Tunable band gap properties of planar metallic fractals

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We report the effect of finer structures, or more levels, in a particular type of planar metallic fractal pattern on the stop bands or band gaps in the transmission spectrum of electromagnetic wave. It is found that the band gaps shift to low frequencies with the increase of levels, showing tunable band gap properties. In addition, we find from both experiments and theoretical calculations that the bands appearing in long wavelength range are determined by individual patterns which compose a 6×6 fractal array, while those bands located in the range of short wavelength are strongly influenced by their neighboring residents. Our experimental observations are in good agreement with those of finite difference time domain simulations. © *2004 American Institute of Physics.*
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Electromagnetic (EM) spectral band gaps can be generated by either photonic band gap (PBG) materials, employing Bragg scattering,[1–5] or by frequency selective surfaces (FSS), owing to their intrinsic resonances.[6] It is well known that the thickness and the lateral dimensions of PBG materials must be a few times larger than those of the wavelength where EM wave is forbidden due to the Bragg mechanism, while ordinary FSS typically operates at one single frequency range. Recently, we have reported that a particular kind of planar conducting fractal exhibits multiple stop bands and pass bands over an ultrawide frequency range.[7] In this article, we report the influence of finer structure (i.e., number of levels of the fractal) on the band gap properties. We find from both experiments and finite difference time domain (FDTD) simulations that the stop bands or band gaps in the transmission spectrum are downshifted to low frequency when fractal levels increase. We also show the interaction of the fractals in the sample, which is made of an array of fractals instead of a single fractal.

Our fractal pattern[7] was generated from a master line, defined as the first level of the structure, placed parallel to the y axis in the xy plane. The $(k+1)^{\text{th}}$ level structure contains 2^k lines, with the midpoint of each perpendicularly connected to the ends of the k^{th} level lines. The length of the $(k+1)^{\text{th}}$ level lines was scaled from that of the k^{th} level line by a factor of 2(1) if k is an even (odd) number. With increasing number of levels, the pattern approaches a space-filling curve that tiles a two-dimensional square. In the experiments, we prepared six sets of metallic fractal patterns. The number of their levels consecutively varies from 8 to 13, while the thickness of the metal lines in all patterns is the same: 0.05 mm and the first-level line length: 17.5 mm. The linewidths were different from one to another (see Table I). A single fractal pattern called a resident was fabricated on a $40\ \text{mm}\times40\ \text{mm}\times0.2\ \text{mm}$ dielectric substrate by the shadowing/masking/etching technique. Each sample was a 6×6 array, consisting of 36 residents with identical levels. The band gap properties were determined by a network analyzer (Agilent 8722ES), where two ports were connected to two double-ridged waveguide horn antennas (HP11966E), one of which acted as microwave generator while another identical receiver horn was placed at a distance of 100 cm from the source. The fractal plate was placed on a stage, 15 cm from the receiving horn, which can be rotated about the z axis by an angle θ (**E** perpendicular to the plane of incidence).

The open circles in Figs. 1 and 2 denote the measured transmission when the linearly polarized plane EM wave is normally incident to the sample with E field along the y axis and the x axis, respectively. Curve $e8y$ in Fig. 1 shows three stop bands, marked as 1, 2, 3, appearing at the frequencies (in GHz) of 0.92, 2.47 and 6.58, respectively. These bands downshift a little to low frequencies when the levels of fractal change from 8 to 9 (see curve $e9y$ for comparison). From curve $e10y$, we note that a new gap marked as 4 appears coming from the high frequency range. We also note that in curve $e11y$, gap 1 disappears and moves out of the measurement range. The further shift of gaps 1, 2 and 3 toward the low frequency is observed as the fractal levels increase up to 13. The same phenomenon occurs in the case that E field is polarized along the x axis, which is shown in Fig. 2.

In order to explain our experimental observation, the effect of increasing levels of the fractal pattern on the locations of the stop bands in the frequency domain is studied by the FDTD method which is usually for microwave simulations.[8–12] The frequency response of the fractal at normal incidence can be modeled by a thin homogeneous plate with an effective dielectric constant of the form

$$\varepsilon_{\text{eff}}(f)=\varepsilon_0+\sum_{\ell}\frac{\beta_\ell}{f_\ell^2-f^2}, \tag{1}$$

where f represents frequency, the index ℓ runs over all band gaps, and ε_0, f_ℓ and β_ℓ are inputting parameters.

In simulation, we replaced the sample, a 6×6 array, with its element (i.e., a resident) and let a plane wave with

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TABLE I. Geometry of the six sets of metallic fractal patterns.

Number of levels	8	9	10	11	12	13
Length of first level (mm)	17.5	17.5	17.5	17.5	17.5	17.5
Linewidth (mm)	0.5	0.5	0.2	0.2	0.1	0.1
Line thickness (mm)	0.05	0.05	0.05	0.05	0.05	0.05

specified polarization direction incident upon the resident due to the existence of polarization dependence on x and y directions. For the microwave frequency we treated the fractal metal as a perfect conductor, and the substrate dielectric constant was taken to be 5.3. In order to achieve convergent results, FDTD cells of different sizes were adopted during the EM space discretization for different patterns. For example, the maximum cell was set to be $0.4\times0.4\times0.5$ mm for the case of sample $e8y$ in Fig. 1. It was observed that band gap frequencies indeed downshifted to low frequencies as the number of levels increased. We have revealed the multiplicity of band gaps and the subwavelength property originating from a series of resonances intrinsic to the metallic fractal pattern.[7] Actually, incident EM field excites currents in the metal lines of the fractal, with the induced current amplitude reaching its maxima at those resonances, which results in weak transmission. In addition, it is clear that every resonance has currents excited mainly along the metal lines of a particular level, flowing toward higher levels of the fractal. Therefore, low frequency resonances are essentially determined by the very long, continuous metal lines starting from the low level in the fractal; while high frequency resonances are governed by other metal lines that start from the high level. So resonances, which correspond to stop bands or band gaps in spectra, will shift to a lower frequency, since the relevant metallic path is longer after more levels (finer structures) are added into the fractal pattern. The simulation results are plotted as the solid lines in Figs. 1 and 2, which are

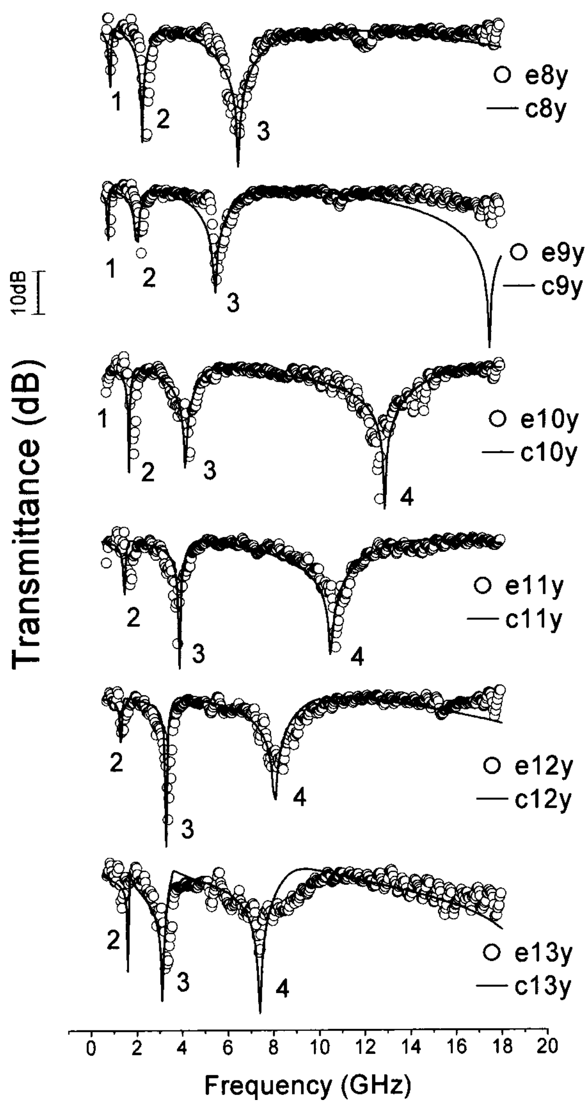

FIG. 1. The measured (circles) and simulated (lines) transmittance for the six sets of metallic fractal patterns. In the legend, the letters "e" and "c" represent "experimental" and "calculated," the integer refers to the number of levels, and "y" means the polarization direction of E field.

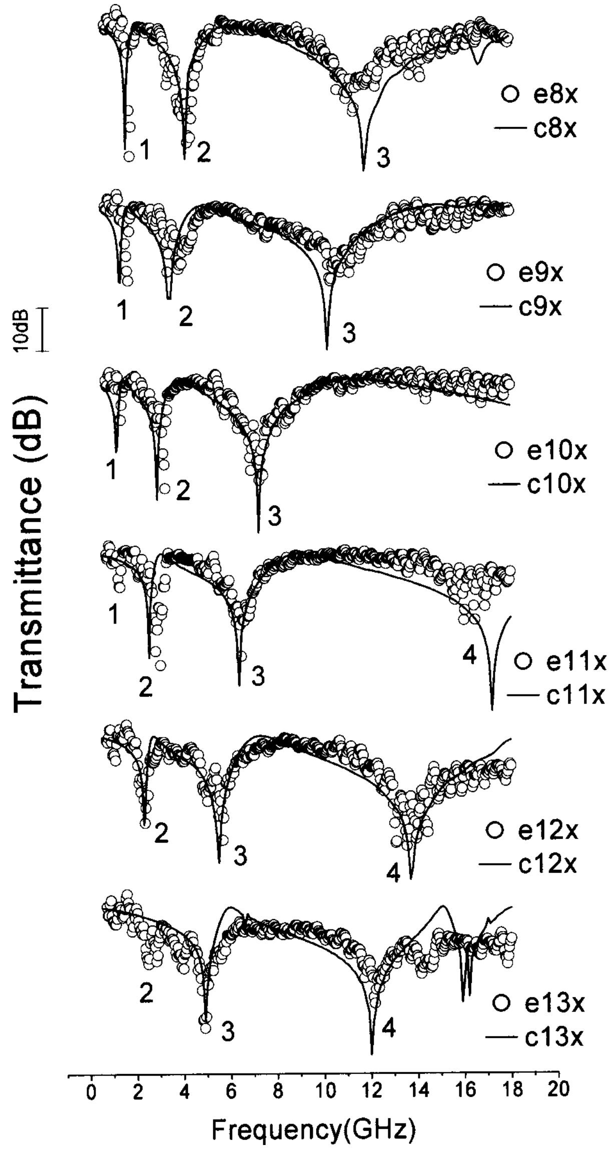

FIG. 2. The measured (circles) and simulated (lines) transmittance for the six sets of metallic fractal patterns. In the legend, the letters e and c represent experimental and calculated, the integer refers to the number of levels, and "x" means the polarization direction of E field.

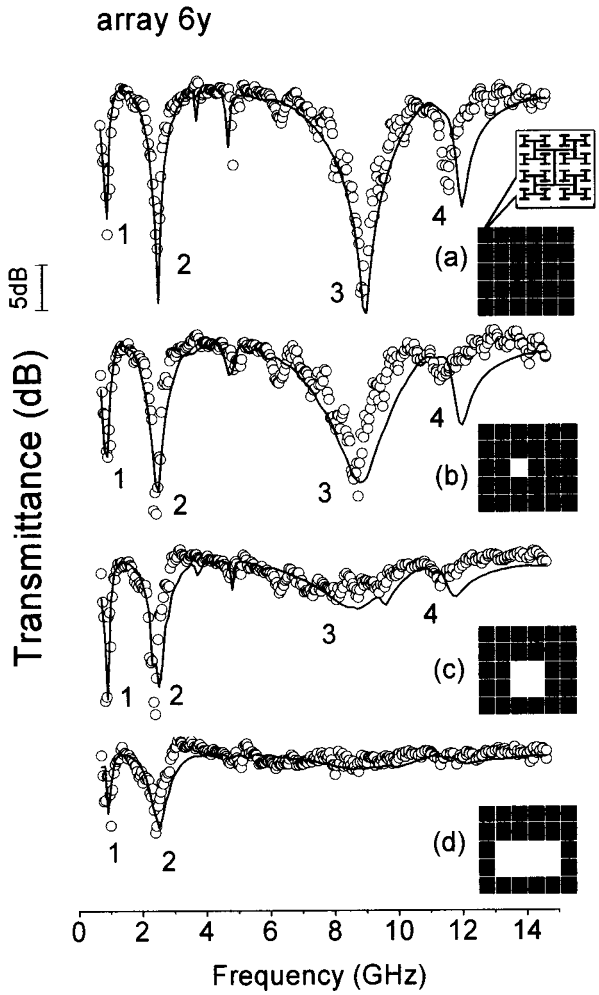

FIG. 3. Curves in (a), (b), (c) and (d) are the measured (circles) and simulated (lines) transmission for a 5×6 array of six-level metallic fractal patterns under four configurations shown as the insets (a black block represents a resident, i.e., a six-level fractal pattern). The polarization of E field is along the y axis.

in good agreement with the experimental observations.

An interesting phenomenon occurs if several residents are removed from such an array. We measured the transmission spectra of an ensemble of six-level fractals under four configurations when the incident E field was polarized along the y direction, which are shown as open circles in Fig. 3. The fractal of six levels has 21.8 mm first-level line length, 0.6 mm linewidth and 0.03 mm thickness, which is deposited on a $42.5\times42.5\times0.45$ mm substrate. A complete 5×6 array produces four stop bands marked as 1, 2, 3 and 4, which can be seen clearly in Fig. 3(a). However, with one resident removed from the array, the band characters of 1 and 2 remain almost unchanged while the stop bands 3 and 4 become weak and shift a little toward the low frequencies, as shown in Fig. 3(b). Figure 3(c) shows that the bands 2, 3 and 4 shift to low frequencies when four residents are missed. Moreover, if eight elements are absent, as in Fig. 3(d), the stop bands 1 and 2 are weakened and bands 3 and 4 become too weak to be recognized. However, we note from Figs. 3(a)–3(d) that bands 1 and 2 (located at low frequencies or long wavelengths) always remain unchanged while bands 3 and 4 change substantially.

We simulated the four cases mentioned above by FDTD in order to explain the experimental observations. Simulation results, the solid lines shown in Fig. 3, agree well with the measured curves, and repeatedly manifest that stop bands are shifted to a lower frequency and the attenuation at stop bands becomes weaker with more residents removed from the fractal array. In particular, we found that the stop band at high frequency is much more sensitive to the configuration than that at low frequency. We attribute this to the fact that in the simulation the stop band located at low frequency (band 1) results from the intrinsic resonance of one individual resident and not strongly associated with its neighbors, while those bands located at high frequency are much strongly influenced by their neighboring situation due to short wavelength. The reason for this is that, for the space-filling curve of the fractal pattern, the length of the continuous metallic line where the resonant current flows is much longer than that of a resident dimension of the fractal, therefore, the impurity of the pattern has almost no influence on the resonate frequencies at low frequency range such as those of gaps 1 and 2, while the wavelengths of gaps 3 and 4 are compatible with the lengths of higher level substructures thus, it is obvious that these bands are strongly affected from high frequency to low in turn, with the area of the blank impurity caused by absent units increasing.

In short, through both experiments and FDTD simulations we studied the effect of increasing the number of levels in the fractal pattern on stop bands in the transmission spectrum, and found that stop bands may shift when the number of levels increased. We also observed that the change of stop bands at high frequency related to global symmetry is more dramatic than that at low frequency related to local order when there are some elements absent from the sample array, which was supported by the simulations. The underlying physics is due to the intrinsic property of our fractal pattern.

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[1] E. Yablonovitch, Phys. Rev. Lett. **58**, 2059 (1987).
[2] S. John, Phys. Rev. Lett. **58**, 2486 (1987).
[3] J. D. Joannopoulos, R. D. Medde, and J. Winn, *Phontonic Crystals* (Princeton University, Princeton, NJ, 1995).
[4] J. S. McCalmont, M. M. Sigalas, G. Tuttle, K. M. Ho, and C. M. Soukolis, Appl. Phys. Lett. **68**, 2759 (1996).
[5] W. Y. Zhang, X. Y. Lei, Z. L. Wang, D. G. Zheng, W. Y. Tam, C. T. Chan, and P. Sheng, Phys. Rev. Lett. **84**, 2853 (2000).
[6] B. A. Munk, *Frequency Selective Surfaces, Theory and Design* (Wiley, New York, 2000).
[7] W. Wen, L. Zhou, J. Li, W. Ge, C. Chan, and P. Sheng, Phys. Rev. Lett. **89**, 223901 (2002).
[8] K. S. Yee, IEEE Trans. Antennas Propag. **14**, 302 (1966).
[9] S. Fan, P. R. Villeneuve, and J. D. Joannopoulos, Phys. Rev. B **54**, 11245 (1996).
[10] A. Taflove and S. C. Hagness, *Computational Electrodynamics: The Finite-Difference Time Domain Method* (Artech, Norwood, 2000).
[11] J. Pendry, J. Phys.: Condens. Matter **8**, 1085 (1996).
[12] Simulations were performed using the software package CONCERTO, developed by Vector Fields Limited, England, 2003.