Computer Animation of Accretion of Neutron Stars

Wang Kei So, Ho Yin Law, Hiu Pan Tang
S.K.H Tsang Shiu Tim Secondary School

Abstract

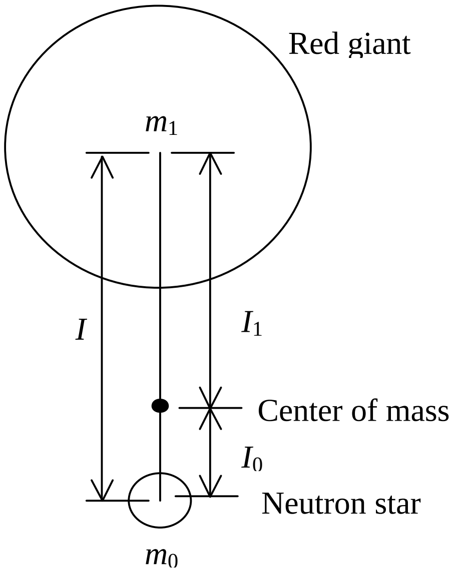
In the research, binary systems consisting of neutron star were studied. After reviewing the basic knowledge of neutron stars, we wrote a computer program to animate and visualize the accretion process on a two-dimensional plane.

1. Introduction

A ***neutron star*** can exist in a binary star system. Since the neutron star has a very high density, its gravitational attraction on its companion star is so strong. Matter from the companion star is lost by being attracted to the neutron star. This phenomenon is called ***accretion***. During accretion, the normal companion star is distorted by the neutron star's gravity and the fluid material on it is absorbed by the neutron star. Before of the energy of the fluid material is converted to radiation energy emitted from the neutron star, these materials will loop around the neutron star to form an ***accretion disc,*** until most of the material is absorbed by the normal companion star.

2. Model of a Binary System Containing a Neutron Star and a Red Giant

In order to make the animation more convincing, we have made reference to the binary star system ***'Hercules X-1'***, which was observed to contain a ***neutron star*** and its ***companion red giant***. We have made use of its data in our computer animation.



As shown in the figure, the distances of the neutron star and red giant from the centre of mass satisfy the equations $I_0 + I_1 = I$ and $m_0 I_0 = m_1 I_1$. Hence they are given by

$$I_0 = \frac{m_1}{m_0 + m_1} I \text{ and } I_1 = \frac{m_0}{m_0 + m_1} I.$$

For the neutron star and red giant to exhibit circular motion, their velocities are given by Newton's 2nd law,

$$\frac{Gm_0 m_1}{I^2} = m_0 \frac{v_0^2}{I_0} \text{ and } \frac{Gm_0 m_1}{I^2} = m_1 \frac{v_1^2}{I_1}.$$

Hence

$$v_0 = \sqrt{\frac{Gm_1 I_0}{I^2}} \text{ and } v_1 = \sqrt{\frac{Gm_0 I_1}{I^2}}.$$

Choosing the centre of mass as the origin, the positions are

Neutron star: $(0, -I_0)$.

Red giant: $(0, I_1)$.

The velocities are

Neutron star: $(v_0, 0)$.

Red Giant: $(-v_1, 0)$.

3. Assumptions and theory applied on the model of accretion

1. The red giant is made up of a fluid of particles. The density of the fluid decreases from its interior to the core. We first assume the total mass of the red giant is concentrated at the centre of mass of the red giant. Following this assumption, we next assume that the fluid particles on the surface, which are very light with respect to the centre of the red giant, have no interactions with each other. The latter assumption can avoid the chance of repulsion between the fluid particles whenever there is a collision.

2. Although the fluid particles on the surface are so light with respect to the centre of the red giant, still the attraction between the particles and the centre is so strong and may make the red giant collapse. In order to restore its original shape, we applied the **Hard Sphere Model of a Red Giant.**

We assume that the particles outside the surface of the red giant cannot penetrate to the interior of the hard sphere. Whenever a fluid particle goes inwards from the surface, we will bring it back to the surface. Furthermore, we discard the radial velocity component which points towards the centre and accept only the velocity component tangential to the surface of the hard sphere. The detailed mathematics of the hard sphere model is explained in Appendix.

This model resembles fluid dynamics in that if some fluid leaves a space, the nearby fluid will automatically replace the space. By this assumption, we can avoid the collapse of the red giant.

3. Furthermore, we found that the fluid particles may eventually leave the accretion disc due to repulsion. If a fluid particle orbits around the neutron star at a very short distance, it may eventually escape.

In order to avoid this, we have added a damping acceleration on the fluid particles. This assumption is based on the fact the kinetic energy of the fluid particles is partly converted to heat energy due to friction. The friction will provide deceleration.

4. The Animation

The animation part consists of a ***C++ programme*** and a ***graph plotting programme called Presto Plot.***

4.1. Computer programming

The computer programming is used to demonstrate and calculate the interactions, relative distances and velocities etc. between the neutron star and the red giant. The programme is based on Newton's law of gravitation and some fluid dynamics. Since there are some limitations on the computer programming, the animation is actually different from the real situation. In order to overcome these, we have made several assumptions on the model of the accretion process described in the previous section.

4.2. Processing of the model of the accretion

The programme starts by initializing all the positions, velocities of the fluid particles, the neutron star and the centre of the red giant. Distances are measured in astronomical units, where 1 AU = 1.5×10^{11} m = average distance between Sun and Earth. Time is measured in years. Masses are measured in solar masses, where 1 solar mass = 2×10^{30} kg.

When the red giant starts orbiting around the neutron star, the fluid particles on the surface of the red giant are attracted towards the neutron star according to the Newtonian dynamics. For every time step, the programme first calculates the acceleration on each particles, then the new velocity and hence the new position. The new positions are then updated and output as a group of new data. For every set of data at a different time, the positions of the particles are plotted to form a picture. These pictures are the animation of the model of accretion.

4.3. Animation processing

Our programme aims to demonstrate and visualize the situation when the Red Giant companion star is distorted by the strong gravitational fields of the Neutron star.

5. Improvements and corrections made to the computer programme

During the testing stage of the computer programs, we found that the following two improvements are needed.

1. At first, each particle of the red giant is initialized to move in orbits around the neutron star with its own orbital velocity. That is, they are moving as individual particles, rather than moving as a single body as a whole. Thus, the particles move apart in a few time steps.

To correct this situation, we have initialized the velocities of the particles of the red giant to be due to the translational motion of the centre of mass of the red giant, plus the rotational motion with respect to the centre of mass of the red giant too.

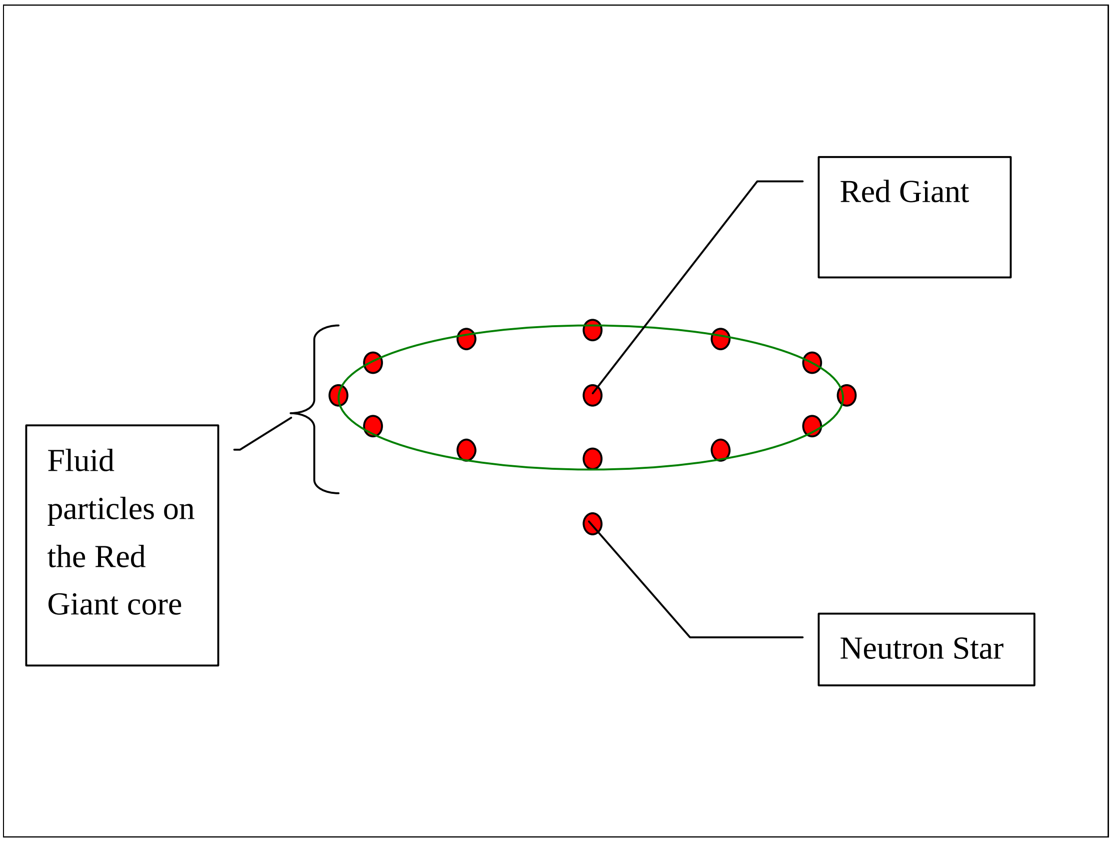
The angular velocity of the rotational motion is the same as that of the orbital motion around the neutron star, so that a point on the surface of the red giant facing the neutron star is always facing it, and a point on the surface of the red giant opposing the neutron star is always opposing it. This is called synchronous rotation, and is observed in many binary systems in astronomy, such as the motion of the Moon around the Earth.

2. Secondly, the programme calculates the new acceleration of each particles for every time step. However, due to the limitation of the graph plotting programme, if the time step set in the programme is too small, the animation of accretion cannot complete. However, if the time interval is enlarged, the error of calculation will also increase.

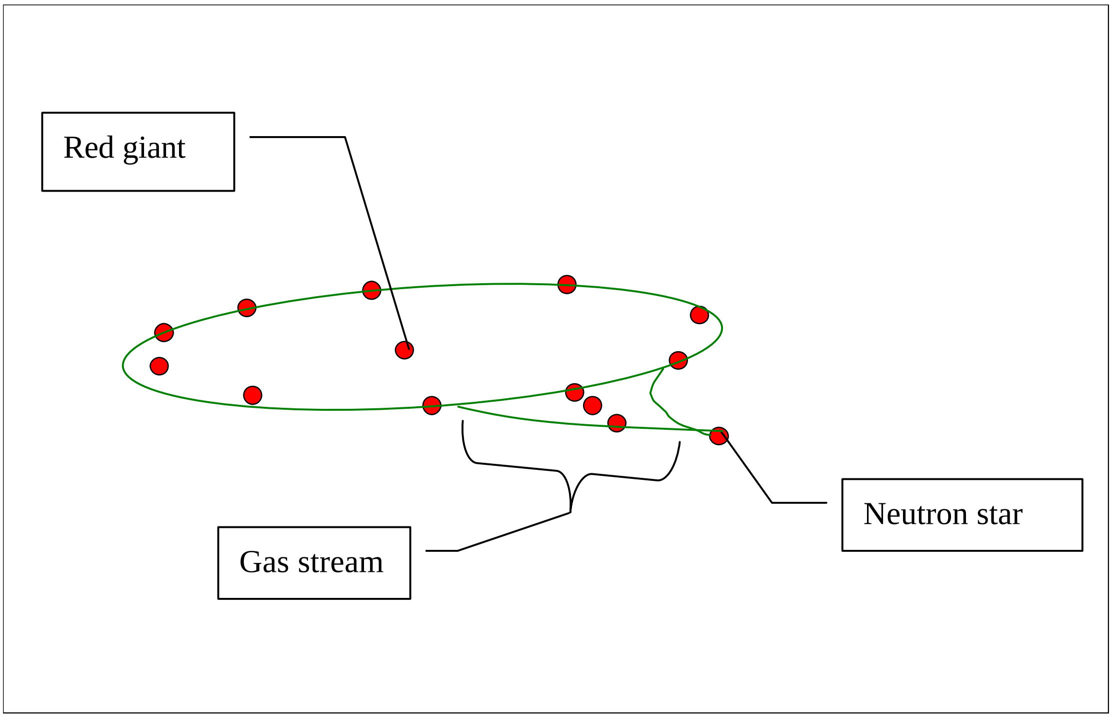
In order to increase the accuracy of this animation, we still divide the time intervals as small as possible, but the new data is only updated when several time intervals have passed. For example, if the time interval is set to be 0.005 units, the new group of data will not be updated for every 0.005 units, but 0.05 units or 0.1 units. This will neither decrease the accuracy nor be restricted by the graph plotting programme.

6. Steps of observations in the animation

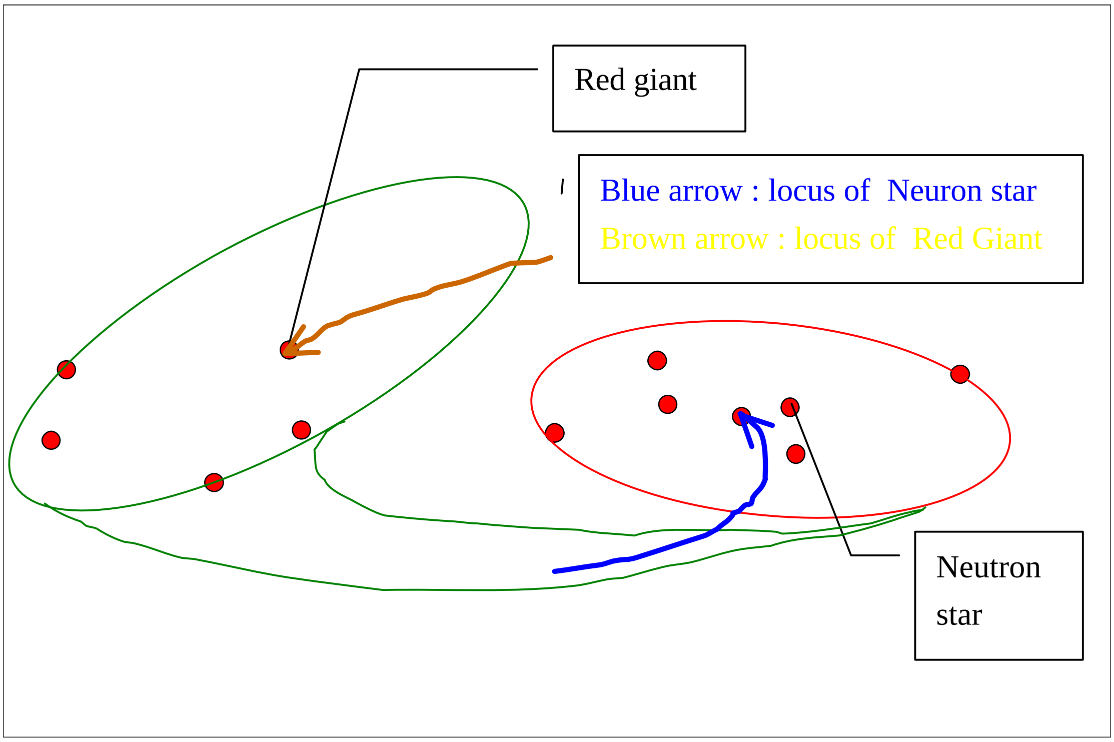
1. Initialize the pattern of the binary star system

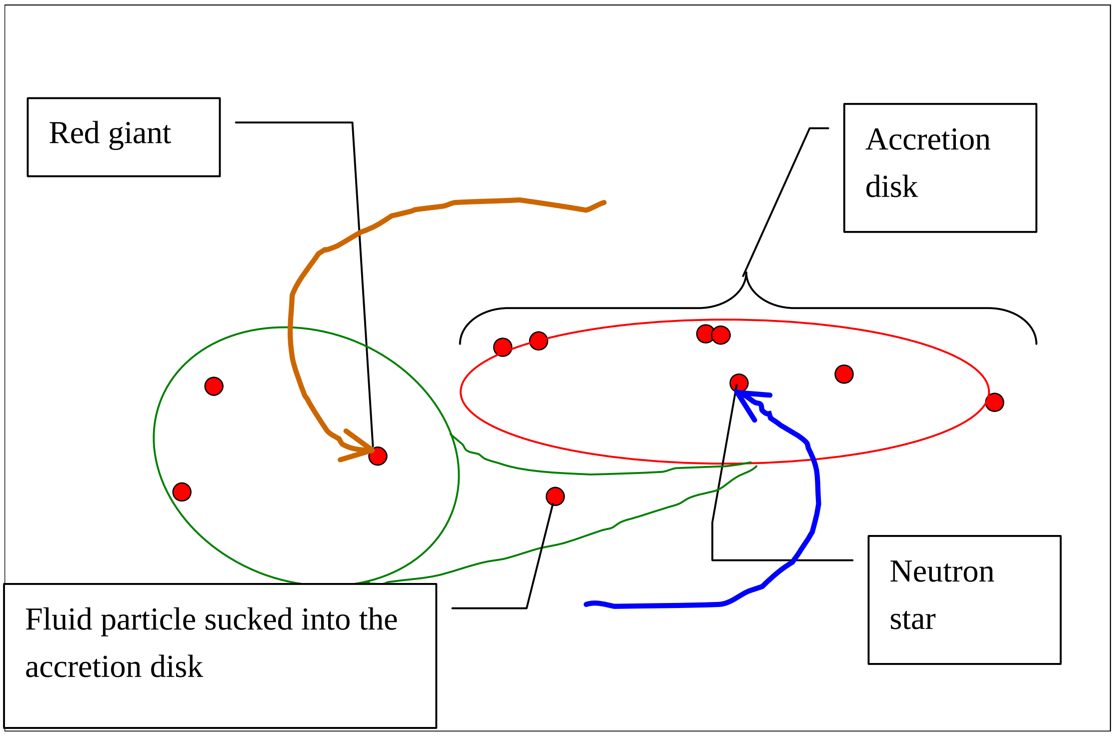


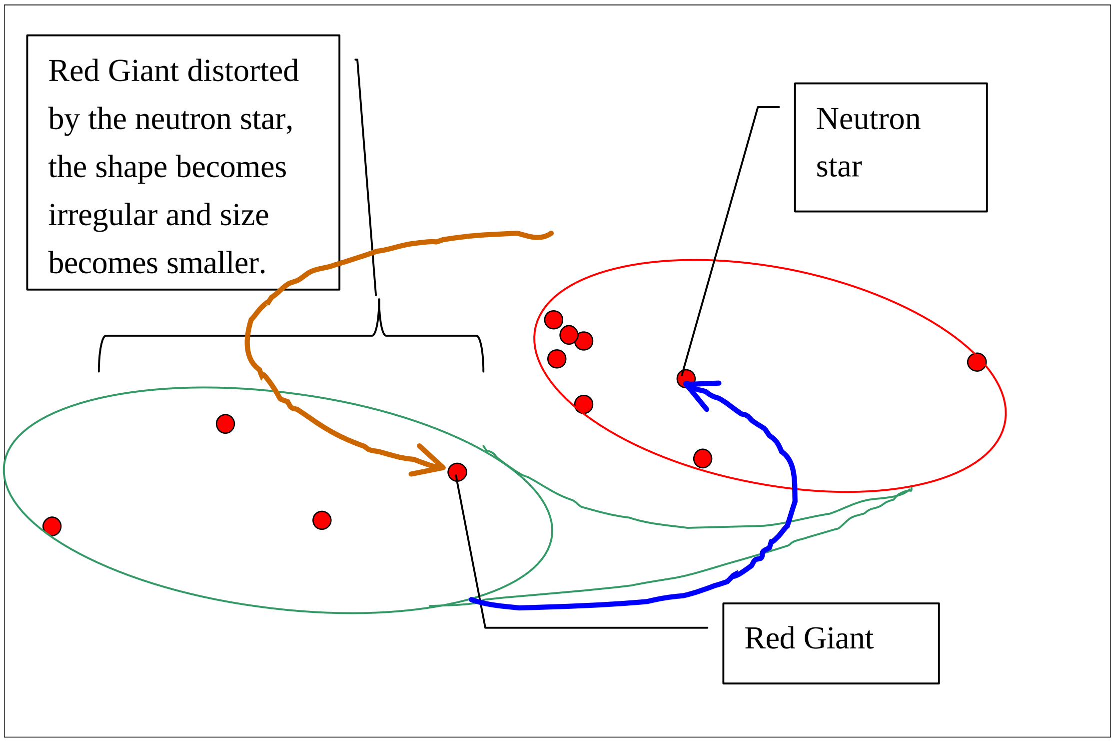
2. When both the Neutron star and the Red Giant start to move, some fluid particles start to leave the Red Giant's core and generate a gas stream

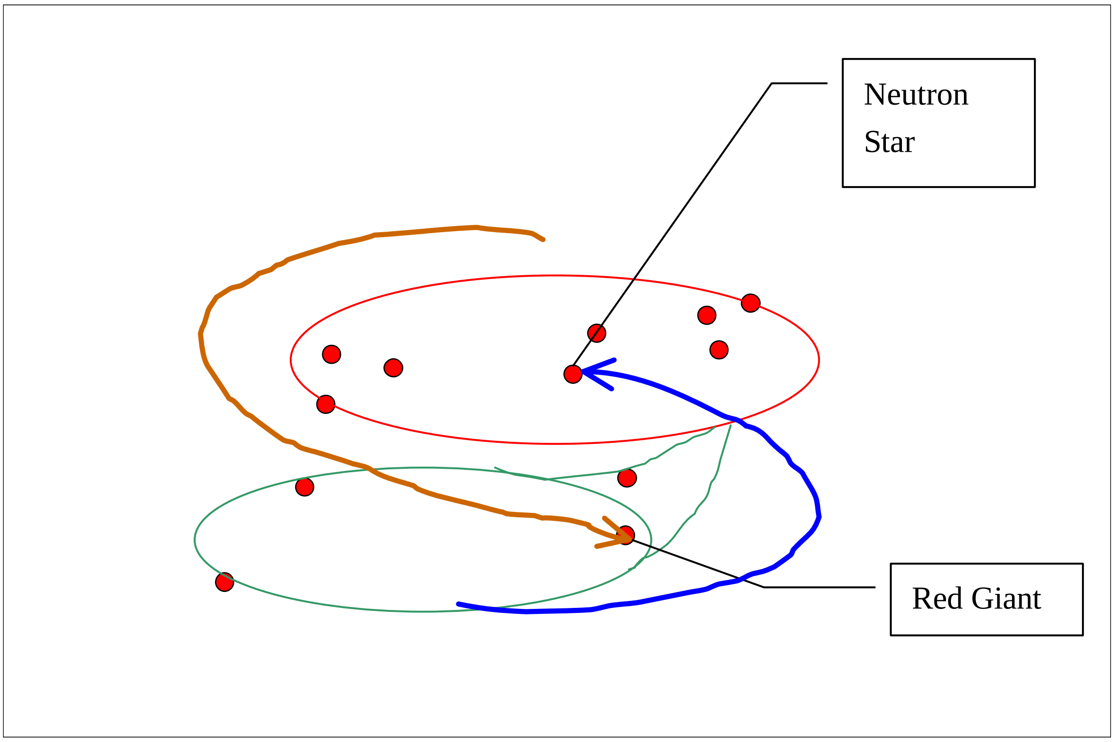


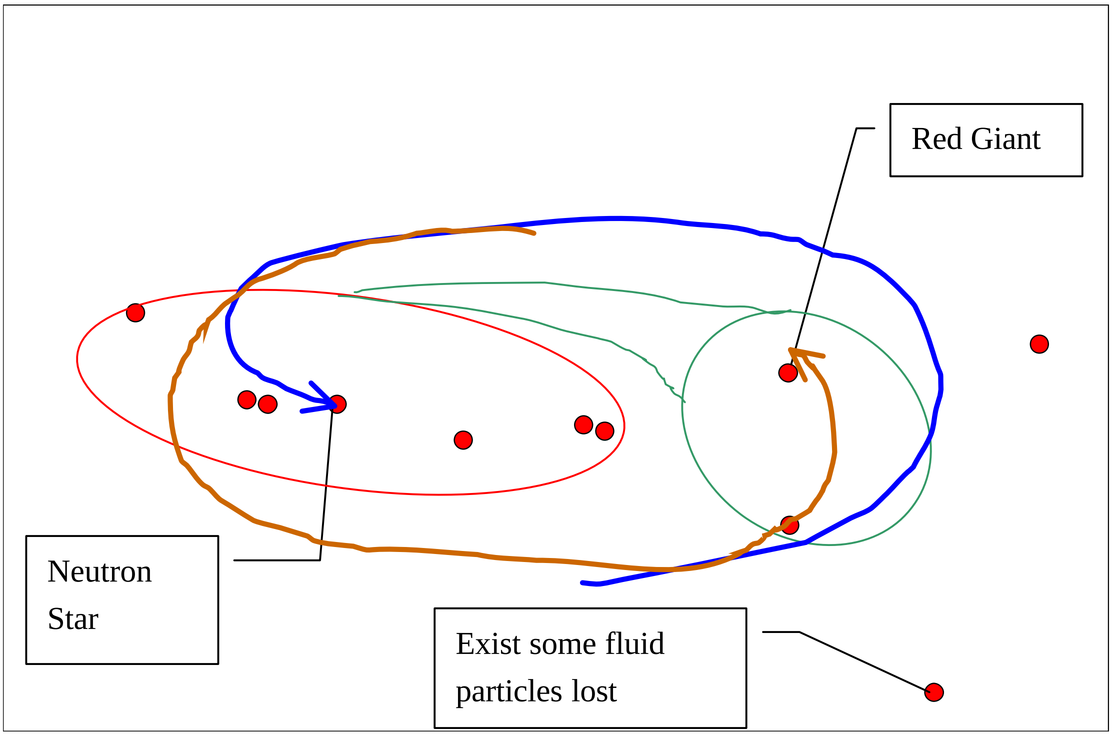
3. When they move further, more fluid particles flow from the Red Giant to the Neutron star's gravitational field. Since the fluid starts to accumulate and friction exists, the fluid particles form a disk which is very luminous. The disk is called accretion disk.

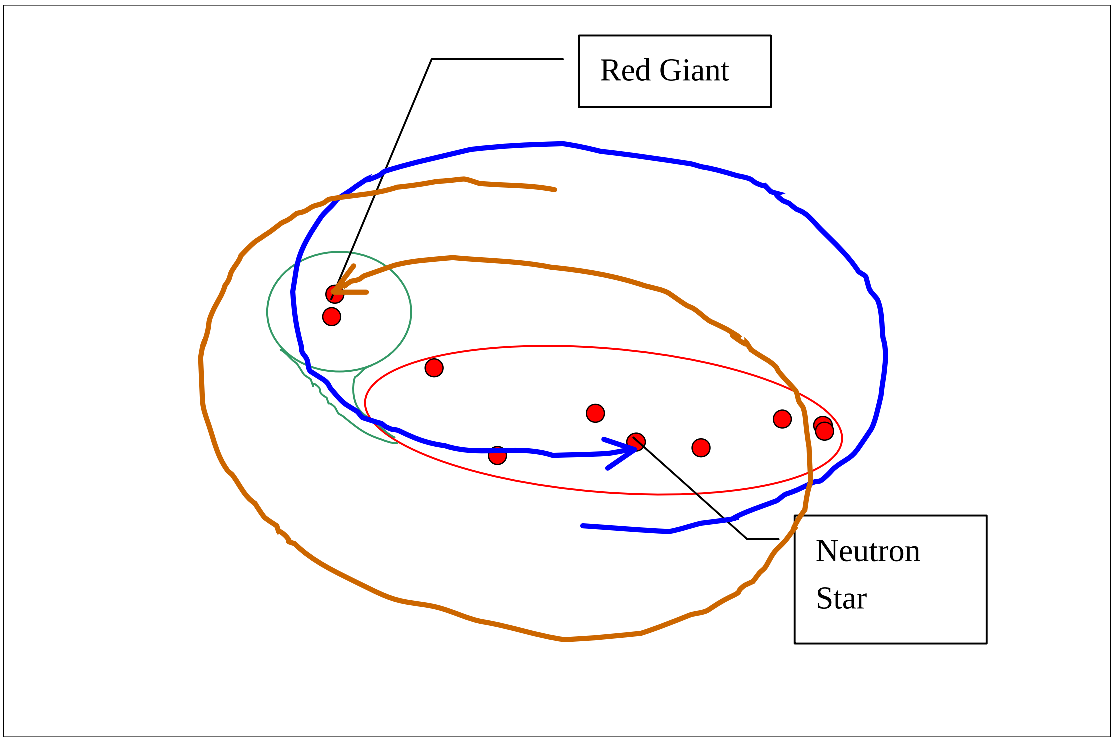



Red giant
Accretion disk
Neutron star
Fluid particle sucked into the accretion disk

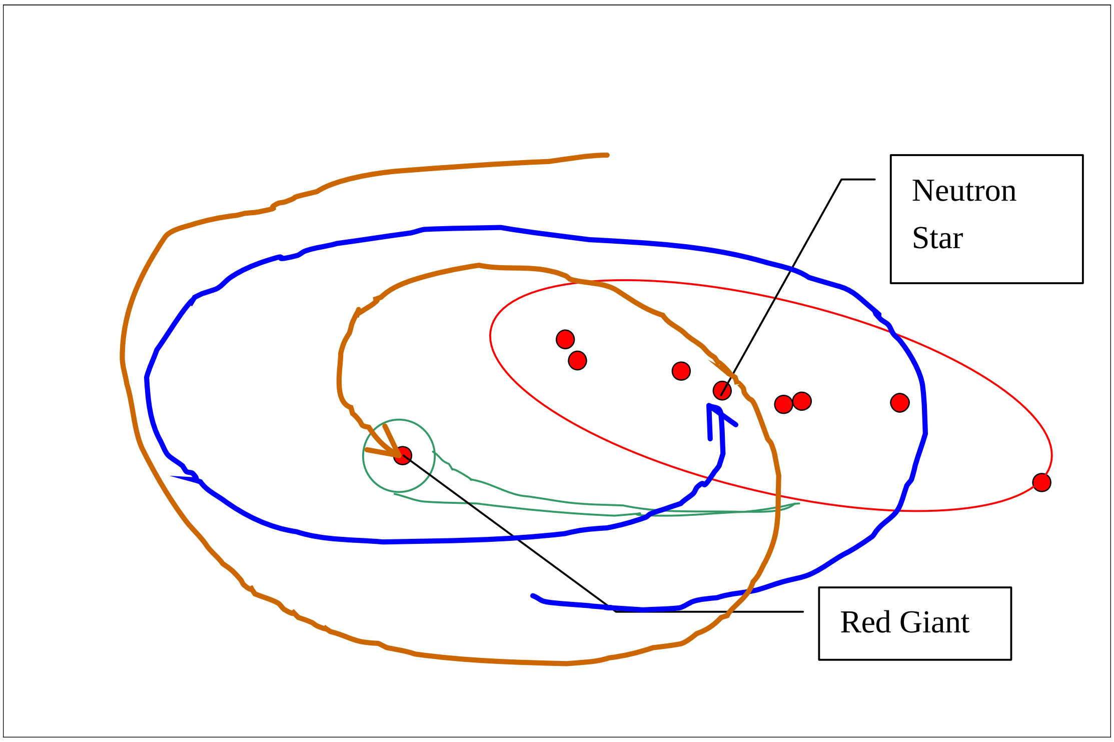

Red Giant distorted by the neutron star, the shape becomes irregular and size becomes smaller.
Neutron star
Red Giant


Neutron Star
Red Giant


Red Giant
Neutron Star
Exist some fluid particles lost



4. The Red Giant finally becomes very small while the Neutron star seems to be expanded. In this later stage, the initial Red Giant slowly becomes small while the initial neutron star slowly becomes massive.



7. Conclusion

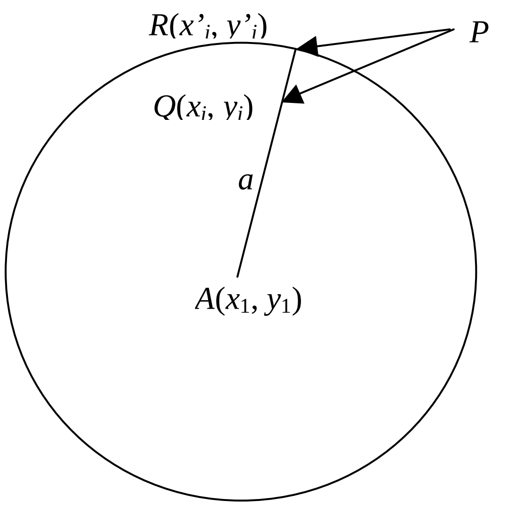
We have successfully simulated the accretion of the Red Giant by its companion neutron star. The simulation process revealed the necessary assumptions and the initial conditions needed to make the simulation work. However, due to our limitations on the knowledge of the neutron star and computer programming, the study is still mainly qualitative. To improve the accuracy of the simulation at a similar scale, it may be necessary to introduce techniques of computational fluid dynamics rather than considering the computed elements as hard particles. This is proposed as a future direction of studies.

Acknowledgments

Dr. K. Y. Michael Wong, Department of Physics, The Hong Kong University of Science and Technology supported this study.

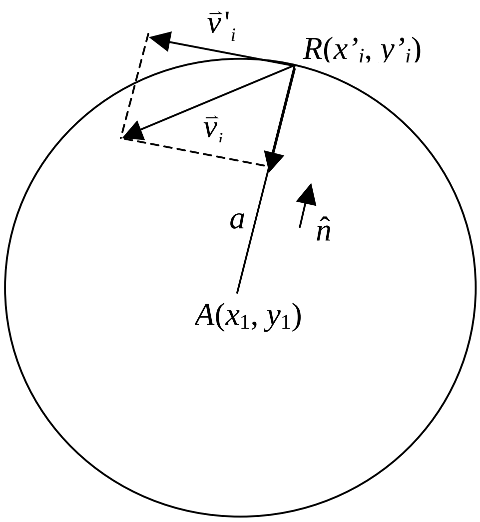
Appendix: The Hard Sphere Model of a Red Giant

As shown in the figure below, if we find that the Newtonian dynamics of the particle at *P* takes it to position *Q* which is inside the hard sphere, we have to relocate the particle to position *R*, which lies on the surface of the hard sphere along the same radius.



Using the length ratios of AR and AQ, the position of R is given by

$$x'_i = x_1 + \frac{a(x_i - x_1)}{\sqrt{(x_i - x_1)^2 + (y_i - y_1)^2}}, \quad y'_i = y_1 + \frac{a(y_i - y_1)}{\sqrt{(x_i - x_1)^2 + (y_i - y_1)^2}},$$



Also, if we find that the Newtonian dynamics of the particle at Q is (v_{ix}, v_{iy}), we should discard the radial velocity component, and accept only the velocity component tangential to the surface of the hard sphere.

Let $\hat{n}$ be the unit vector lying along the radial direction. Then the radial component of the velocity has a magnitude of $\vec{v}_i \cdot \hat{n}$.

Hence the tangential component of the velocity is $\vec{v}'_i = \vec{v}_i - (\vec{v}_i \cdot \hat{n})\hat{n}$. Note that $\hat{n} = \frac{\vec{r}'_i - \vec{r}_1}{|\vec{r}'_i - \vec{r}_1|}$, where $\vec{r}'_i$ and $\vec{r}_1$ are the position vectors of R and A respectively. Hence

$$v'_{ix} = v_{ix} - \frac{v_{ix}(x'_i - x_1) + v_{iy}(y'_i - y_1)}{(x'_i - x_1)^2 + (y'_i - y_1)^2}(x'_i - x_1),$$

$$v'_{iy} = v_{iy} - \frac{v_{ix}(x'_i - x_1) + v_{iy}(y'_i - y_1)}{(x'_i - x_1)^2 + (y'_i - y_1)^2}(y'_i - y_1).$$

Reference

V. M. Lipunov, Astrophysics of Neutron Stars, Springer-Verlag, Berlin (1992).