Growth Rates of Band Pattern Formation in a Rotating Suspension of Non-Brownian Settling Particles

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Abstract

We report an experimental study of axial band formation in a settling suspension of non-Brownian glass spheres of 200 micrometers in diameter. These particles are uniform in size and density and are suspended in an aqueous solution of glycerin with a controlled viscosity ranging from 20 to 50 centipoise. The suspension completely fills a horizontal rotating cylinder. The dynamic patterns that form within the cylinder are a result of interplay between gravitational, centripetal and hydrodynamic forces. Using a video imaging technique, we measure the concentration profile of the particles along the rotating axis of the cylinder and obtain the growth rate of the band formation as a function of the solution viscosity and the rotation rate of the cylinder. The measured growth rate is compared to a proposed theory but does not completely validate it. While the theory is supported by experimental data to some extent, much remains to be learned about this unique and dynamic system.

Keywords: pattern formation, settling particles, hydrodynamics.

1. Introduction

Complex interactions occasionally result in organized phenomena. Though these patterns and structures are often clear, symmetric, and easily observed, our comprehension of them will remain limited until they are investigated. Important advances for fluid science have occurred during the research of spatio-temporal patterns and structures, and our grasp of fluid properties becomes increasingly important as science and technology evolves. The structures formed in viscous coating flows have received recent attention, and their application is extremely important to lubrication and industrial processes [1]. The coating flow in a partially filled, horizontal, rotating cylinder, which is strongly affected by the free surface, develops an extensive assortment of interesting behaviors [2-3]. When the fluid medium in the cylinder is replaced with sand, intriguingly similar patterns arise [4]. Granular seeded flow systems exhibit further patterns that are curiously comparable to their simple fluid flow counterparts, but are far less understood.

Recent experiments have shown that granular flow systems in a partially filled, horizontal, rotating cylinder exhibit axial banding [5-7]. These structures were in part attributed to the presence of the free surface caused by the partial filling of the tube. But later, similar density fluctuations along the symmetry axis were observed in a completely filled horizontal rotating cylinder when attempts were made to suspend a solution for surface-free crystal growth [8]. In these experiments, negatively, neutrally, and positively buoyant particles showed signs of axial banding. Recently a variety of patterns and structures were encountered for a settling suspension of uniform non-Brownian particles in a completely filled horizontal rotating cylinder [9].

The distinction between granular and pure fluid systems is the introduction of hydrodynamic interactions, which are essentially caused by an object moving through a fluid. The object must displace fluid before it can move forward, and the resulting fluid relocation creates forces that act on other objects suspended in the same medium. The non-Brownian hard spheres used in our experiment create hydrodynamic interactions that, in conjunction with gravitational and centripetal forces, eventually lead to the formation of structures. This intense and complex interplay presents a unique challenge to statistical physics.
The experiment offered in this paper concerns the formation of axial bands (Figure 1). The growth of these bands is measured and quantitatively compared to a proposed theory on the development of axial segregation in a cylindrical centrifuge [10].

2. Proposed Theory

In brief, the theory states that hydrodynamic effects on the particle trajectories result from differential centrifuging between particles located at different radii, gravity having negligible effect. After the suspended particles reach a homogeneous state the induced hydrodynamic interaction between two particles creates an attractive force between the particles, resulting in axial instabilities. Wall screening and lubrication forces caused by the cylinder wall are also considered. Equation (1) is then deduced for the growth rate of the axial bands observed in the concentration profile of the cylinder:

\[
\gamma_k = \frac{\phi \left( \rho_s - \rho_f \right) \Omega^2 R^2}{8\mu} f(k)
\]  

(1)

Where \( \gamma_k \) is the mode growth rate, \( \phi \) is the particle volume fraction, \( \rho_s \) is the density of the solid sphere, \( \rho_f \) is the density of the suspending fluid, \( \Omega \) is the rotation rate of the cylinder, \( R \) is the radius of the cylinder, \( \mu \) is the viscosity of the medium, and \( f(k) \) contains the wavelength dependence of the growth rate. This theory, when applied, generates the proportional relationship for the mode growth time, \( \gamma_k^{-1} \), shown in equation (2), where \( a \) is some constant:

\[
\gamma_k^{-1} = a \mu \left( \frac{2\pi}{\Omega} \right)^2
\]  

(2)

The theory incorporates no adjustable parameters, and it should also be noted that this theory is only applicable to particles denser than the suspending fluid, although axial banding has been observed with neutral and buoyant particles. This article will show that while this theory correctly predicts some of the experimental data, much remains to be learned about this unique and dynamic system.

3. Experimental Setup

The experiment employs a horizontal Plexiglas cylinder with an adjustable length set to \( L = 22.7 \) cm and an internal radius \( R = 0.95 \) cm. The cylinder is completely filled with an aqueous solution of 76.96 mass percent glycerin. Uniform glass spheres of radius \( a = 200 \) \( \mu m \) and density \( \rho_s = 2.4 \) \( g/cm^3 \) are dispersed in the solution with a particle volume fraction \( \phi = 0.0229 \), and the size variation of the particles is less than 8%. A small amount of detergent is added to prevent particle aggregation. All air is evacuated from the cylinder. The cylinder rotates about its axis of symmetry and is propelled by a thermally isolated stepper motor with \( 2.5\times10^{-4} \) steps per rotation. An indexer is constructed to provide a stable pulse train to the stepper driver with the final rotation period \( T_R \) resolution being \( 2.5\times10^{-4} \) plus or minus \( 7.5\times10^{-6} \) seconds per rotation. The cylinder is housed within a Plexiglas box that serves dual purposes. Water is pumped through the box for temperature control from a refrigerated recirculator that has an accuracy of plus or minus approximately one degree Celsius, which in turn controls the viscosity of the solution within the cylinder. The Plexiglas box also reduces the refraction problems encountered when observing the Plexiglas cylinder. The cylinder, box, and stepper motor are all mounted on an aluminum rack that can be leveled with accuracy (Figure 2).
The glass spheres are initially settled along the bottom of the cylinder, and at startup the cylinder is immediately spun at the desired rotation rate $W$. During the experiment we use standard video imaging software to gather data on the spatial distribution of particles in the $-z$ plane (cylinder profile). A video CCD camera records particle images (Figure 1) using backlit illumination. The glass spheres are blue in order to absorb light. The captured images are each analyzed by calculating an average density value for every vertical column of pixels (Figure 3).

The resulting amplitudes are compared, plotted against time, and filtered to reduce inherent noise (see Figure 5). The apparent exponential growth is then fit to an equation by first isolating the exponential exponents and solving them to generate a parabolic curve of the second order (Figure 4). A parabolic equation is then fit to these points and plotted to compare with experimental measurements (the red line of Figure 4).

The parabolic equation is put into equation (3), which is then plotted against the corresponding experimental data. The equation was found to have the best fit with a parabolic exponent as shown in Figure 5. Saturation was defined as the plateau reached after growth and was set to one for consistency.

$$\varphi_{saturation} = 1 - \exp[ A + B \times + C \times^2 ]$$  

(3)
Data was gathered at various rotation periods and viscosities as shown in Figure 6 within the region found previously to exhibit the strongest axial band formation. The viscosity error shown resulted from the error caused by the temperature control. Though attempted, adequate data could not be gathered beyond 50 centipoise, perhaps due to the induced viscosity error.

4. Results and Discussion

General band pattern growth behaved as shown in Figure 7, where each line represents the typical result for the viscosity shown. As viscosity increased, the pattern intensity took longer to develop and reach saturation. An interesting jump occurs between 40 and 50 centipoise that contributes to a significant trend found within the band phase and noted in other phases as well.
Further analysis displays this interesting trend in viscosity changes. Figure 8 illustrates the asymptotic behavior of the growth time discovered as viscosity approaches 60 centipoise. When coupled with the high viscosity error discussed earlier, this characteristic helps to explain the instabilities encountered when measurements were attempted at 55 and 60 centipoise.

![Figure 8 Growth Time vs. Viscosity](image)

Rotation rate did not greatly influence growth time until higher viscosities, as seen in Figure 9. The sharp increase in growth time from 40 to 50 centipoise is again made evident, and adds to questions about the high viscosity band phase.

![Figure 9 Growth Time vs. Rotation Rate](image)

Comparing the direct results to the theory demonstrates the need for a more advanced model. In Figure 10, the dashed line represents the theory that growth time is directly proportional to viscosity times the square of the period. While data taken near the center of the band phase (as seen in Figure 6) somewhat matches the theory, there are no adjustable parameters to account for the variations from the edges of the band phase. The experimental results do not completely validate the proposed theory.

![Figure 10 The theory (dashed line) only corresponds to data in the center of the band phase (see Figure 6).](image)
5. Summary

We have shown that a theory proposed for the growth rates of axial band formation in a rotating suspension of non-Brownian settling particles falls short of being the perfect answer. Further theoretical study will be required, though the proposed theory does very well for a first attempt at understanding this system. Investigation of this phenomenon has also led to interesting discoveries about the nature of the band formation region, especially the unstable behavior encountered beyond 50 centipoise in viscosity. As this experiment has only focused on a small part of this system, further exploration of this phenomenon is necessary in order to possess a full understanding of these rich and dynamic patterns.

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References

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