Surface Wave on Water

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Abstract

Surface wave is an everyday phenomenon that is not discussed in secondary school textbooks. It is very different from most other waves in that its velocity is strongly dependent on water depth and wavelength. In order to observe this interesting phenomenon, we have built a large water tank (6ft x 4ft x 2ft) to investigate the relation between wavelength and wave velocity under different water depths. Our measured wave velocity deviates significantly from the theory when the wavelength is small (< 2 cm). By adding an agent to reduce the surface tension, we noticed that the wave velocity increases with surface tension. The effect is noticeable only for small wavelengths.

The wake generated by boats of different sizes travelling at different velocities was observed in a field trip. The wavelength, direction of the wave crests and direction of the wave group were reported.

I. Introduction

A wave is a disturbance that transfers energy progressively from point to point in a medium. The disturbance may take the form of elastic deformation or variation of physical quantities such as pressure, electric or magnetic intensity, or temperature.

Waves are usually analyzed in terms of sinusoidal waves. There are several physical quantities used to describe sinusoidal waves. The **amplitude** (A) is the maximum displacement of the wave's disturbance. The **wavelength** (λ) is the distance between peaks or troughs. The **period** (T) is the time elapsed for two consecutive crests to pass over a given point. **Frequency** (f) is the number of complete wave cycles produced or travelling through in one second. The **velocity** (v) is the distance travelled by the wave crest in a given unit of time. This velocity is also called the phase velocity (v_{ph}).

We now describe two interesting phenomena related to surface waves.

1. Why waves break?

Have you ever seen waves break? If you have, do you know why? Indeed, it is related to the depth of water on which the waves are travelling [1]. If the water depth is less than one-twentieth of the wavelength,

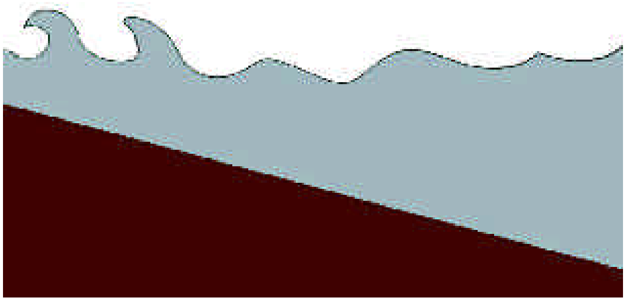

Fig.1. An illustration showing the breaking of waves as surface waves travel to shallow water.

all waves travel at nearly the same speed. The shallower the water, the slower they go.

As a series of wave crests approaches the shore, the leading one reaches shallow water earlier and its speed decreases. Since the rear wave crests are still in deep water, they travel faster and so they move closer to the crests in the front. Thus the wave and its energy are squeezed in one narrow zone. With more energy, water at the crest travels ahead faster. Yet, the wave as a whole continues to slow down due to the shallower water. Eventually, the velocity of water at the crest is faster than the wave. The crest gets ahead of the wave, and the wave breaks. Figure 1 illustrates the phenomenon.

The theory on surface wave velocity has been known for over 100 years [2]. For example, the author of the book "Exploring Ocean Science" stated that $v_{ph}=1.25*\lambda^{1/2}$ for deep-water waves (water depth > $\lambda/2$). For shallow-water wave (water depth < $\lambda/20$), $v_{ph}=3.13*d^{1/2}$, where d is the water depth. A formula covering both deep- and shallow-water waves is given in Ref.[2].

2. The wake of a ship/boat - does the pattern consist of just two straight lines?

Have you ever observed the wake pattern produced by a ship or a boat? When the velocity of the boat is greater than that of the wave, a wake pattern is observed. Interestingly, the pattern does not consist of just two straight lines, which form a triangle on the water surface. Instead, the pattern is shown in Figure 2. It consists of a wave group formed by a series of short lines that mark the wave crests. The wave group and the line of wave crest make angles θ_1 and θ_2, respectively, to the direction that the boat is moving.

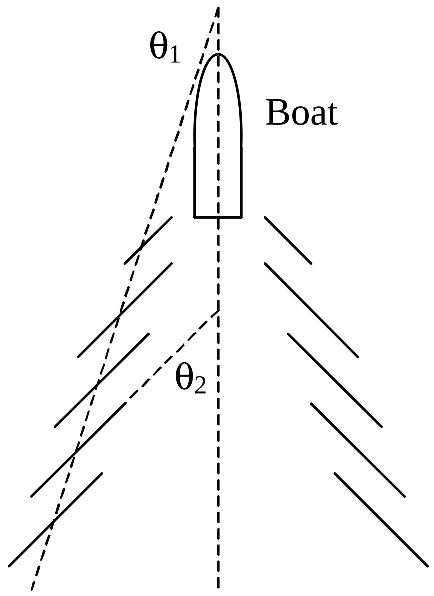

Fig.2. The wake produced by a small boat. θ_1 (θ_2) is defined to be the angle between the boat's velocity and the wave group (wave crest).

We have made a field trip to Tolo Harbour to observe the wake produced by public ferry and motorboat. The motorboat speed can be varied at will. The boat speed, wavelength, angle of the wave group and angle of the wave crest were measured. Here are the data collected by us.

Velocity (m	Wavelengt	Wavelengt	Angle of wave group θ_1 (degree)	Angle of wave crest θ_2 (d

s - 1)	h (f t)	h (m)		eg ree)
2.8	(2.5)	(0.76)	(20)	(40)
3.8	(6)	(1.8)	(22)	27
4.9	(8)	(2.4)	19	38
8.8	(7)	(2.1)	(15)	11
12.5	(7)	(2.1)	8	13

Table 1: Information collected during an observation on the wake generated by boats. θ_1 measures the direction of the wave group and θ_2 measures the direction of the wave crests in the wake. The values in brackets are estimates obtained by direct on-site observations. Other values are more accurate and were measured after the process was recorded with a video camera.

We found that the angle of the wave crest (θ_2) is roughly twice the angle made by the wave group (θ_1). Moreover, when the velocity of the boat increases, both angles decrease while the wavelength increases. Larger boats are able to create waves of greater wavelength. The theory behind all these is beyond our understanding. On private discussions with our supervisor (HF Cheung), he pointed out that the angle-doubling effect can be explained starting from the relation between wave velocity and wavelength. Since the phase velocity depends on the wavelength, wave group actually travels at a different velocity called the group velocity. For surface wave on water, the group velocity is only half of the phase velocity typically.

II. Objectives

We want to measure the relation between wavelength and velocity of waves in a tank for different depths of water. In relation to this, we also like to study the effect of surface tension. To relate the study to the real world, we also observed the propagation of water waves caused by boats in the Tolo Harbour, and the results have already been presented in the last section.

III. Method

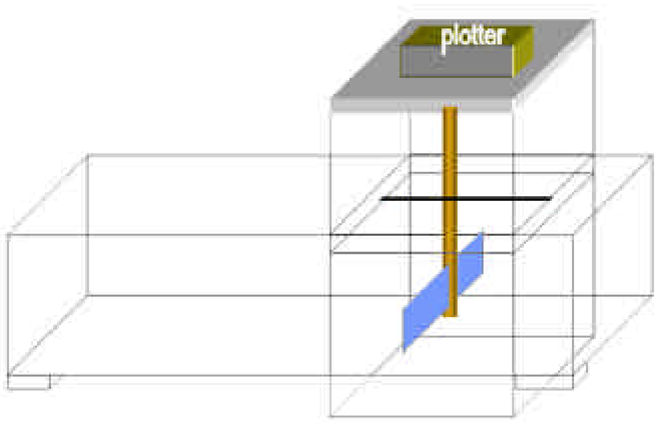

Fig.3. A sketch of the water tank built by us.

We have built a big water tank (6 ft × 4 ft × 2 ft) using thick clear Acrylic sheets (see photos in Appendix 2). The motor of a x-y plotter was used to generate water waves in the tank. An electronic signal generator fed a sinusoidal signal to the motor and so controlled the frequency of the waves. A lamp or fluorescent light acted as parallel light source to project the waveform onto a screen. The floor acted as the screen in our experiment. Other equipment included an electronic timer to measure the period, a meter ruler to measure the wavelength, and a video camera to record the wave motion.

Methods to measure wavelength

We used different methods to measure the wavelength. In all of these methods a video camera was used to record the wave motion. Frames from the video picture were selected and then the wavelength was measured using a meter ruler.

Method 1 used standing waves. The signal generator was adjusted so that a standing wave was produced. Method 2 used waveform projection. The waveform was projected onto the floor. Method 3 measured the distance traveled by the wave in a certain period of time. From there, the velocity and wavelength were deduced. Method 4 used a long straight light source. Light from this source was reflected from the water surface, which caused distortion on the image of the light source. The wave motion was then recorded onto the video picture and the wavelength was subsequently measured (see photos in Appendix 2).

<u>Method to measure surface tension coefficient</u>

A capillary tube must be rinsed with a suitable cleaning agent (chromosulphuric acid) before the measurement. The height of the capillary rise (h) was measured with a meter ruler. The diameter (2r) of the tube was measured with a travelling microscope. We now give a brief description of the principle involved. When there is a capillary rise, the force of gravity is balanced by the surface tension. The forces are:

Upward force (surface tension) = $2\pi r\gamma$

Downward force (force of gravity) = mg = $\pi r^2 h\rho g$.

From this, the surface tension coefficient can be calculated from $\gamma = r\rho gh/2$. We first measured the surface tension of water. Then ethylene glycol was used to modify (lower) the surface tension of water in our tank and the surface tension was again measured.

Diameter of capillary pore (2r) = 1.2 mm

Density of water (ρ) = 1000 kg/m^3

Gravitational acceleration (g) = 9.8 m/s^2

Capillary rise (h) = 23 mm (water)

Capillary rise (h) = 10 mm (with ethylene glycol)

The results for the surface tension coefficient measurements are:

γ = 68 mNm^{-1} (water)

γ = 29 mNm^{-1} (with ethylene glycol)

From date book [3], the surface tension of water against air is

Temperature (°C)	20	25
γ (mNm^{-1})	72.75	72.0

Our measured value of the surface tension coefficient agrees well with the data-book value.

The largest error was in the diameter. That error is estimated to be roughly 5%.

IV. Results of our experiment on the velocity of surface wave

We have made measurements of the wave velocity for water depths from 2.5 cm to 26 cm. In most cases the uncertainty was about 5 %. Occasionally the uncertainty was up to 15 % for waves with highly erratic waveforms. We present results only for the most interesting water depths. More results are given in Appendix 1. The theoretical wave velocity was obtained from a formula in Ref.[2].

Table 2: Wave velocity for water depth = 0.025m

Period (s)	Wavelength (m)	$k = 2\pi/\lambda$	$w = 2\pi f$	v_ph (m/s)	v_ph_predicted
1.559	0.767	8.20	4.03	0.492	0.495
1.045	0.490	12.8	6.01	0.469	0.494
0.714	0.360	17.5	8.80	0.504	0.490
0.513	0.250	25.1	12.2	0.487	0.475

0.359	0.151	41.6	17.5	0.421	0.427
0.252	0.0933	67.3	24.9	0.370	0.361
0.181	0.0586	107	34.7	0.324	0.296
0.125	0.0329	191	50.3	0.263	0.225
0.090	0.0214	294	69.8	0.238	0.182
0.063	0.0160	393	99.7	0.254	0.158
0.045	0.0116	543	140	0.257	0.134

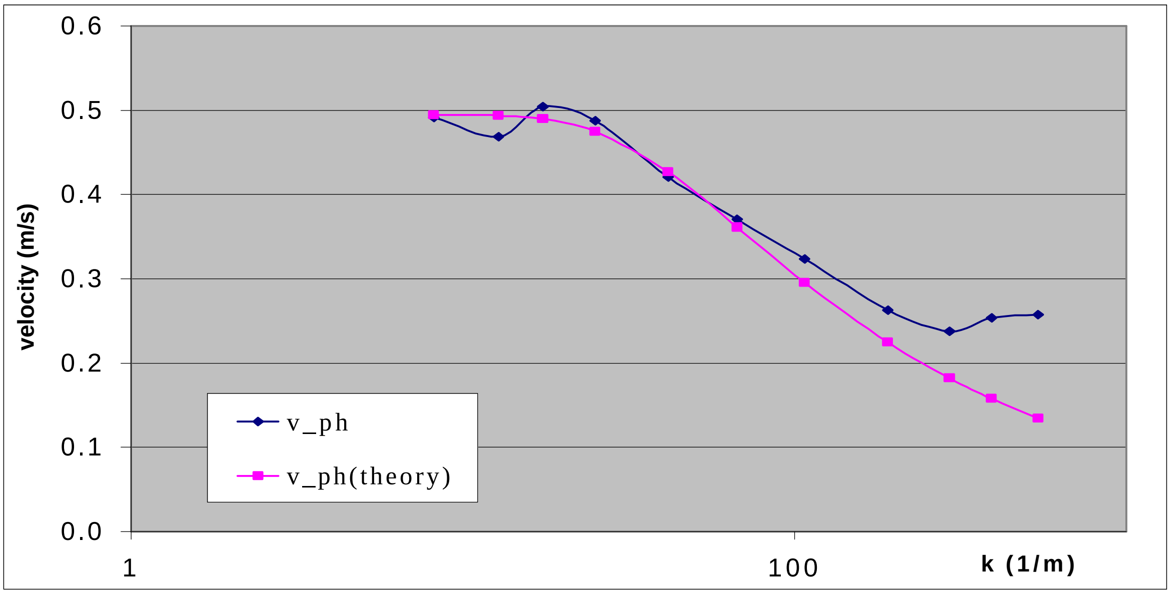

Fig.4. Graph of the wave velocity for water depth = 0.025m

Table 3: Wave velocity for water depth = 0.040m (surface tension $\gamma = 68\ mNm^{-1}$)

Period	Wavelength	k	W	v_ph	v_ph_predicted
0.364	0.187	33.7	17.3	0.513	0.497
0.251	0.102	61.8	25.0	0.405	0.388
0.181	0.0577	109	34.7	0.319	0.297
0.125	0.0328	191	50.3	0.263	0.226
0.09	0.0219	286	69.8	0.244	0.185
0.063	0.0153	411	99.7	0.243	0.154
0.045	0.0116	541	140	0.258	0.135

Table 4: Wave velocity for water depth=0.040m (with ethylene glycol, $\gamma = 29\ mNm^{-1}$)

Period	Wavelength	k	w	v_ph	v_ph_predicted
0.364	0.187	33.6	17.3	0.514	0.498

0.251	0.101	62.5	25.0	0.400	0.386
0.181	0.0560	112	34.7	0.309	0.293
0.125	0.0295	213	50.3	0.236	0.214
0.09	0.0189	332	69.8	0.210	0.172
0.063	0.0123	510	99.7	0.196	0.139
0.044	0.009	698	143	0.205	0.118

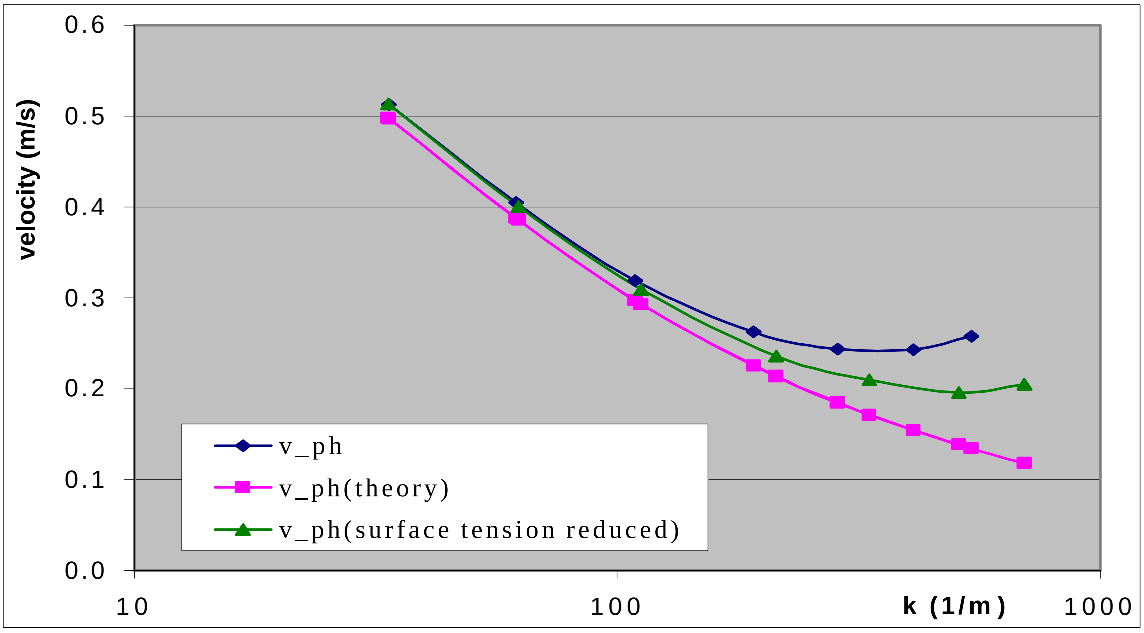

Fig.5: Graph of the wave velocity for water depth = 0.040m

V. Discussion

From Figure 4, we found that the experimental velocity agrees with the theoretical velocity for long waves, including both shallow-water and deep-water waves. Experimental error (5-15%) can account for the differences between the experimental and theoretical values. However, for short waves (λ < 2cm), we found that the differences are large and cannot be accounted for by experimental error alone.

When the measurements were repeated after the addition of ethylene glycol to reduce the surface tension, we found that the data (Figure 5) are in better agreement with theory for short waves. We found from Ref.[1] that when the wavelength is very small, surface tension instead of gravity becomes the dominant restoring force, and serves to pull raised regions down and depressed regions up. Thus, the phase velocity increases with increased surface tension.

VI. Conclusion

We conclude that the measured wave velocities agree with the theoretical values when the wavelength is large (> 2cm). When the wavelength is small, the wave velocity is affected by surface tension. We also found that the wake produced by a boat consists of parallel lines. The angle of the wave crest is roughly twice the angle of the wave group.

Acknowledgements

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References

[1] Keith Stowe, *Exploring Ocean Science*, 2nd edition (John Wiley and sons, 1996).

[2] Horace Lamb, *Hydrodynamics*, 6th Edition (Dover, 1945).

[3] Kaye and Laby, *Table of Physical and Chemical Constants*, 16th edition (Longman, 1995).

Appendix 1 – More data on the wave velocity

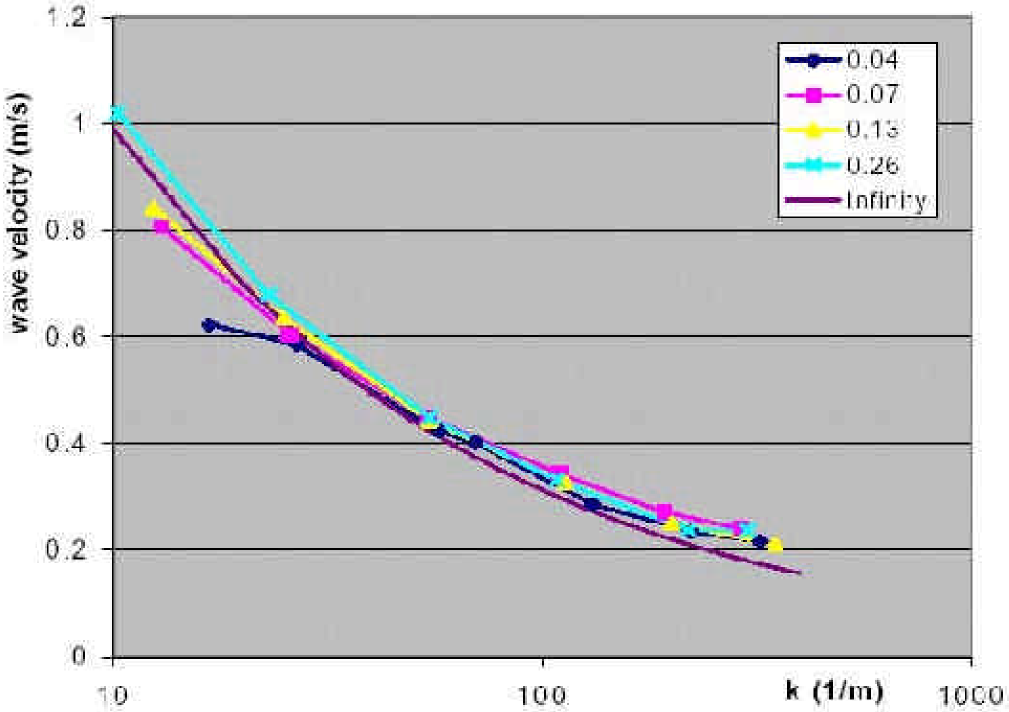

Fig 6. The measured wave velocity for water of depths 0.04m, 0.07m, 0.13m, and 0.26m. The theoretical results for infinite water depth are also shown. When the wavelength is much greater than the water depth, the wave velocity tends to a constant, as shown by the data for the 0.04m water depth. At shorter wavelengths, the wave velocity is affected by surface tension.