The Computer Modeling of Fermi Acceleration of Cosmic Rays

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Abstract: Cosmic Ray has been a hot topic of research for nearly a century. Several ways have been proposed to accelerate the interstellar particles to high energy, and one of these is acceleration by the shockwave produced from the supernovae through collisions between the interstellar particles and the particles in the supernova remnants [1]. What we are interested in is how cosmic ray can be accelerated to that high energy (>10^{10} eV). Here, we will model the collision of the interstellar particles in the shockwave of a supernova and show how the particles can be accelerated.
Key Words: Fermi acceleration, interstellar particles, supernova, supernova remnant

I. Introduction

Cosmic Rays are subatomic particles bombarding the Earth. They were first discovered to be from outerspace when Victor Hess flew a balloon to an altitude of about 16000 ft and found a very penetrating radiation coming from outside atmosphere [2]. In 1949, Enrico Fermi proposed a mechanism, called the Fermi Mechanism, which can explain the high energy level of the Cosmic Rays [3]. According to this model, the interstellar particles collide with the shockwave from a supernova. By the magnetic field and collisions with particles in the supernova remnant, the interstellar particles can be accelerated to an extremely high velocity to become what we call "Cosmic Ray". It is still believed to be a major mechanism of producing highly energetic Cosmic Ray.

II. Modeling

In this project, we simulate the Fermi mechanism by using the computer. In our model, we consider the process of the acceleration to be one-dimensional collisions. We simulate how a particle collides with two oscillating walls repeatedly. Here, the walls represent the planes of the collisions and interactions between the particle and the dense matter in the shockwave produced in a supernova. In addition, there are several assumptions in the model. First, all the collisions are assumed to be perfectly elastic [4], i.e.,

$$\frac{1}{2}mv_{before}^2 = \frac{1}{2}mv_{after}^2 \qquad (1),$$

where $v_{before}(v_{after})$ is the velocity of a particle with mass m before (after) a collision with the wall, as seen in the wall's rest frame. By combining *(1)* with the law of conversation of momentum, we have

$$\begin{cases} v^{particle}_{after} = \dfrac{m-M}{m+M} v_{before} & (2), \\ v^{wall}_{after} = \dfrac{2m}{m+M} v_{before} & (3), \end{cases}$$

where m and M represent the mass of the particle and the wall, respectively [7].

As $M \to \infty$, Equations *(2) and (3)* become

$$\begin{cases} v^{particle}_{after} = -v_{before} \\ v^{wall}_{after} = 0 \end{cases}$$

The velocity of the particle only changes in direction but not its magnitude:

$$v_{after} = -\left| v_{before} \right|,$$

in the wall's rest frame.

Actually, there are two situations causing the 'reflection' of the particle. One is the collision between the particle and the matter in the shockwave and the other is the deflection caused by the irregular magnetic fields produced by the supernova. However, in both cases, the above assumptions still hold. In the actual condition, the matter transported in the shockwave is in very high density. It, therefore, acts as an elastic wall. In the case of deflection, the irregular magnetic field acts as a ' magnetic mirror', which deflects the particle to the opposite direction. As the work done by the magnetic field is perpendicular to the direction of its motion, the particle's speed does not change. Therefore, we can regard these as perfectly elastic collisions. Although it seems that the particles have not been accelerated yet, the speed in fact changes after the transformation from the wall's (shockwave's) frame to the laboratory's frame.

For simplicity, the walls are assumed to be performing simple harmonic motion. We define

$$\begin{cases} L(t) = L_o + A\sin(?\,t + ?_i) & (4), \\ U(t) = A?\cos(?\,t + ?_i) & (5) \end{cases}$$

where the position of the wall, $L(t)$, and the velocity of the wall $U(t)$ are all set to be functions of time, t, and L_o, A, ? and ? are constants. When a collision occurs, the position of the particle will be equal to that of one wall, i.e. $X(t)=L(t)$. We use the computer to solve the equation and get the value of t as a solution. By putting t into *(5)*,

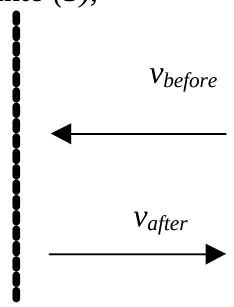

Fig. 1a) Collision in the wall's rest frame: the wall is at rest.

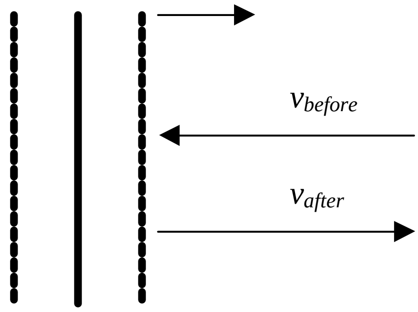

Fig. 1b) Collision in the laboratory's frame: the wall is moving at a velocity, U.

we can get the velocity of the wall and thus v_{after} can be calculated.

III. Transformation

All the results that we derived above are in the wall's frame. Therefore, transformation to the laboratory's frame is needed. We define the velocities of the particle before and after the collision to be v_o' and v' in the lab frame *(Fig. 1a* and *Fig. 1b)*.

i.e. $v_o \to v_{before}$

$u \to U$

$v \to v_{after}$

According to special relativity, we have

$$\begin{cases} v_o' = \dfrac{v_o + u}{1 + uv_o} & (6) \\ v' = \dfrac{v + u}{1 + uv} & (7) \\ v = -v_o & (8) \end{cases}$$

We therefore have

$$v_o = \frac{v_o' - u}{1 - uv_o'} \quad ,$$

$$v = \frac{v' - u}{1 - uv'} \quad ,$$

$$\frac{v' - u}{1 - uv'} = -\left(\frac{v_o' - u}{1 - uv_o'}\right)$$

Finally, we can express v' in terms of u and v_o', which are all in the lab frame:

$$v' = \frac{2u - u^2 v_o' - v_o'}{1 - 2uv_o' + u^2} . \qquad (9)$$

IV. Simulation of Supernova

In order to make the model more reliable, we use actual data of supernova remnants in the parameters. We first consider the initial velocities of the particles. As the interstellar particles are nearly at rest in the galaxy, we can put the velocity of the supernova remnant to be the initial velocity [5] [8]. Then, we have to think about the velocity of the particles in the supernova remnants, i.e. the velocity of the wall in the model.

As the temperature of the supernova remnant can be high, we get the velocity of the wall by using the gas law [9],

i.e. $\dfrac{1}{2}mv^2 = \dfrac{3}{2}\dfrac{R}{N_A}T$

$$\bar{v} = \sqrt{\frac{3RT}{mN_A}} \qquad (10)$$

where $\bar{v}$ is the root-mean-square speed of the particles in the supernova remnants. Also, the velocity of the wall depends on the cosine function. So, the velocity can be expressed as

$$v = \frac{2(Aw) \times \int_{\frac{p}{2}}^{\frac{3p}{2}} \cos wt \quad dt}{2p} \qquad (11)$$

By combining *(11)* and *(12)*,

$$A = \sqrt{\frac{3RT}{mN_A}} \times \frac{p}{2}$$

where mN_A is the molar mass of the particles.

The other parameters in the model are generated by a random number generator.

V. Results and Discussion

We first consider the walls in the model to be anti-phased and with the same angular frequency and amplitude.

1. Various initial velocities

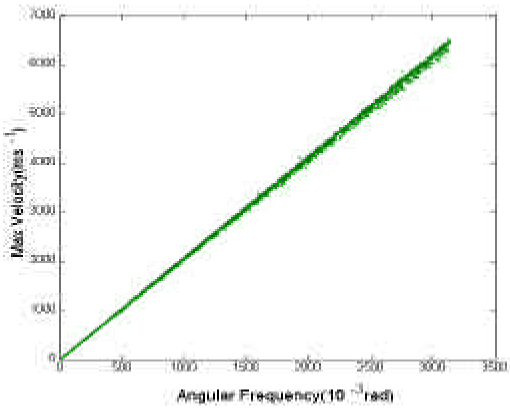

Fig. 2a) Particle with constant initial velocity

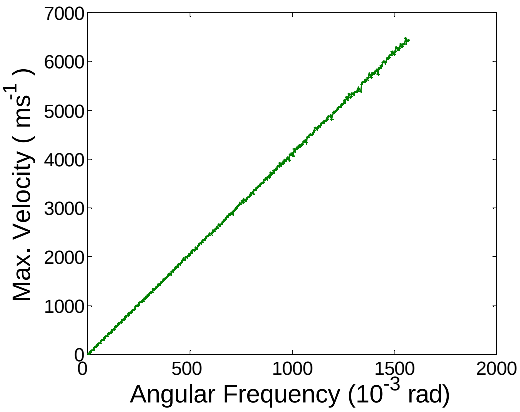

Fig. 2b) The v_m-? graph of the model, with the initial velocity of the particle directly proportional to the angular frequency ? .

Fig. 2a shows that the angular frequency is directly proportional to the maximum velocity of the particle. However, there are some points that are slightly off the straight line. On the other hand, in *Fig. 2b*, all the points lie on the same straight line. So, we infer that the dispersion of the data depends on the initial conditions.

We first consider a set of particles in models with different angular frequencies. If we put the initial velocity to be

$$u = kw \quad ,$$

where k is a constant,

$$\frac{l}{wk} = \frac{t'}{w} \quad ,$$

where l is the distance traveled by the particle and $t' = tw$, we have

$$Aw\cos(wt) = Aw\cos(t')$$

$$L_o + A\sin(wt) = L_o + A\sin(t')$$

Since t' is taken to be the "time unit" where it is the time for the particle traveled with w=1, it should be a constant value. The particle should collide with the elastic walls in the same position and the acceleration is different according to the angular frequency, i.e. $a = wj$, where j is a constant value according to the velocity of the wall. The velocity corresponding to different angular frequencies after the collision should be in ratio according to the angular frequency. The velocity can also be written in teams of $u = kw$. Therefore, the particles will continue to be at velocity kw and collision at the same position with the others in different angular frequency.

It proved that there is no resonance as the angular frequency is varied.

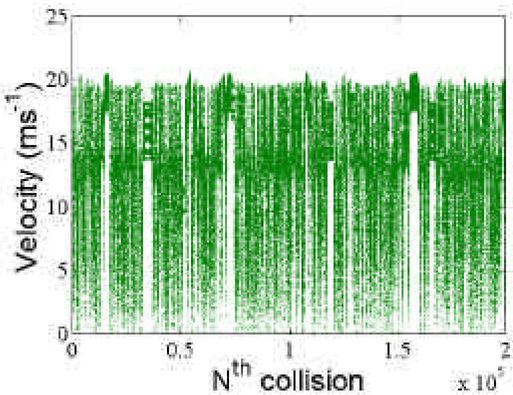

Fig. 3a) The velocity changes in the process of collision. The initial velocity is less than 19.5 ms^{-1}.

In *Fig. 3a*, the velocity of the particle is changing with the number of collisions. When its velocity rises to a relatively high value, of about 19ms^{-1} it will drop or remain at the value. Therefore, we tried to use an initial velocity which is a little bit higher than that value.

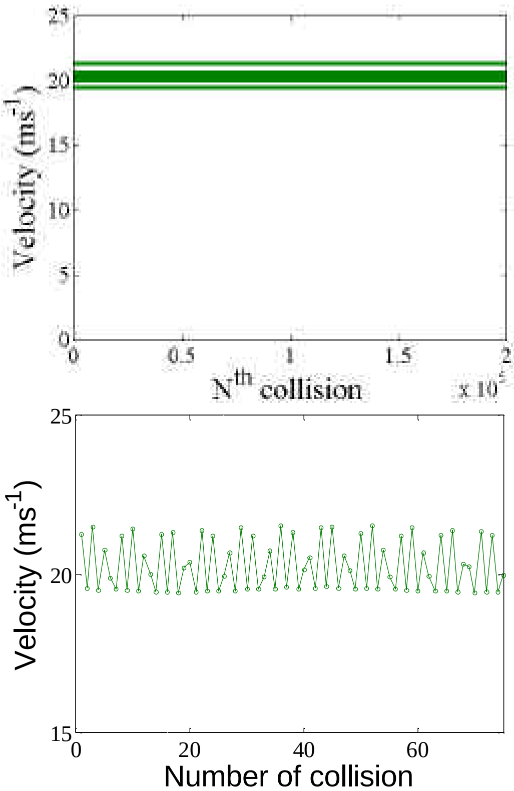

Fig. 3b) The velocities of the particles flow into a band.

When the initial velocity of the particle is greater than a particular value, about 19ms^{-1} in this case, the velocity of the particle will fall into a band *(Fig. 3b)*. The velocity will remain in the band indefinitely.

2. Various path lengths

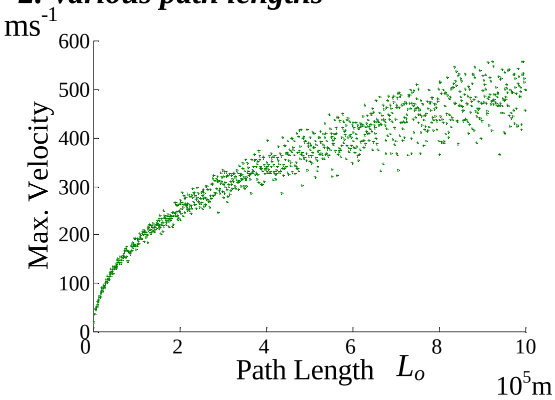

Fig. 4) The maximum velocity against path length

In *Fig. 4*, it can be seen clearly that the longer the path the higher is the maximum velocity. There is a limit to the maximum velocity, which is the same phenomenon as in *Fig. 3a*.

3. Various phase differences between the two walls

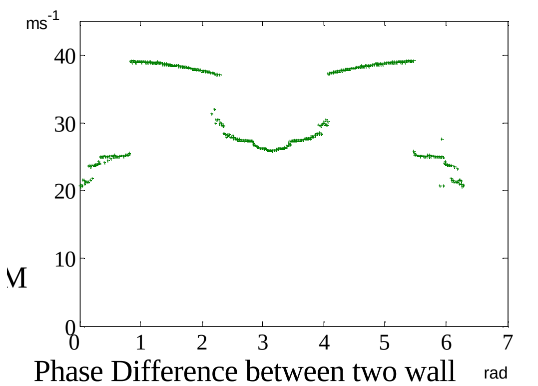

Fig. 5) The maximum velocity against angular frequency of the walls

In *Fig. 5*, we observe very interesting phenomenon. The graph plotted is symmetric about an w difference equals to p . This is because the velocity and position of the wall are,

$$Aw\cos(wt+q_i)=Aw\cos(wt-(2p-q_i))$$

$$A\sin(wt+q_i)=A\sin(wt-(2p-q_i))$$

the angular frequency can be expressed as $(2p-q_i)$, so the symmetric phenomenon appears.

4. Energy distribution

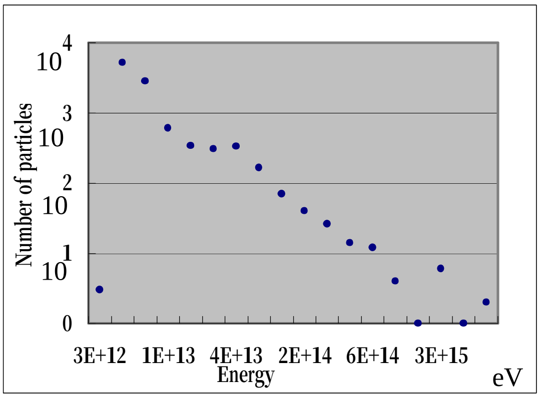

Fig. 6a) The distribution of the particle energy in Newtonian mechanics

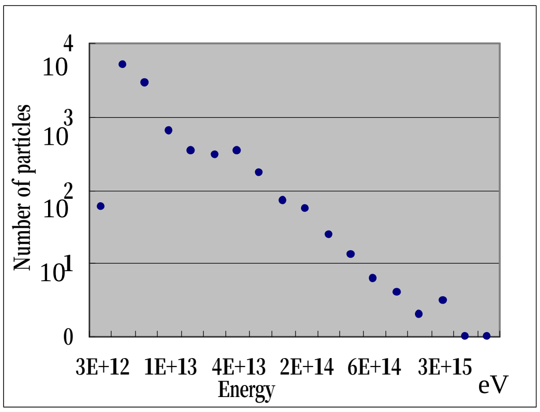

Fig. 6b) The distribution of the particle energy taking into account special Relativity

In *Fig. 6a* and *Fig. 6b*, we show the distribution of the energy of 10000 particles in Newtonian mechanics and Special Relativity. Obviously, the tendency and the shapes of the graphs are almost the same, which means that relativity only has a small effect on the model. On the other hand, a very large number of particles concentrated around the energy 3.9×10^{12} eV. This demonstrates that Fermi acceleration is able to perform acceleration to a high energy.

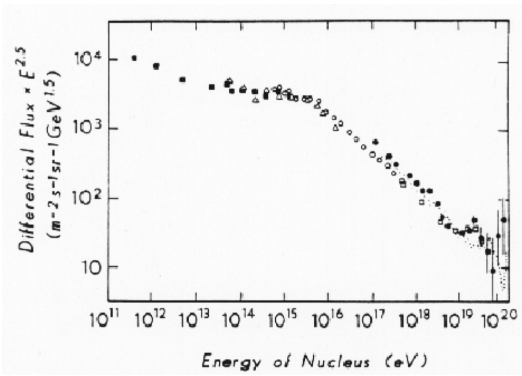

Fig. 6c) Cosmic ray spectrum with real cosmic ray data [6]

Fig. 6c is the energy distribution of real cosmic ray particles. The main characteristic of the distribution is the segment of a segment of straight line, which means the flux of cosmic rays is proportional to the energy to some power. This characteristic is also shown in *Fig. 6a* and *Fig. 6b*, which reveals the modeling is in a level similar to reality. However, there are still some problems with it. The particles in the model cannot be accelerated to an extremely high energy value. This is partly caused by our assumption that the particles are hydrogen particles and the acceleration by magnetic fields are not included in our model. Also, we only considered the Fermi Acceleration in one-dimension, whereas a more realistic model should be three-dimensional. These may be the reasons for the limitation of acceleration.

VI. Conclusion

As we see from the results, it is possible to produce highly energetic cosmic rays with energy greater than 10^{10} eV. Also, our model, though very simple, can simulate the actual situation by producing a number of particles that fit the distribution. Our model produces an approximate power law relating the flux and the energy, in agreement with observed data. In addition, we found that the maximum velocity of the particle is proportional to the angular frequency of the oscillation of the wall. As the angular frequency depends on the speed of the shockwave, we conclude that the maximum velocity of particles is dependent on the nature of the supernova, i.e. the greater amount of the energy released from it, the more energetic cosmic rays will be produced.

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VII. Reference

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