Experimental determination for resonance-induced transmission of acoustic waves through subwavelength hole arrays

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We measured acoustic transmissions through subwavelength hole arrays fabricated on brass plates at normal and oblique incidence. It is found experimentally that the transmission phenomena for the hole array in thin plate case are analogous to the previously observed enhanced transmission of electromagnetic waves [Ebbesen et al., Nature (London) 391, 667 (1998)]. While for the hole array in thick plate case, the transmission peaks of acoustic wave occur well below Wood’s anomalies, and the spectrum characteristics reveal a Fabry–Pérot-like resonance. An effective fluid approach is conceived and it can well describe the transmission properties of the hole array in thick plates within a range of incidence angle. © 2008 American Institute of Physics. [DOI: 10.1063/1.2951457]

I. INTRODUCTION

Enhanced transmission of light through a lattice of subwavelength holes fabricated on a metallic film was observed by Ebbesen and co-workers in 1998,1–8 where the optical transmission can be much larger than the area fraction of the holes at specific frequencies. Since then, the remarkable phenomenon has inspired a tremendous amount of attention and works on resonant transmissions of electromagnetic (EM) waves through various apertures on either metallic or dielectric structure.9–14 Phenomenologically, various observed transmission resonances are associated with two geometrical factors: structural factor emerging globally from the lattice periodicity and aperture factor owned locally by the individual unit.15–18 Structural-factor-related resonances typically have the transmission wavelength comparable to the lattice constant and are dependent strongly on the incidence angle. In sharp contrast, aperture-factor-related resonances have the wavelength determined mainly by the transversal/longitudinal dimensions of the aperture and are not sensitive to the incidence angle.

In this paper, we investigated experimentally the acoustic wave transmissions through a two-dimensional array of subwavelength holes, considering that acoustic and EM waves share a lot of wave phenomena. However, they have something in difference. In nature, acoustic wave is a scalar longitudinal wave in inviscid fluids, while EM wave is a vector transverse wave. Consequently, a subwavelength hole has no cutoff for acoustic wave, but does for EM wave, which underlies the distinct transmissions of acoustic/EM waves through a hole in an ideally rigid/conducting screen. The acoustic transmission approaches a constant, $8/\pi^2$, whereas the EM transmission goes to zero, with decreasing the ratio $d/\lambda$ (hole diameter/wavelength).19,20

Transmission/diffraction by an acoustical grating is an old problem, and the previous investigations addressed some cases: one-dimensional periodic slits in a rigid screen,21 a single hole in a thick wall,22 and a one-dimensional grating composed of parallel steel rods with finite grating thickness.23 Here we measured the acoustic transmissions through a two-dimensional array (square lattice) of subwavelength hole fabricated on either thin or thick brass plates at normal and oblique incidence. It is found that the acoustic transmission phenomenon for the hole array in thin plates is analogous completely to the case of EM wave,1,2 except for the transmission phase. For the hole array in thick plates, the transmission peaks are related to the Fabry–Pérot-like (FP-like) resonances inside the holes and can occur to the frequencies well below Wood’s anomalies. An effective medium model is derived, where the material parameters of the new effective fluid is scaled from those of the hole-filled fluid by a geometrical value. The effective fluid model may describe well the transmission properties of the hole array with large thickness within a range of incidence angle.

II. SAMPLES AND MEASUREMENTS

In our experiments, a square lattice of circular holes with diameter, $d$, and lattice constant, $a$, was drilled on a smooth brass plate of thickness, $t$. The schematic picture of the unit cell is illustrated in the inset of Fig. 1. The measurements of far field transmissions of acoustic waves in the ultrasonic frequency regime were performed in a large water tank. Two immersion transducers were employed as ultrasonic generator and receiver, and the sample was placed at a rotation stage located between the two transducers at an appropriate distance. The sample could be rotated so that the oblique incidences were measured. The ultrasonic pulse was incident upon the sample and the transmitted signal was collected by the receiver, collinear with the incident wave. Transmission magnitude, $T$, and transmission phase, $\varphi$, of the sample were...
obtained by normalizing the Fourier transformed spectra of the transmitted signal through the sample, $|A_i(f)|\exp\{j\varphi_i(f)\}$, with respect to the signal through the water background (without the sample in place), $|A_b(f)|\exp\{j\varphi_b(f)\}$, where $f$ is the frequency and $j^2=-1$. Consequently, $T=|A_i(f)/A_b(f)|$ and $\varphi=\varphi_i(f)-\varphi_b(f)-2\pi ft/c$ ($c=1490$ m/s being the speed of acoustic wave in water).24

In this paper, the term “transmission” when referring to the spectrum means the amplitude ratio, $T$, of the transmitted and the incident waves. To figure out whether the transmission is enhanced, we need calculate the ratio of transmitted energy flow to the energy flow incident on all holes. Within a unit cell, the ratio can be expressed by

$$I_a/\pi d^2/4 = |A_i/A_b|^2 \pi d^2/(4a^2) = T^2 \xi = T \xi^{-1}$$

where $I_i (A_i)$ and $I_b (A_b)$ denote the intensities (amplitudes) of transmitted and incident acoustic waves, respectively, and $\xi = \pi d^2/(4a^2)$ is the area fraction of the holes. We call the squared transmission magnitude $T^2$ as transmittance $T$ representing the acoustic intensity transmission. If $T/\xi > 1$, then the enhanced transmission is obtained. Thus, in the following discussions we use the transmittance spectrum whenever comparing with the area fraction of the holes.

III. RESULTS AND ANALYSES

A. The thin plate case

First we measured the acoustic transmission of a hole array with the parameters $d=0.5, a=1.5$, and $t=0.5$ mm. Figure 1 shows the transmittance of the hole array at normal incidence, compared to the transmittance of a smooth brass plate with identical thickness. For the smooth brass plate, very low transmittance is seen because of the acoustic impedance mismatch ($\eta_{\text{brass}}/\eta_{\text{water}}=25$). It is noticed that the transmittance rises at lower frequencies, which indicates a thin brass plate can block acoustic waves of very long wavelength or very low frequency. This fact is different from the EM case where a sheet of metal as thin as skin depth works well. For the hole array, a pronounced peak is seen at 0.85 MHz and followed by a transmittance zero close to 1.0 MHz which is just Wood’s anomaly $\lambda=a$. The peak has the transmittance (68%), much larger than the area fraction (8.7%) of holes occupation in the array structure, and shows an acoustic transmission enhancement through the hole array, similar to the EM case.1

We also investigated the dependence of the transmission peak on the lattice constant. Figure 2(a) shows the normal transmissions of the hole arrays with identical lattice constant $a=2.0$ mm and different hole diameters. The transmission peak and two Wood’s anomalies (pointed by arrows) are identified at $\sim 0.75$ and $\sim 1.1$ MHz. With the larger diameter holes ($d=1.2$ mm), the peak becomes more pronounced. For comparison, the transmission curve of the array of $a=1.5$ mm is replotted in Fig. 2(b). It is clear to show that the peaks and Wood’s anomalies downshift to lower frequencies as the lattice constant increases. In Fig. 2(a), we also plotted the measured transmission phase $\varphi$ for the hole array of $d=1.2, a=2.0$, and $t=0.5$ mm, and found $\varphi=-0.98\pi$ at the peak frequency. The approximate $-\pi$ phase change reveals the oscillations of the acoustic field on the front and rear surfaces of the plate are out of phase, which is distinct from the corresponding characteristic in the EM case. For EM wave transmitted through a hole array, the hole acts as barrier due to the transmission frequency much lower than the cutoff frequency of the hole, and the wave has to tunnel through the hole in a form of evanescent field. So the phase change of the EM wave across the holey film/plate assumes nearly zero.25 This difference in transmission phase bares the distinct behaviors of a hole to acoustic and EM waves, again.

Figure 3 shows the transmission spectra at oblique incidence measured with the incident angle $\theta$ varying from 0° to 25° for the hole array of $d=1.2, a=2.0$, and $t=0.5$ mm. The obliquity occurred along the [1,0] direction of the array, as illustrated in Fig. 3(a). The transmission map is plotted as a

![Image](https://example.com/image.png)
function of both the frequency and the incidence angle. The predicted variation of Wood’s anomalies versus angle is plotted as solid lines and is superposed on the map. Derived from the conservation of momentum, the variation relation reads

$$\alpha f^{(l,m)} = (l \sin \theta + \sqrt{l^2 + m^2 \cos^2 \theta})/\cos^2 \theta,$$

(2)

for the Wood’s anomaly $f^{(l,m)}$ of order $(l,m)$. It is seen from the map that the measured shifting of Wood’s anomalies with the incidence angle agrees well with the solid lines. On the other hand, the peaks exhibit a strong angle-dependent behavior in the same way as Wood’s anomalies.

In recent investigations, it is demonstrated that the structure factor (SF) resonance can be responsible for enhanced transmissions of EM waves through subwavelength hole arrays.$^{15,16}$ However, for the role of any surface wave, supported by the grating medium, playing in the enhanced optical transmission process, there are different views.$^{26–28}$ We have concluded that the acoustic surface wave at the brasse-water interface might play no role in the present transmission phenomenon, and shown that the SF resonance holds for acoustic waves by generalizing the proof of EM waves.$^{29,30}$ The SF resonance has some spectral features: the resonant wavelength is determined essentially by the lattice constant and is very sensitive to the incidence angle with accompanied by Wood’s anomalies. Here, the experimental results for the enhanced acoustic transmission through the hole array in the 0.5 mm thick plate manifests the features of SF resonance.

**B. The thick plate case**

When the plate thickness becomes larger, the situations begin to divide for two types of waves. For EM wave, the transmission peak will diminish after the metallic film/plate becomes thick enough because the holes have the cutoff. In sharp contrast, there is no cutoff for acoustic waves to propagate through the holes. When the thickness is large enough, for instance $t=2.3$ mm, there can be multiple transmission peaks well below the Wood’s anomaly, as shown in Figs. 4(b) and 4(c). Their spectra show the typical characteristics of FP resonance. In fact, these transmission peaks are caused by standing-wave-formed resonances of the acoustic wave establishing inside the hole channel. However, these resonances undergo a tuning, to some degree, by diffractive evanescent waves parasitical to a grating, and consequently deviate from the ordinary FP conditions while the plate thickness becomes comparable to the lattice constant.$^{29}$ Under the limit of thick plate (the ratio $a/t \rightarrow 0$), they coincide with the FP resonance conditions.

For a very small $a/t$ ratio, these resonance wavelengths are much larger than the lattice constant, allowing us to take a view of effective media. Here we employ a simple argument in the same fashion as the EM case$^{31}$ with the assumption of brass plate being rigid, and find that the structure of an array of holes fabricated in a rigid plate and filled with a fluid (mass density $\rho_0$ and bulk modulus $\kappa$) may be viewed as an effective fluid with the same thickness, effective mass density $\tilde{\rho}_0$ and bulk modulus $\tilde{\kappa}$, as shown schematically in Fig. 4(a). It is known that the acoustic wave is characterized by the pressure field, $p$, and velocity field, $u$. Averaging the pressure field in the holes, we get the effective pressure field in the effective fluid $\tilde{p} = \tilde{\rho} \tilde{u}$. Requiring the acoustic energy flow across the surface to be the same for the hole array and the effective fluid, i.e., $pu \cdot \pi d^2/4 = \tilde{p} \tilde{u} \cdot \tilde{a}^2$, we obtain the effective velocity $\tilde{u} = \tilde{u}$. Also the total acoustic energy for both systems are required to be the same, $(\frac{1}{2} \tilde{p}_0 \tilde{u}^2 + \frac{1}{2} \tilde{\rho} \tilde{v}^2)/\kappa$ $\pi d^2$ $/4t = (\frac{1}{2} \tilde{\rho}_0 \tilde{u}^2 + \frac{1}{2} \tilde{p}_0^2/c\tilde{k}) \tilde{a}^2 t$, which gives us the effective parameters $\tilde{\rho}_0 = \tilde{\rho} \tilde{\rho}_0$ and $\tilde{\kappa} = \tilde{\kappa} \kappa$. Thereafter, the acoustic speed and impedance of the effective fluid are $\tilde{v} = \tilde{v} \sqrt{\tilde{\kappa}/\tilde{\rho}_0}$ and $\tilde{\eta} = \tilde{\xi} \eta$, where $v$ and $\eta$ are the acoustic speed and impedance of the filling fluid, respectively. The relations indicate the acoustic speed remains unchanged and the impedance is scaled by a factor of the area fraction of holes for the effective fluid. It is noted that an analytical derivation also produces the similar conclusion.$^{32}$

The above argument is applicable under long wavelength limit ($a/\lambda \rightarrow 0$) where diffractive evanescent waves are negligible. For the sample with thickness comparable to lattice
constant, the diffractive evanescent waves tune the FP resonances and the resultant transmission resonances can occur for channel length \( \sim 16\% \) thinner than required by the FP resonances. Superficially, this diffractive effect is substitutable by a slowing of acoustic wave propagation inside the holes, i.e., \( \nu = 0.84c \). Based on the above argument, the effective fluid equivalent to a 2.3 mm thick sample has the acoustic speed \( \bar{\nu} = 0.84c \), or the acoustic refractive index \( n = \frac{c}{\bar{\nu}} = 1.19 \), and the effective impedance \( \bar{\rho} = \frac{\rho}{\bar{\nu}} = 0.84 \). Based on these two effective parameters at hand, we calculated the transmission spectra, both magnitude and phase, of the effective fluid layer at normal incidence according to the formula

\[
T = \frac{1}{\cos(k_0\bar{\nu}t) + \frac{j}{2} \left( \frac{\bar{n}}{\eta_{\text{water}}} \right) \sin(k_0\bar{\nu}t)},
\]

where \( k_0 = \frac{2\pi f}{c} \) is the wavenumber of the incidence wave. The calculated results (solid lines) are shown in Figs. 4(b) and 4(c) to compare to the experimental data (open circles) for two samples with identical thickness of 2.3 mm and identical lattice constant of 2.0 mm but different hole diameters of 1.2 and 0.5 mm. Good agreement between the calculations and the experiments is seen at the frequencies below Wood’s anomaly (0.75 MHz), which verifies the applicability of the effective fluid model at normal incidence.

Likewise, we measured the transmission of the hole array, \( d = 1.2, a = 2.0, \) and \( t = 2.3 \) mm, at oblique incidence. The obliquity occurred along the [1,0] direction of the array. Figure 5 shows the measured results and the angle-dependent transmission magnitude plotted as a function of the wave frequency and the incidence angle for the hole array \( d = 1.2, a = 2.0, \) and \( t = 2.3 \) mm. The obliquity occurs along the [1,0] direction of the array. The solid line superposed are the variation curve of Wood’s anomaly (−1,0) with the incidence angle. The open circles superposed denote the variation of the transmission peaks, calculated from the FP resonance condition of the effective fluid layer at oblique incidence.
variation of Wood’s anomaly (−1, 0) (solid line). Two flat-bands appear below 0.75 MHz and they are the two peaks, the first and second FP resonances, in Fig. 4(b) where the phase values indicate the order of FP resonances. The open circles superimposed are the variations of the transmission peaks of the effective fluid layer which are obtained from the FP resonance condition at oblique incidence, $\sin[k_0 n \sqrt{1 - (\sin \theta/n)^2}] = 0$.

The agreement between the calculated variations of the transmission peaks and the measured results indicates the applicability of the effective fluid model persists to a range of incidence angle. The discrepancy at $\theta > 15^\circ$ for the second FP transmission peak is due to the emerging of the (−1, 0) diffraction order nearby, and the red flatband is seen to terminate upon crossing with Wood’s anomaly (−1, 0) in the figure. Where the cross happens, there will be the strong resonance of the array and FP-like resonance localized at each hole, which possibly gives rise to the 0.48 MHz band at $\theta = 25^\circ$.

### IV. CONCLUSION

We investigated experimentally the acoustic transmission through the subwavelength hole arrays fabricated on brass plates at normal and oblique incidence within ultrasonic frequencies regime. The transmission phenomena for the hole array in thin brass plates, analogous to the observed enhanced transmission of EM waves through subwavelength hole arrays in a metallic film, exhibit the transmission enhancement (transmittance being larger than the area fraction of the holes). At the peak frequency, the transmission phase is nearly −$\pi$, indicating the out-of-phase oscillations of the acoustic field at two surfaces of the plate. However, for the hole array in thick brass plates, the transmission peaks of acoustic waves are related to the FP-like resonances inside the holes and therefore occur well below Wood’s anomaly since a hole has no cutoff frequency for acoustic propagation. Consequently, the structure can be viewed as a new fluid with effective mass density and bulk modulus scaled, under long wavelength limit, by a factor of area fraction of the holes. The effective medium model describes well the transmission properties of the hole array within a range of incidence angle.

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16F. J. García de Abajo, Rev. Mod. Phys. 79, 1267 (2007).


24With the deduction of $2\pi ft/c$, the transmission phase $\phi$ refers to the phase difference of the wave between the front and rear surfaces of the sample.

25We did the experimental measurements and numerical simulations for microwave transmission through subwavelength hole array samples, and the results show an $\sim 0$ phase change for the enhanced transmission peak, which will be reported elsewhere.


30Here, acoustic surface wave refers to a type of elastic interface wave, Stoneley wave, which can be excited at the interface between a solid material and a fluid, though the surface-wave-like mode induced by the structure factor may be defined on structured surfaces made of ideally conducting/rigid material, see Ref. 16.
