Simulation of Eclipsing Binary Star Systems

Boris Yim[1], Kenny Chan[1], Raphael Hui[2]
[1]Wah Yan College Kowloon
[2]Diocesan Boys' School

Abstract

This report briefly introduces the information on eclipsing binary star systems. We made a program which simulates the orbital motion and perform light curve plotting. A detailed description on our project is provided. We also performed a simulation on a famous eclipsing binary, Algol, and compared it with the actual experimental result.

1. Introduction [1]

Half or more of all stars in the universe are in orbit around another star or stars. In most of these multiple-star systems, there is a type of system which consists of two stars only, known as a *binary star system*, whose components may be separated by a large fraction of a light year, or they may be almost touching. In binaries, individual stars orbit in elliptical orbits around a common center of mass. The more massive component, which is not necessarily the brighter of the two stars, has the smaller orbit; the relative size of each star's orbit is inversely proportional to its mass.

One type of binary system is known as *eclipsing binaries*, in which the eclipse of one star by another is the key to identify its binary nature. In such systems, an eclipse occurs because the stars are fairly close to each other and their orbits are seen more or less edgewise. Thus their periodic motion causes first one star and then the other to pass between its companion and us, temporarily cutting off all or part of the eclipsed star's light. Consequently there will be a decrease in the apparent brightness of the system each time an eclipse occurs. The resultant light curve of the system depends on the brightness and size of both stars which provides very useful information for deriving a model of the system.

The aims of our project are to simulate the orbital motion and changes in luminosity of an eclipsing binary system with a computer program, and try to study the relationship between the light curve and the characteristics of binary stars with the aid of the program. By using a real example, we will compare the output results of the program with actual experimental results.

2. The Physics of Binary Stars

2.1. Basic Assumptions

In order to simplify the calculations involved in the program, we assume that *all stars in the binary systems involved in the program are spherical and are blackbodies.* Also, *the limb darkening effect*, *material transfer phenomenon between the stars* and *distortion of stars are neglected*.

2.2. Period of the system

The period of a binary system t is given by

$$t = \sqrt{\frac{d^3}{M_A + M_B}} \tag{1}$$

where d is the average separation of the stars in astronomical units, M_A and M_B are respectively the masses of the two stars, namely A and B, of the binary system, measured in units of solar masses.[2]

2.3. Position of the stars at a particular time

The orbit of the binary stars can be computed using Newtonian mechanics [3]. Firstly, for a given time t, *the* eccentric anomaly ψ of the true relative orbit can be found by the equation

$$wt = \Psi - e \sin \Psi . \tag{2}$$

This can be solved by using the following iteration formula derived from Newton's Method.

$$\Psi_n = \Psi_{n-1} - \left(\frac{wt + e \sin \Psi_{n-1} - \Psi_{n-1}}{e \cos \Psi_{n-1} - 1} \right) \tag{3}$$

where e is the eccentricity of the orbit, w is the angular velocity derived from t , and t varies from 0 to 2π. [3]

Then the real distance r is calculated, from the definition of eccentric anomaly, by the following formula.

$$r = d(1 - e \cos \Psi) \tag{4}$$

After that, the polar angle q is found by the elliptical orbit equation.

$$q = \cos^{-1}\left(\frac{\cos \Psi - e}{1 - e \cos y} \right) \tag{5}$$

The coordinates of star A in the orbital plane (x_A', y_A') and that of star B (x_B', y_B') are then respectively given by

$$\begin{pmatrix} x_A' \\ y_A' \end{pmatrix} = \begin{pmatrix} \dfrac{-r\cos q}{1+\dfrac{M_A}{M_B}} \\ \dfrac{-r\sin q}{1+\dfrac{M_A}{M_B}} \end{pmatrix} \tag{5a}$$

$$\begin{pmatrix} x_B' \\ y_B' \end{pmatrix} = \begin{pmatrix} \dfrac{r\cos q}{1+\dfrac{M_B}{M_A}} \\ \dfrac{r\sin q}{1+\dfrac{M_B}{M_A}} \end{pmatrix} \tag{5b}$$

Finally, they are transformed into the coordinates of star A and star B in the true orbit, i.e. (x_A, y_A, z_A) and (x_B, y_B, z_B) respectively, as follows

$$\begin{pmatrix} x_a \\ y_a \\ z_a \end{pmatrix} = \begin{pmatrix} \cos g\cos a - \cos b\sin a\sin g & -\sin g\cos a - \cos b\sin a\cos g & \sin b\sin a \\ \cos g\cos a + \cos b\sin a\sin g & -\sin g\cos a + \cos b\sin a\cos g & -\sin b\sin a \\ \sin b\sin g & \sin b\cos g & \cos b \end{pmatrix} \begin{pmatrix} x_a' \\ y_a' \\ 0 \end{pmatrix} \tag{6a}$$

$$\begin{pmatrix} x_b \\ y_b \\ z_b \end{pmatrix} = \begin{pmatrix} \cos g\cos a - \cos b\sin a\sin g & -\sin g\cos a - \cos b\sin a\cos g & \sin b\sin a \\ \cos g\cos a + \cos b\sin a\sin g & -\sin g\cos a + \cos b\sin a\cos g & -\sin b\sin a \\ \sin b\sin g & \sin b\cos g & \cos b \end{pmatrix} \begin{pmatrix} x_b' \\ y_b' \\ 0 \end{pmatrix} \tag{6b}$$

where a is the angle of inclination with z-axis as the axis-of rotation, b is the angle of inclination with y-axis as the axis-of rotation, and g is the angle of inclination with x-axis as the axis of rotation. (Refer to Fig. 1)

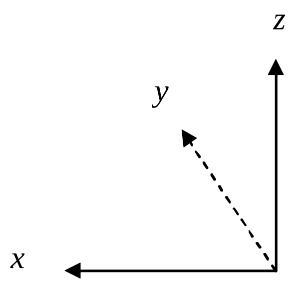


Fig. 1 *- The directions of the axes of rotation. The y-axis is pointed into the paper.*

2.4. Observable power **[4]**

To discuss the various cases of eclipses, we let d = the distance between the centers of the stars, projected onto the plane transverse to the line of sight. Also, a = the radius of the larger star, b = the radius of the smaller star, F_a = *light flux from the surface of the larger star and* F_b = *light flux from the surface of the smaller star.*

Case 1: d > a + b (No eclipse)

This is the full phase. The entire surfaces of both stars are not blocked. The observable power P is

$$P = F_a pa^2 + F_b pb^2 \tag{7}$$

Case 2: a – b < d < a + b (Eclipse occurs)

In this case, only the projected area not being shaded by the front star contributes to the observed light. Assuming that the smaller star is eclipsing the larger star, the unshaded area of the eclipsed star is given by

$$\begin{aligned} &a^2(p - a) + a^2 \cos a \sin a - b^2 b + b^2 \cos b \sin b \\ &= a^2(\cos a \sin a + p - a) + b^2(\cos b \sin b - b) \end{aligned} \tag{8a}$$

where a and b (Refer to Fig. 2) can be obtained by the cosine law.

$$a = \cos^{-1}\frac{a^2 + d^2 - b^2}{2ad} \tag{8b}$$

$$b = \cos^{-1}\frac{b^2 + d^2 - a^2}{2bd} \tag{8c}$$

Therefore, the observed power from the eclipsed star (in this case, P_a) is

$$P_a = F_a\left[a^2(\cos a \sin a + p - a) + b^2(\cos b \sin b - b)\right] \tag{9}$$

And the total observable power P is the observable power from the eclipsed star and its full-phase partner, i.e.

$$P = F_a\left[a^2(\cos a \sin a + p - a) + b^2(\cos b \sin b - b)\right] + F_b pb^2 \tag{10}$$

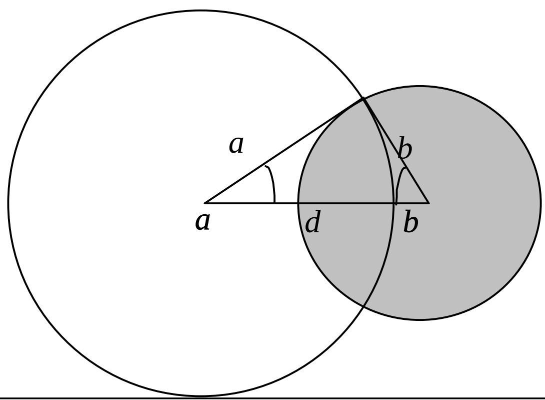


***Fig.2** - Diagram of the binary star during partial eclipse*

Case 3: d < a – b (The entire smaller star is eclipsing the larger one)

The total observable light is contributed by the whole smaller star and the non-eclipsed part of the larger star. Therefore the observable power P is

$$P = F_a p\,(a^2 - b^2) + F_b p b^2 \tag{11}$$

Case 4: d < a – b (The smaller star is completely eclipsed by the larger one)

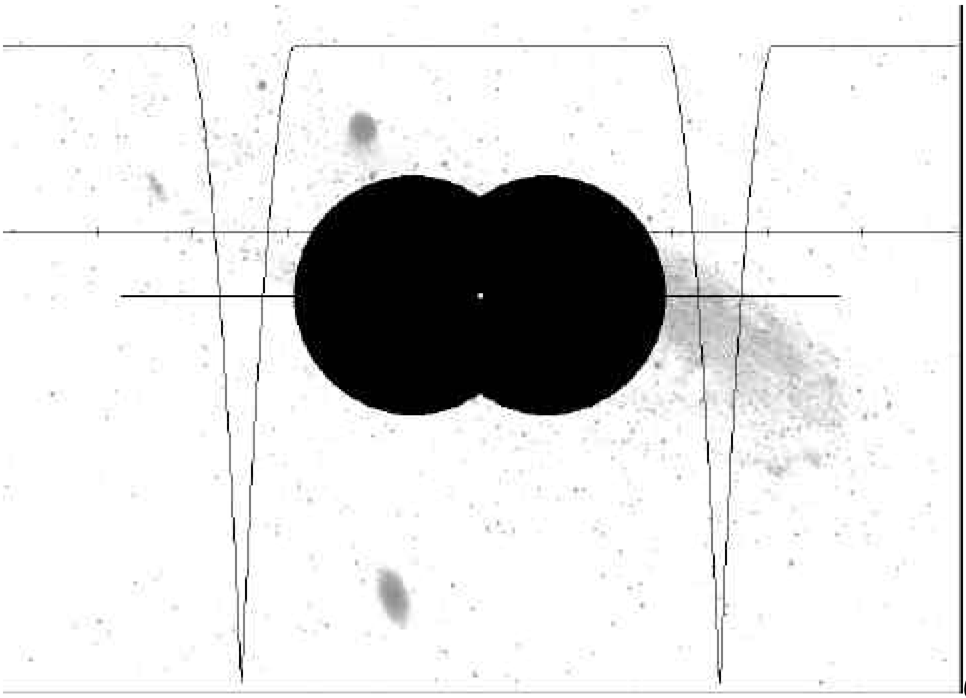
The total observable light is solely contributed by the whole larger star. Therefore the observable power P is

$$P = F_a p a^2 \tag{12}$$

3. Study of the properties of binary systems

3.1. Relationship between star radii and resultant light curve

In an eclipsing binary system, if both stars have similar radii, minima with sharp bottom are observed. (Refer to Fig. 3a) If the difference of radii of the two stars is significant, the minima are flat at the bottom. (Refer to Fig. 3b) It is worth noticing that the duration of the flat bottom is the time for the smaller star to travel out from the back of the larger star. Hence, the ratio of the flat-bottom duration to the eclipse duration reveals the ratio of the radii of the two stars.



(a)

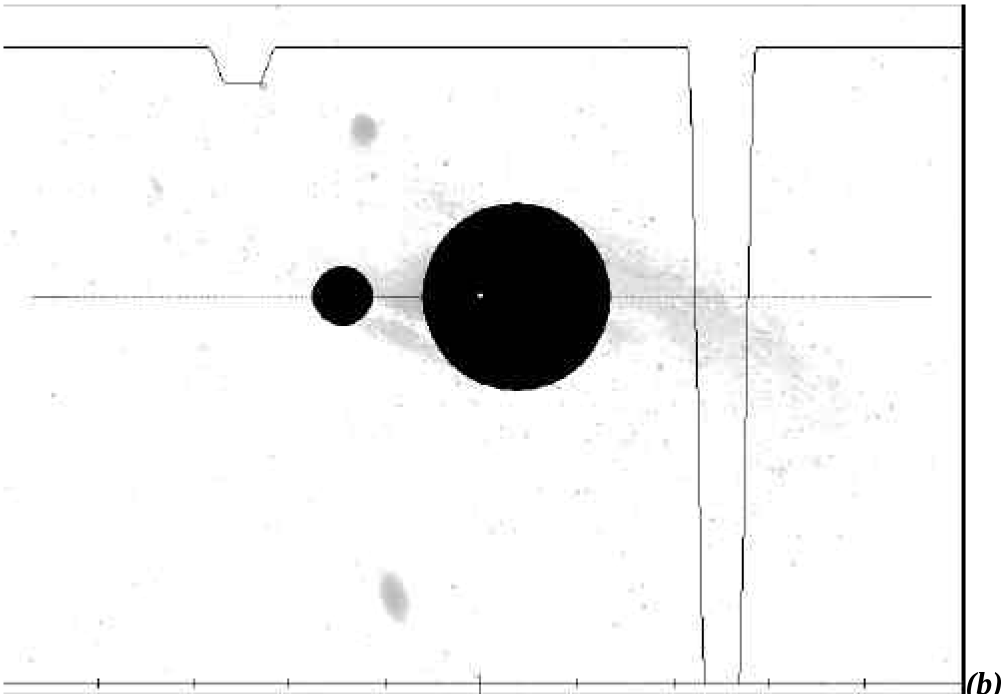
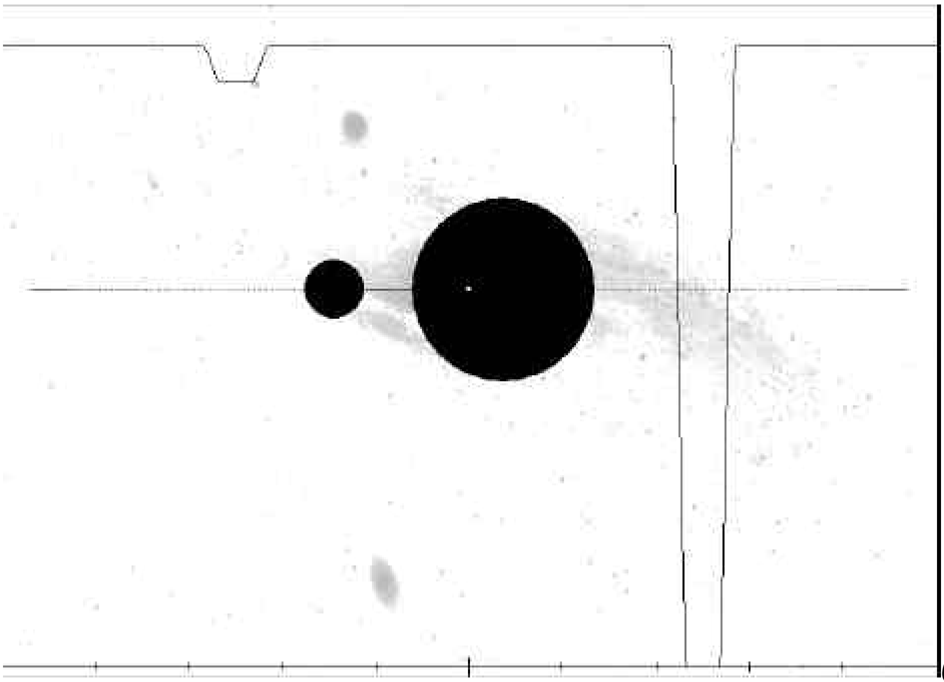


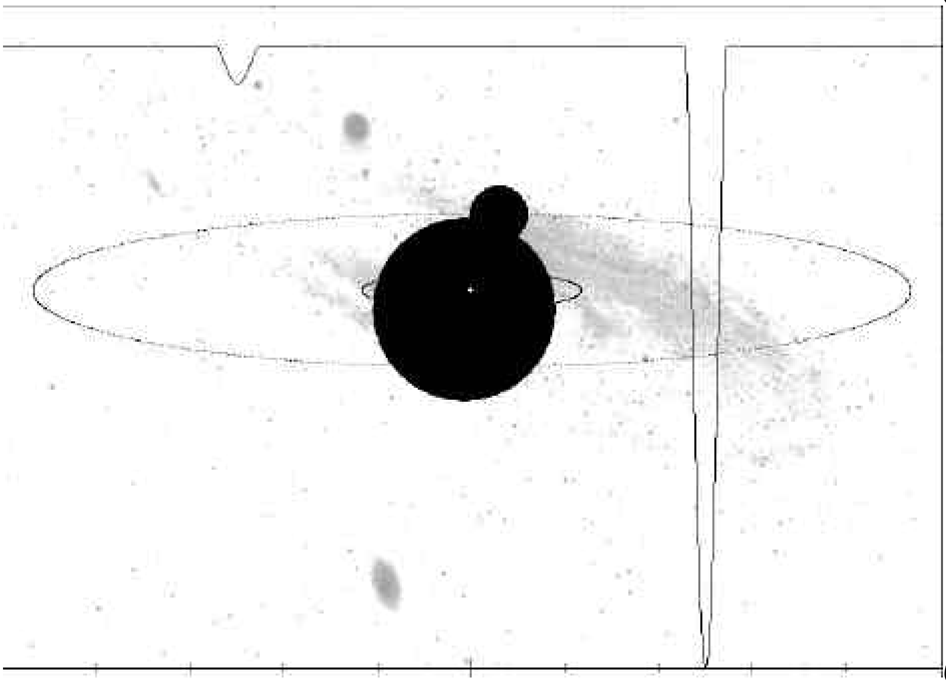
***Fig. 3** - Light curves of binary system with stars of **(a)** similar masses and **(b)** masses with significant difference*

3.2. Relationship between inclination of orbit and resultant light curve

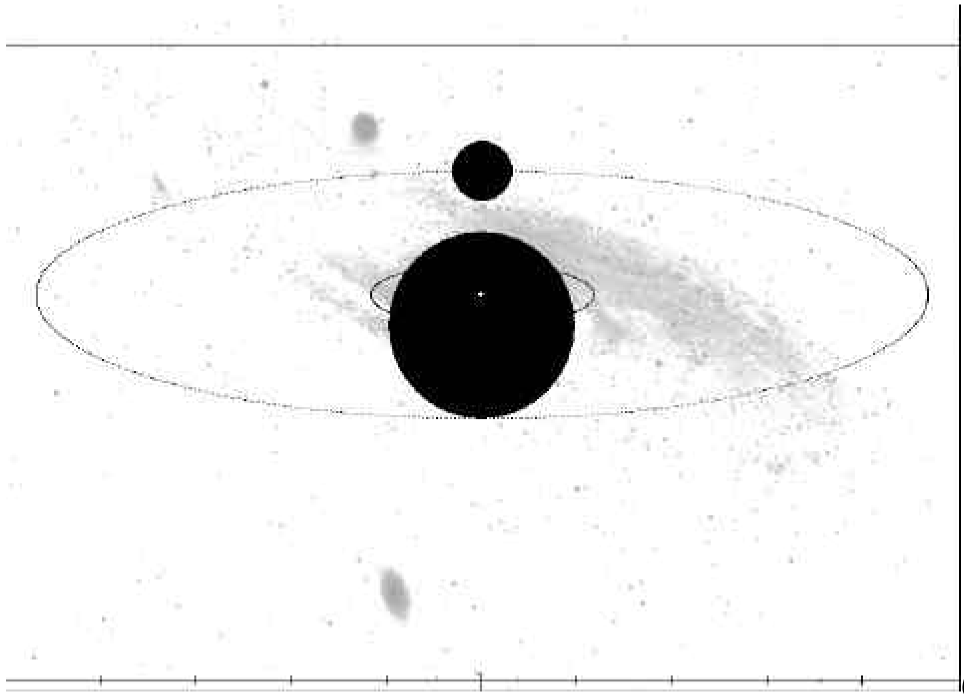
If the orbit of a binary system has no or very small inclination, total eclipse occurs and flat bottom minima will be observed. (Refer to Fig. 4a) If the inclination increases, the flat bottom will disappear, and a sharp bottom is observed, because the smaller star is no longer fully eclipsed. (Refer to Fig. 4b) If the inclination continues to increase, no eclipse will occur and hence no periodic change of the light curve is observed. (Refer to Fig. 4c)



(a)



(b)

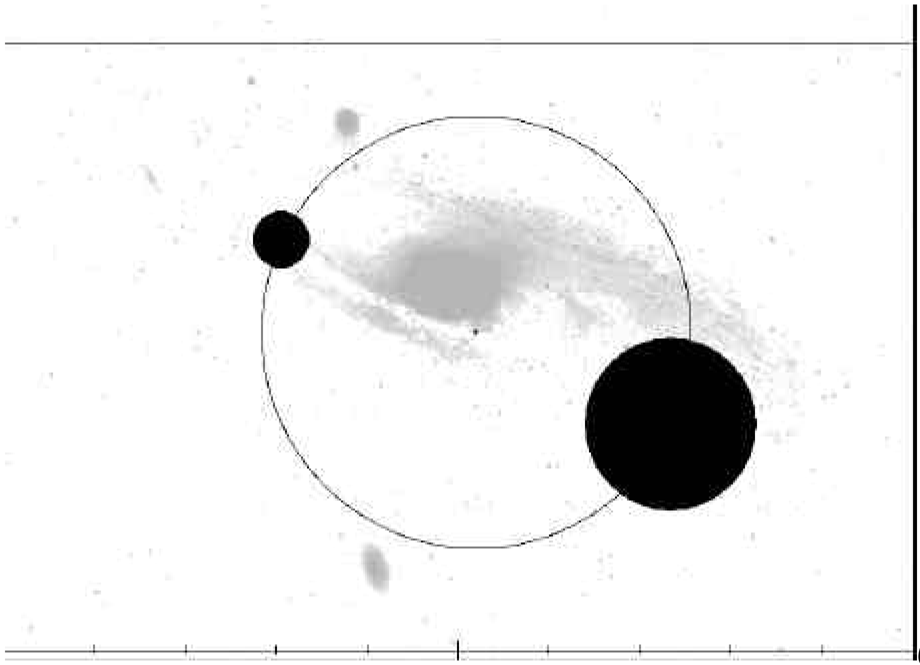


(c)

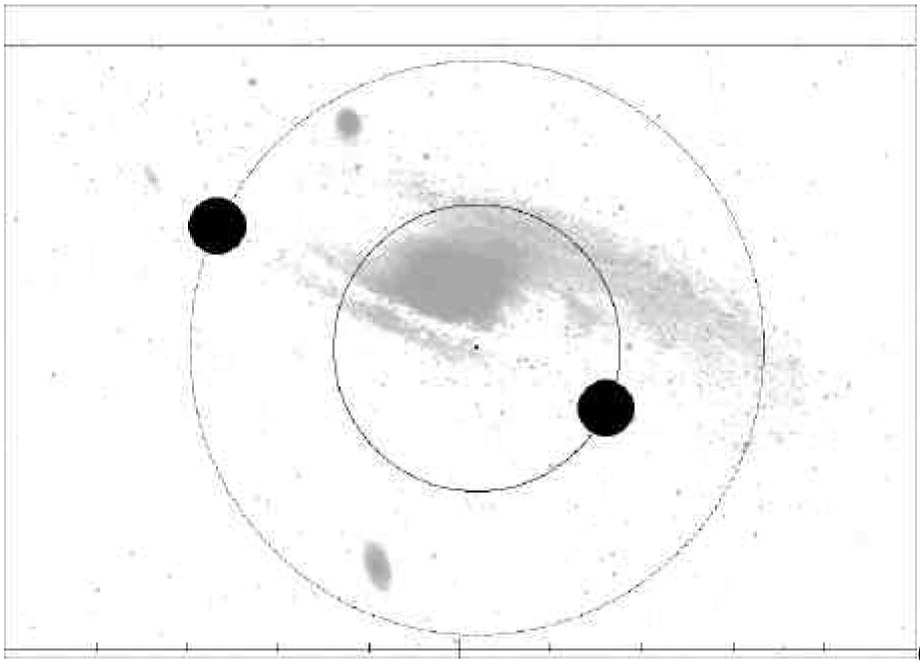
Fig. 4 - *Light curves of binary system with orbits of* ***(a)*** *no inclination,* ***(b)*** *small inclination and* ***(c)*** *large inclination. In this example, both stars have masses with significant difference.*

3.3. Relationship between masses and resultant orbit

If both stars have similar masses, their orbits will be the same with their radii identical, no matter what their volumes are. (Refer to Fig. 5a) However, if their masses differ a lot, the radii of their orbits will also be different. The orbit of the more massive star will have a shorter radius than that of the less massive one. For example, if the mass of a star is twice of its companion, the radius of its orbit is half of its companion. (Refer to Fig. 5b)



(a)



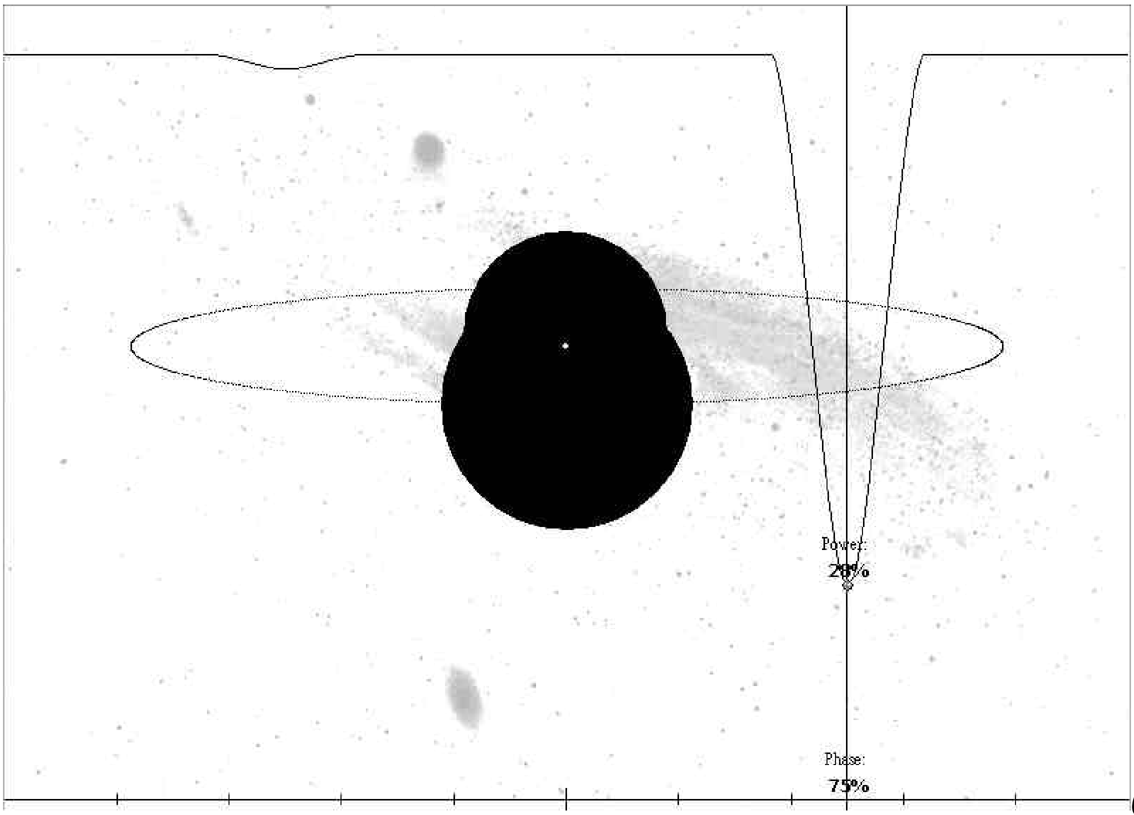
(b)

Fig. 5 *- Orbits of binary system with stars of* ***(a)*** *identical masses but different volume, and* ***(b)*** *masses with 3-time difference but same volume. The orbit with smaller radius is the one of more massive star.*

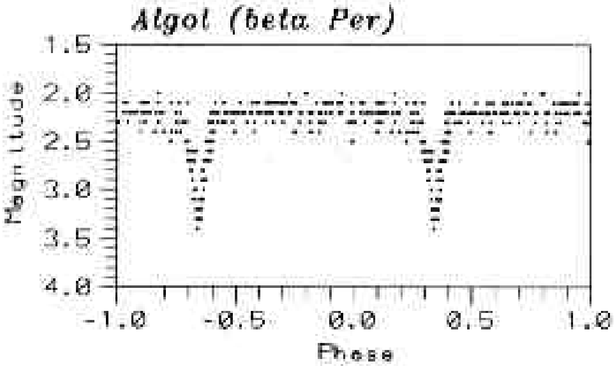
3.4. Example - Algol (Persei b)

Algol is a famous eclipsing binary system. In fact, it is the first eclipsing binary to be discovered. It is known that Algol A is a main sequence star of spectral type B8 (surface temperature=12000K), with mass of $3.59_{\odot}$ and radius of $2.88_{\odot}$. Algol B is a subgiant of spectral type K2 (surface temperature=4888K) with mass of $0.80_{\odot}$ and radius of $3.54_{\odot}$. The average separation between Algol A and Algol B is about 15 solar radius (1.04×10^{10} m). The eccentricity is nearly zero (i.e. the orbit is circular), and the orbit is inclined with angles of inclination α=0°, β=-7.69° and γ=0°. [5]

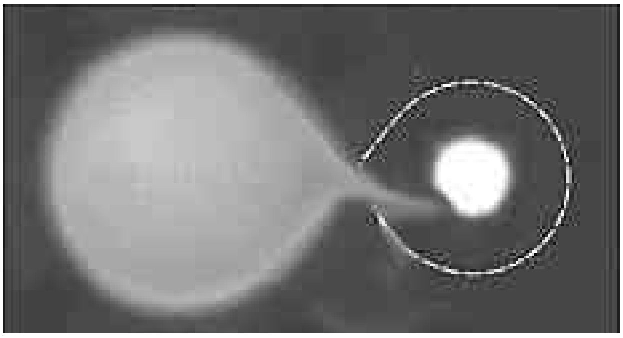
After running the program, it is found that the period is about 3.21 days. The light curve has two minima occurred at 0.5p and 1.5p. It shows that the minimum at 0.5p (minimum *a*) is much shallower than that at 1.5p (minimum *b*). Minimum *a* only has a decrease in brightness of 2%, while minimum *b* has a decrease in visual brightness of 71%. The difference between the maximum and minimum magnitude of the system is about 1.36, which agrees quite well with the observational result. (Refer to Fig. 6a, b) However, the discrepancy between observational result and result from the program is inevitable, because we have neglected the effect of distortion and material transfer. Algol is actually a semidetached binary system where the volume of Algol B is greater than its boundary of the Roche Lobe. Significant transfer of materials occurs, which distorts the shape of Algol B (Refer to Fig. 6c).



(a)



(b)



(c)

Fig. 6 *- Light curves of Algol. In* ***(a)****, the one simulated by the program, it shows that the absolute minimum of the light curve is at t=1.5p, with remaining brightness of 28% only (i.e. 72% decrease in brightness). In* ***(b)****, it shows the light curve of Algol based on experimental data (Adapted from http://www.aavso.org/vstar/vsotm/0199.stm). In* ***(c)****, it shows the material transfer from Algol B to Algol A.*

4. Conclusion

In this project, we have successfully simulated the orbital motion and changes in luminosity of an eclipsing binary system with a computer program, and studied the relationship between the light curve and the characteristics of binary stars with the aid of the program. The program has been proved successful by comparing the output results of the program with actual experimental results of Algol. We believe that the program can be further developed to simulate more than two stars and the assumptions be accounted for. Finally, we sincerely hope the program can ease the work of all who engage in the field of astronomy.

5. Acknowledgements

We would like to thank Dr. K.Y. Michael Wong of Hong Kong University of Science and Technology, our project supervisor, for his guidance and support. (and lunch ☺)

6. Appendix: Derivation of the Luminosity of a Star

Consider a ring of width *a dq* on the surface of the larger star, where *a* is the radius of the star (Refer to Fig. 7). Then the area of the ring *dA* is given by

$$dA = 2prad q \tag{13}$$

where r is the radius of the ring. The power transmitted from the ring dP_0 is

$$dP_0 = 2Fprad q \tag{14}$$

where *F* is the energy flux from the surface of the star.

However, this power is transmitted at an angle of *q* to the line of sight. Hence the observable power from the ring *dP* is

$$dP = 2Fpra \cos q dq \tag{15a}$$

Since $r = a \sin q$, we have

$$dr = a \cos q dq = \sqrt{a^2 - r^2} dq \tag{15b}$$

Hence *dP* becomes

$$dP = 2Fprdr \tag{15c}$$

The total observable power of the star is the area projected onto the plane transverse to the line of sight. In full phase of the star, the observable power of the star *P* is

$$P = \int_0^a dPdr = \int_0^a 2Fprdr = Fpa^2 \tag{16}$$

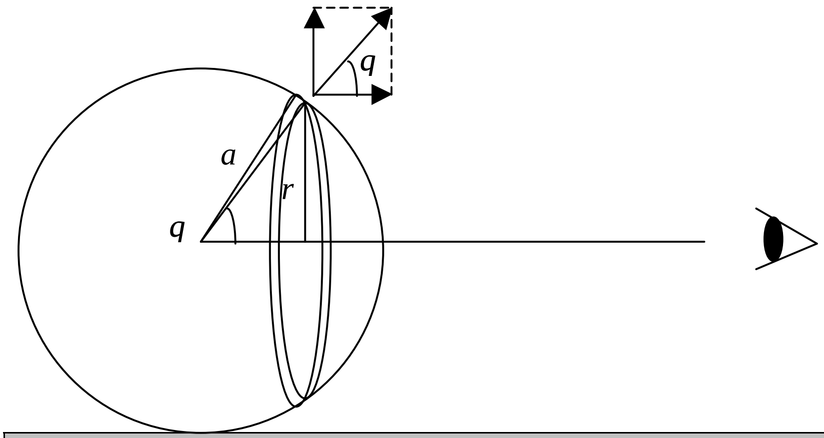


Fig.7 *- Diagram for deriving the power of a star*

7. Reference

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[5] http://www-astro.physics.uiowa.edu/~rlm/lam-page/research/algol/.