the fact that over a considerable range of frequencies there are two TM modes as shown in the band structure, Fig. 3(d). Because of this degeneracy, the two states inevitably interfere with each other, leading to the “disorder” seen in the field pattern. However, as one of the TM mode’s dispersion curves displays a (upper) band edge at the Γ point, its wavelength is inevitably larger, implying a much less disordered field pattern at the higher (8.94 GHz) frequency range.

We have also studied another sample comprising a 6 × 6 simple square lattice composed of metallic cylinders with diameters of 25 mm and lattice constant of 43 mm. The local field distribution, mapped using the same approach as before, is shown in Fig. 4(a). The pertinent band structure is shown in Fig. 4(b). From the separation between the TM and TE modes at the same frequency, we find that at 5.72 GHz there can be a fairly large birefringence. Indeed, from the field pattern shown in Fig. 4(a) the birefringence in this case is larger than those found in the previous sample comprising smaller-diameter cylinders.

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From our calculated band structure it is found that the magnitude of the birefringence is directly related to the separation between the TE mode (the O-component) and the TM mode (the E-component) at the same frequency—larger the separation, larger the magnitude of the birefringence. Thus in Figs. 3(a)–3(c) we see that as the frequency increases from 8.06 to 8.50 to 8.94 GHz, the angle of deviation between the two modes varies from 6° to 4° and then back to 6°. This variation is noted to correspond very accurately with the band structure predictions, based on the separation between the TE and TM modes, as shown in Fig. 3(d).

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