Discrete Flavor Symmetries in D-brane models

fernando marchesano

Motivation

- What does String Theory has to say about flavor?
  - Hierarchies in mass matrices

    example: F-theory GUTs

    Diego & Gianluca's talks

  - Flavor symmetries

    in particular discrete flavor symmetries

    Reviews:
    Ishimori et al '10
    Altarelli & Feruglio '10

- Discrete flavor symmetries are are used in BSM model building to
  - Explain quark textures and lepton masses and mixings
  - Avoid FCNC in the MSSM

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Questions:

How generic are discrete flavor symmetries in s.t.?
What is their origin? 4d field theory description?
Which kind of groups & reps appear?

Motivation

- To answer these questions, we must learn to realize discrete symmetries in string theory

- However, quantum gravity does not seem to like global symmetries

  see e.g. Banks & Seiberg '11

  Banks & Dixon '88

  - microscopic arguments in string theory

  - general arguments in black hole evaporation

and so, in the context of string theory in order to realize exact symmetries one should look for discrete gauge symmetries

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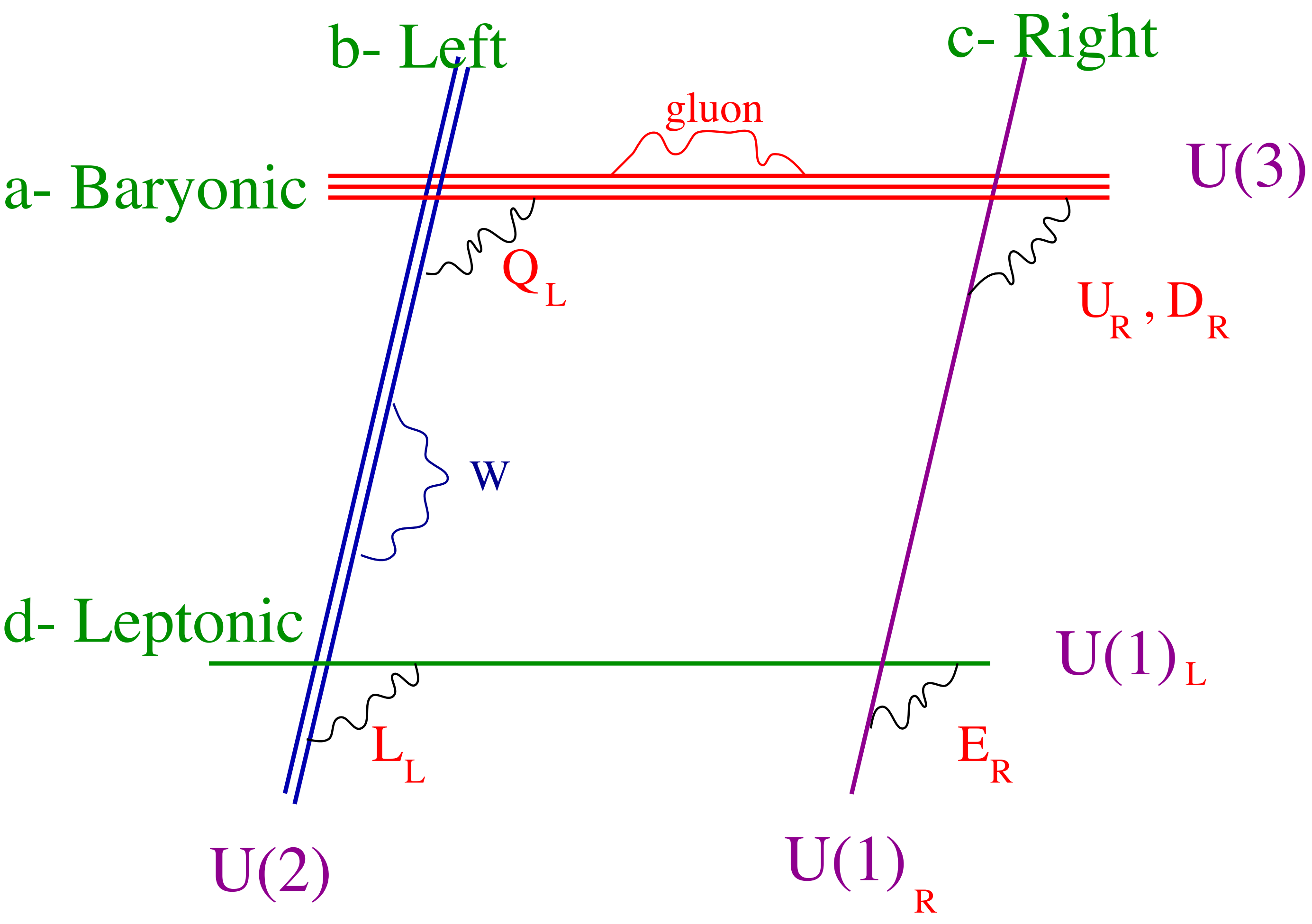
and so, in the context of string theory in order to realize exact symmetries one should look for discrete gauge symmetries

- Examples in the literature:
  - Z_N symmetries $\subset$ anomalous U(1)'s — *coming soon*
  - Compactifications with fluxes — *Mikel's talk*
  - Compactifications with torsion cycles — *Cámara, Ibáñez, F.M. '11*

DGS in D-brane models

- Semi-realistic D-brane models generically contain U(1) gauge symmetries beyond $U(1)_Y$



DGS in D-brane models

- Semi-realistic D-brane models generically contain U(1) gauge symmetries beyond $U(1)_Y$

- Most of them acquire a mass via a Stückelberg mechanism

$$\mathcal{L} \supset kB \wedge F \quad \Rightarrow \quad \mathcal{L}_{\text{Stk}} = \frac{1}{2}(d\phi + kA)^2 \qquad (d\phi = *_4 dB)$$

$$\Rightarrow \quad M_{U(1)} \sim M_s$$

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- Such symmetries are broken by D-brane instantons, which generate effective couplings forbidden by the U(1) symmetry

$$W \sim \Phi^{nk} e^{-2\pi n T} \qquad T = \rho + i\phi$$

invariant under

$$\begin{array}{ccc} A & \rightarrow & A + d\lambda \\ T & \rightarrow & T + ik\lambda \end{array}$$

➡ Mechanism to generate suppressed couplings (Yukawas, neutrino Majorana masses ...)

Blumenhagen, Cvetic, Weigand '06

Ibáñez & Uranga '06

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invariant under

$$A \to A + d\lambda$$
$$T \to T + ik\lambda$$

- However, if k is non-trivial, they still have to preserve a residual $\mathbb{Z}_k$ gauge symmetry ⇒ some couplings are forbidden at all levels

Berasaluce-González et al. '11

Couplings and symmetries

- Consequence: The symmetries of a compactification and their nature are relevant for the structure of couplings in the effective theory.
- Previous example: D-brane U(1) symmetries are made massive by a Stückelberg mechanism, only broken by non-perturbative effects $\rightarrow$ to a subgroup $\mathbb{Z}_N$

Tree level	Y_{ijk}
Non-perturbative	$Y_{ijk}\ e^{-2\pi T}$
Forbidden	0

Discrete Gauge Symmetries in 4d QFT

Discrete gauge symmetries in 4d

- Basic Lagrangian for a $\mathbb{Z}_k$ gauge symmetry

$$\mathcal{L} = \frac{1}{2}(d\phi - kA)^2 - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} \qquad \phi \sim \phi + 1$$

axion

- Gauging of a shift symmetry by a U(1)

$$A_\mu \to A_\mu + \partial_\mu \lambda \qquad \phi \to \phi + k\lambda$$

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- Dual description:

$$\mathcal{L}' = \frac{1}{2}H \wedge *H + kB \wedge F + \frac{1}{2}F \wedge *F \qquad (d\phi = *_4 dB)$$

we can read the remaining $\mathbb{Z}_k$ symmetry from the coefficient of the BF coupling

Banks & Seiberg '11

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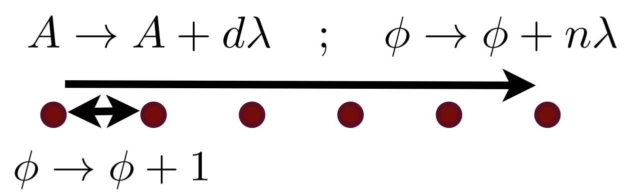
$$A_\mu \to A_\mu + \partial_\mu \lambda \qquad \phi \to \phi + k\lambda$$

- On the other hand, we can interpret k as a winding number between the $S^1 = \mathbb{R} / \Gamma$, where the axion lives and the $U(1) = S^1 = \mathbb{R} / \Gamma'$ of the gauge theory



Discrete gauge symmetry = $\dfrac{\Gamma}{\Gamma'} = \mathbf{Z}_k$

= identifications of ϕ not taken into account by the gauge symmetry

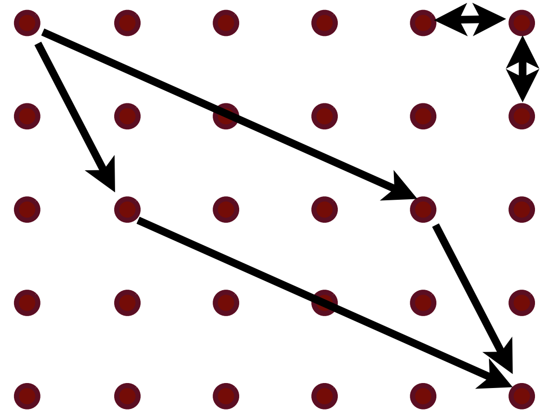
Discrete gauge symmetries in 4d

- **Multiple Abelian** case:

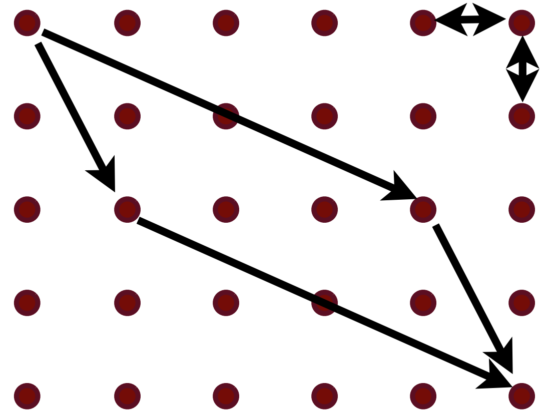
$$(\partial_\mu \phi^a - k_i^a A_\mu^i)(\partial_\nu \phi^b - k_i^b A_\nu^i)\, \eta^{\mu\nu} \delta_{ab}$$

$$P = \frac{\Gamma}{\Gamma'} \rightarrow |P| = \det k$$

$$\vec{\phi} \simeq \vec{\phi} + \vec{k}_i \qquad \phi_a \simeq \phi_a + 1$$



Discrete gauge symmetries in 4d



- Multiple Abelian case:

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- Non-Abelian case:
  - Axion-like scalars with non-commuting shift symmetries

$$\partial_\mu \phi^a \partial^\mu \phi^b G_{ab}(\phi) \qquad \phi^b \rightarrow \phi^b + \epsilon^A X_A^b \qquad [X_A, X_B] = f_{AB}^C X_C$$

Axionic manifold → group manifold or quotient by discrete subgroup M/Γ

  - Gauging of the axionic manifold: $\partial_\mu \phi^a \rightarrow \partial_\mu \phi^a - k_i^a A_\mu^i$
  - Discrete gauge symmetry again given by: $P = \dfrac{\Gamma}{\Gamma'}$

An example

- Simple example: Heisenberg group $\mathscr{H}_3$

$$[X_1, X_2] = X_3$$

- Axionic Lagrangian:

$$G_{ab}(\phi)\partial_\mu\phi\partial^\mu\phi = \mathcal{K}_{ab}\eta^a_\mu\eta^{b\,\mu}$$

$$\begin{aligned} \eta^1_\mu &= \partial_\mu\phi^1 \qquad \eta^2_\mu = \partial_\mu\phi^2 \\ \eta^3_\mu &= \partial_\mu\phi^3 + \frac{1}{2}(\phi^1\partial_\mu\phi^2 - \phi^2\partial_\mu\phi^1) \end{aligned}$$

- $\mathscr{H}_3$ non-compact but $\mathscr{H}_3/\Gamma$ compact → twisted 3-torus

$$\Gamma : \begin{cases} \phi^1 \to \phi^1 + 1 & \phi^3 \to \phi^3 - \frac{\phi^2}{2} \\ \phi^2 \to \phi^2 + 1 & \phi^3 \to \phi^3 + \frac{\phi^1}{2} \\ \phi^3 \to \phi^3 + 1 & \end{cases}$$

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- Upon gauging: $P = \frac{\Gamma}{\Gamma'} = (\mathbf{Z}_k \times \mathbf{Z}_k) \rtimes \mathbf{Z}_k$
  - k=2 → P = D_4
  - k=3 → P = Δ (27)

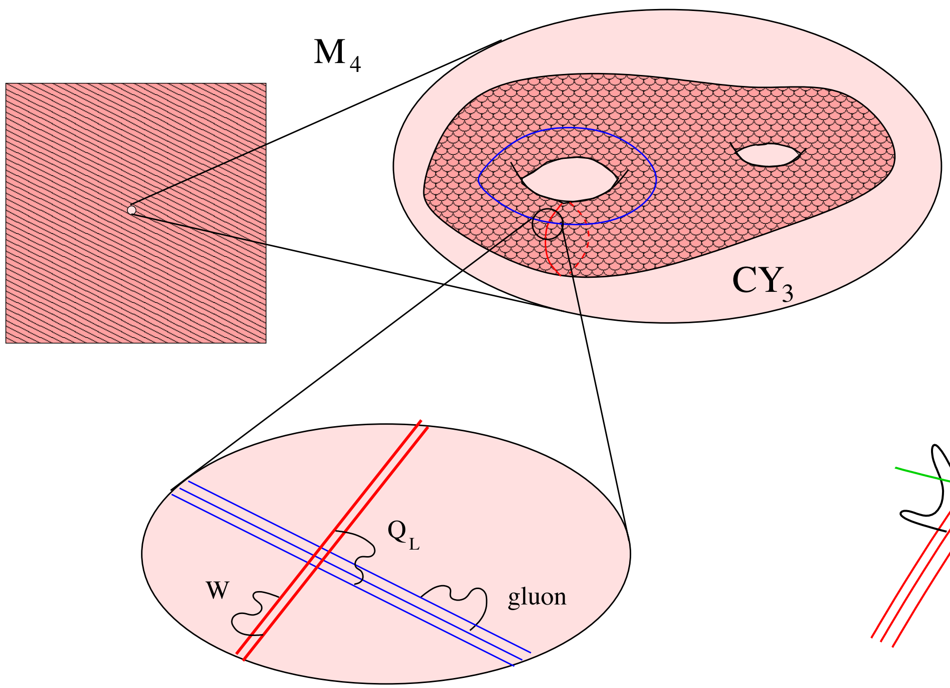
$$T_1^k = T_2^k = T_3^k = 1$$

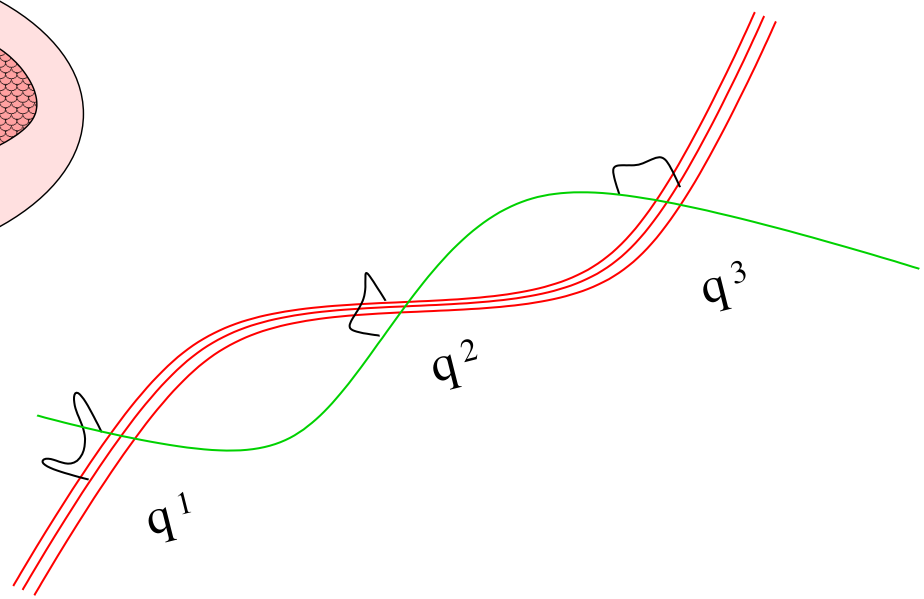
$$T_1 T_2 = T_3 T_2 T_1$$

Discrete Flavor Symmetries from D-branes

DFS & intersecting branes

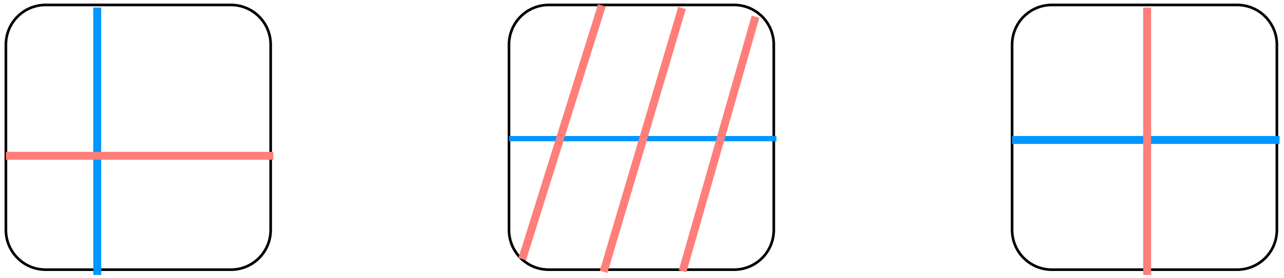
- The discrete symmetries obtained from anomalous U(1)'s are Abelian and flavor-independent
- One may however also obtain flavor discrete symmetries. These symmetries may be non-Abelian and contain the previous Abelian symmetries as a subgroup.
- Simple mechanism for family replication: intersecting D-branes



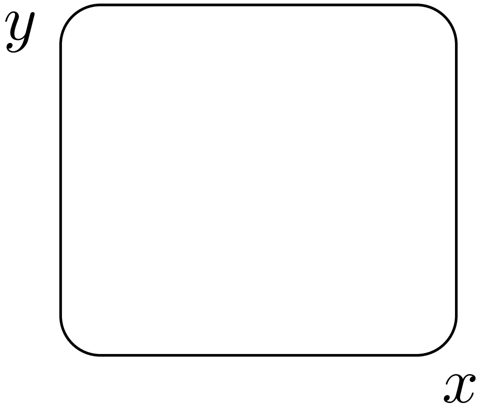


D6-branes on T^6

- Simplest case: intersecting D6-branes on $T^2 x T^2 x T^2$



- On each T^2



$$\left.\begin{array}{l} V_\mu^x \sim g_\mu^x \\ V_\mu^y \sim g_\mu^y \end{array}\right\} \to \quad U(1) \times U(1)$$

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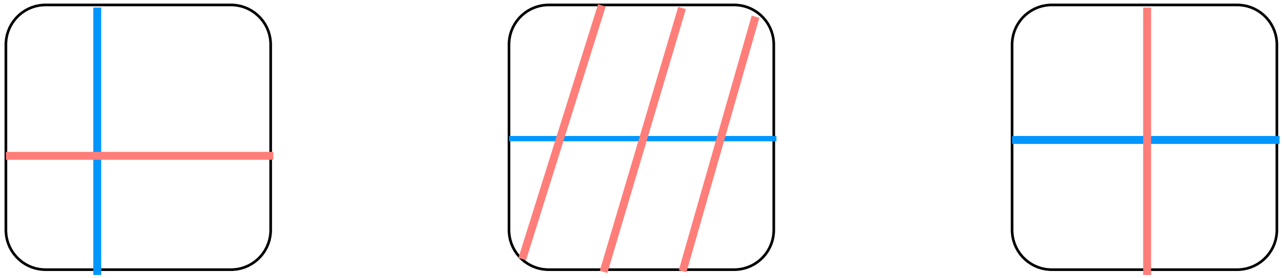

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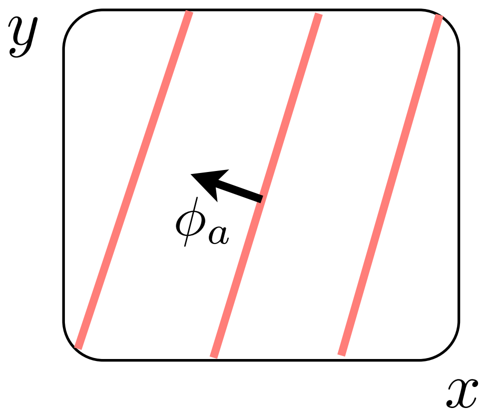

$(n_a, m_a) = (1, 3)$

$$\left.\begin{array}{l} V_\mu^x \sim g_\mu^x \\ V_\mu^y \sim g_\mu^y \end{array}\right\} \rightarrow \quad U(1) \times \mathbb{Z}_q$$

$$\mathcal{L}_{\rm St} = \frac{1}{2}\left(\partial_\mu \phi_a - m_a V_\mu^x + n_a V_\mu^y\right)^2$$

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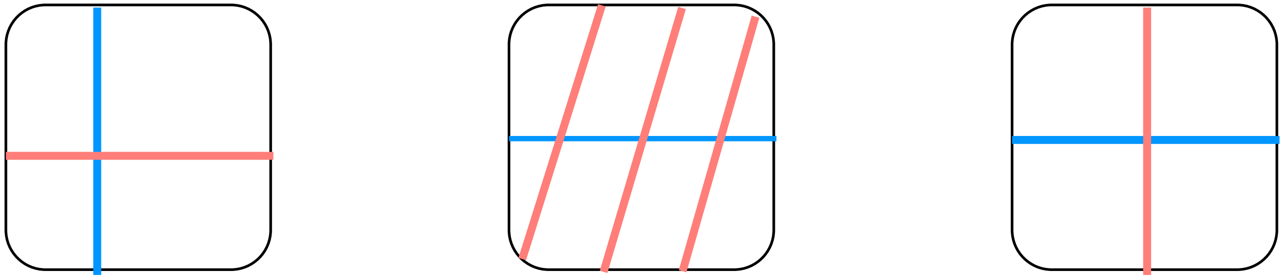

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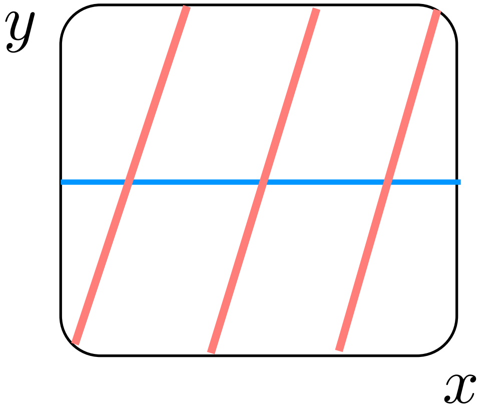

$$(n_a, m_a) = (1, 3)$$

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$$\left.\begin{array}{l} V_\mu^x \sim g_\mu^x \\ V_\mu^y \sim g_\mu^y \end{array}\right\} \rightarrow \quad P = \mathbb{Z}_{I_{ab}}$$

$$\mathcal{L}_{\rm St} = \frac{1}{2}\left(\partial_\mu \phi_a - m_a V_\mu^x + n_a V_\mu^y\right)^2 + \frac{1}{2}\left(\partial_\mu \phi_b - m_b V_\mu^x + n_b V_\mu^y\right)^2$$

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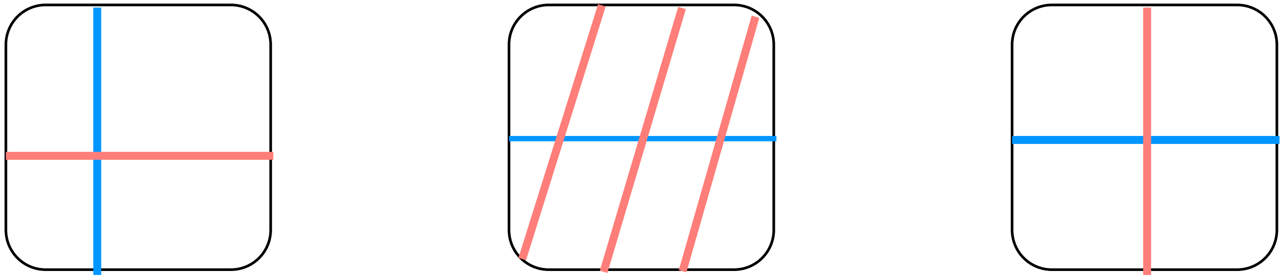

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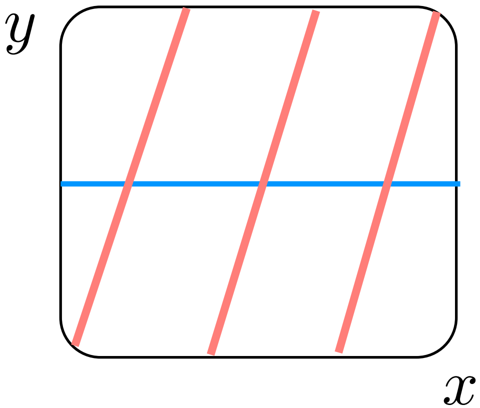

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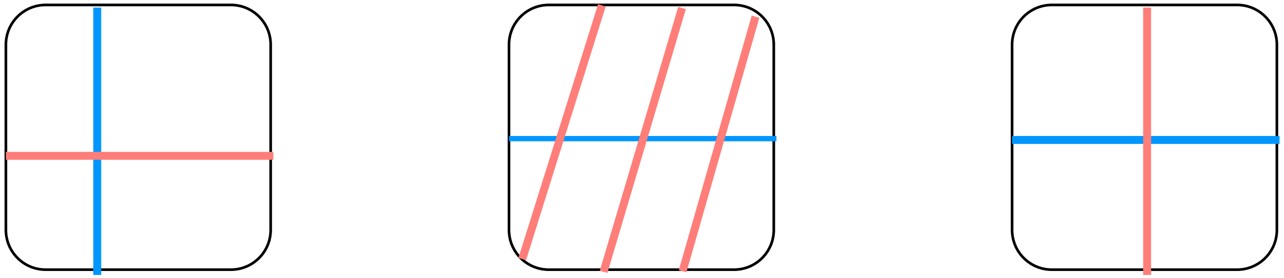
$$g_{\mathcal{T}} = \begin{pmatrix} & 1 & & & \\ & & 1 & & \\ & & & \ddots & \\ & & & & 1 \\ 1 & & & & \end{pmatrix}$$

D6-branes on T^6

- Simplest case: intersecting D6-branes on $T^2 \times T^2 \times T^2$



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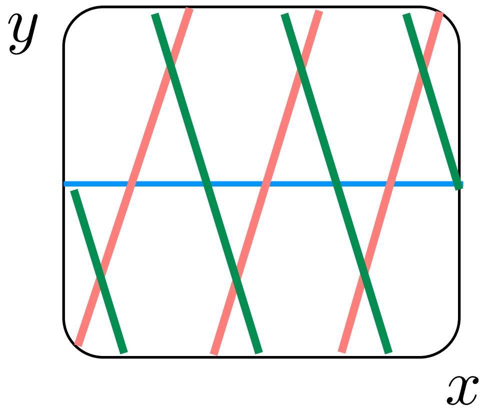

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$$\left.\begin{array}{l} V_\mu^x \sim g_\mu^x \\ V_\mu^y \sim g_\mu^y \end{array}\right\} \rightarrow \quad P = \mathbb{Z}_d$$

$$d = \text{g.c.d.}(I_{ab}, I_{bc}, I_{ca})$$

$$\begin{aligned} \mathcal{L}_{\text{St}} = & \frac{1}{2}\left(\partial_\mu \phi_a - m_a V_\mu^x + n_a V_\mu^y\right)^2 \\ & + \frac{1}{2}\left(\partial_\mu \phi_b - m_b V_\mu^x + n_b V_\mu^y\right)^2 \\ & + \frac{1}{2}\left(\partial_\mu \phi_c - m_c V_\mu^x + n_c V_\mu^y\right)^2 \end{aligned}$$

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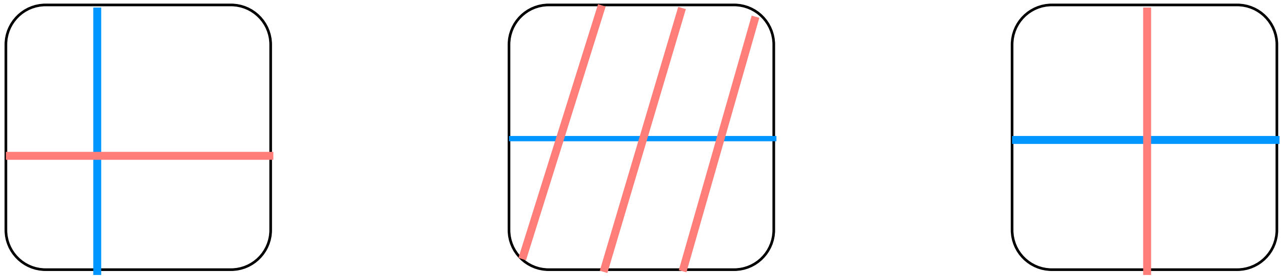
- Simplest case: intersecting D6-branes on $T^2 x T^2 x T^2$



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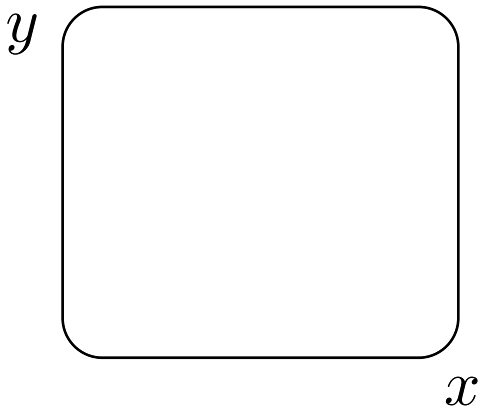

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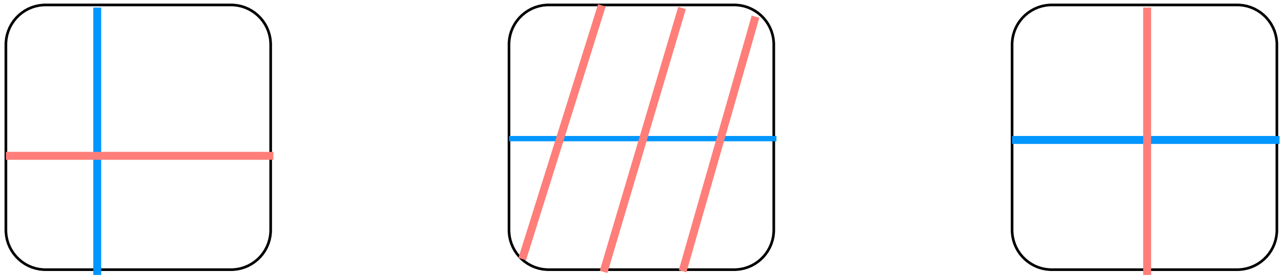

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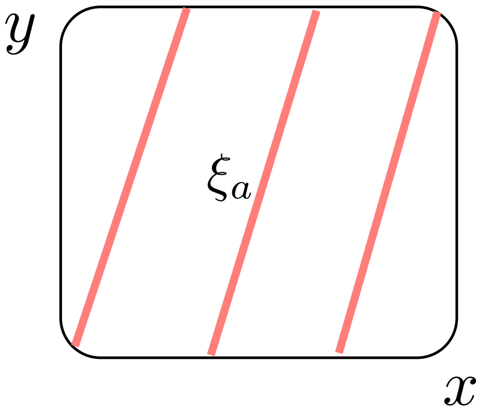

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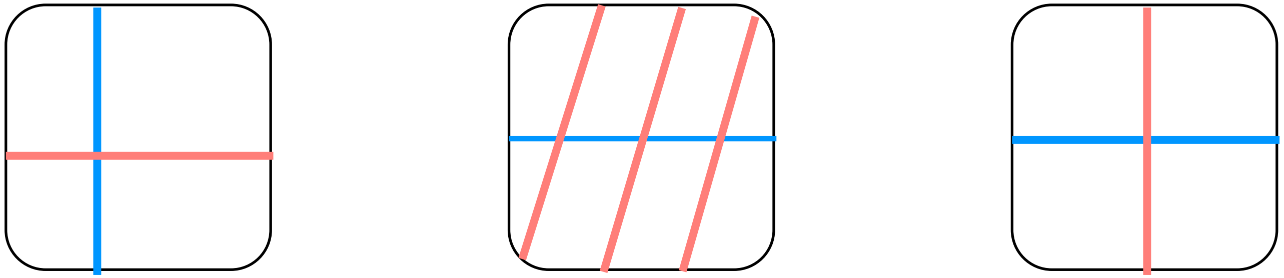
D6-branes on T⁶

- Simplest case: intersecting D6-branes on $T^2 x T^2 x T^2$



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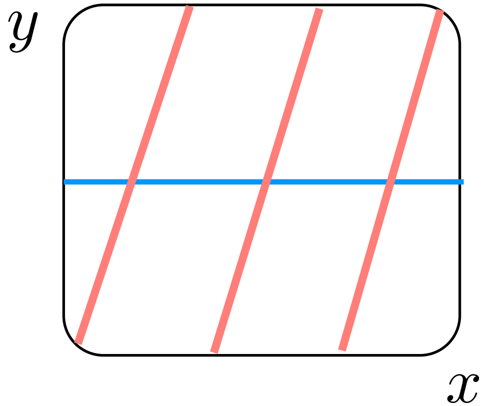

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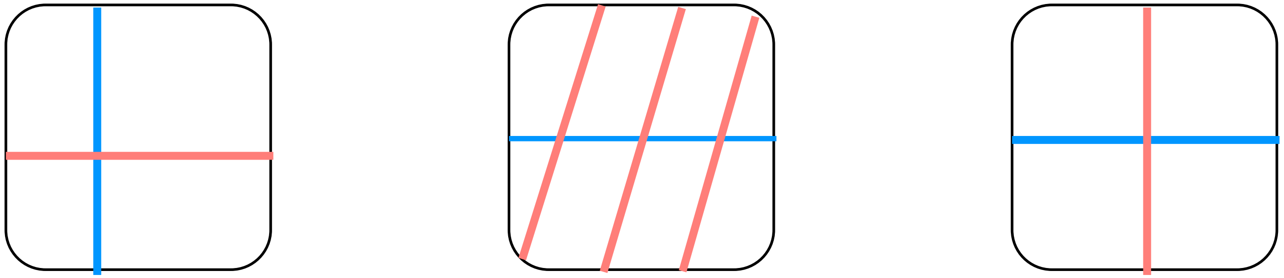

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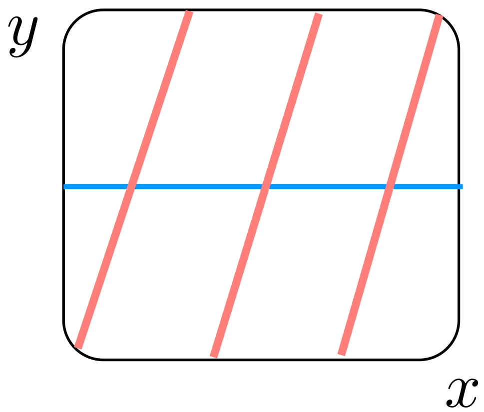

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$$g_W = \begin{pmatrix} 1 & & & & \\ & \omega & & & \\ & & \omega^2 & & \\ & & & \ddots & \\ & & & & \omega^{I_{ab}-1} \end{pmatrix} \qquad \omega = e^{\frac{2\pi i}{I_{ab}}}$$

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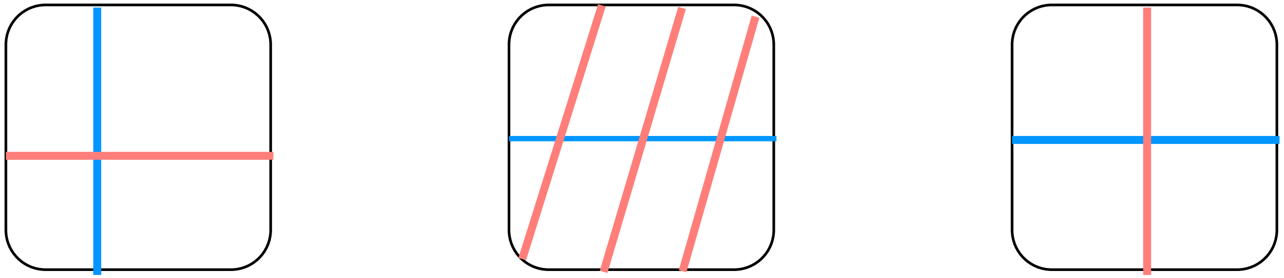

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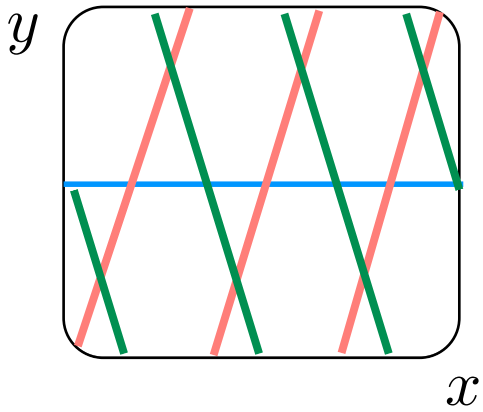

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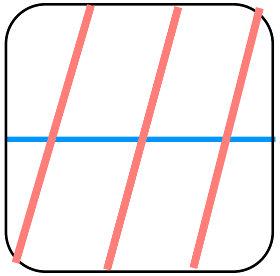
D6-branes on T^6

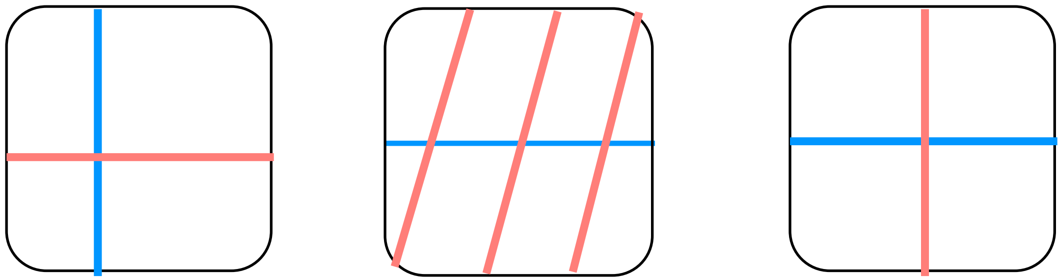
- Flavor group from each T^2



$$P = (\mathbb{Z}_N \times \mathbb{Z}_N) \rtimes \mathbb{Z}_N = H_N$$

$$g_{\mathcal{T}} = \begin{pmatrix} & 1 & & & \\ & & 1 & & \\ & & & \ddots & \\ & & & & 1 \\ 1 & & & & \end{pmatrix} \qquad g_{\mathcal{W}} = \begin{pmatrix} 1 & & & & \\ & \omega & & & \\ & & \omega^2 & & \\ & & & \ddots & \\ & & & & \omega^{N-1} \end{pmatrix}$$

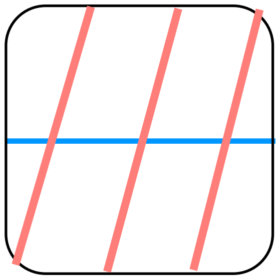
- In total



$$P = H_{N_1} \times H_{N_2} \times H_{N_3}$$

D6-branes on T^6

- Flavor group from each T^2



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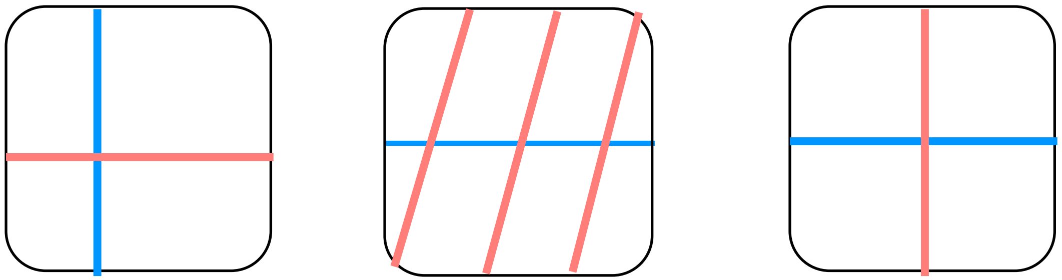


$$P = H_{N_1} \times H_{N_2} \times H_{N_3}$$

- Several D6-branes:

$$P = H_{d_1} \times H_{d_2} \times H_{d_3}$$

$$d_i = \text{g.c.d.}(I^i_{ab}, I^i_{bc}, I^i_{ca}, \dots)$$

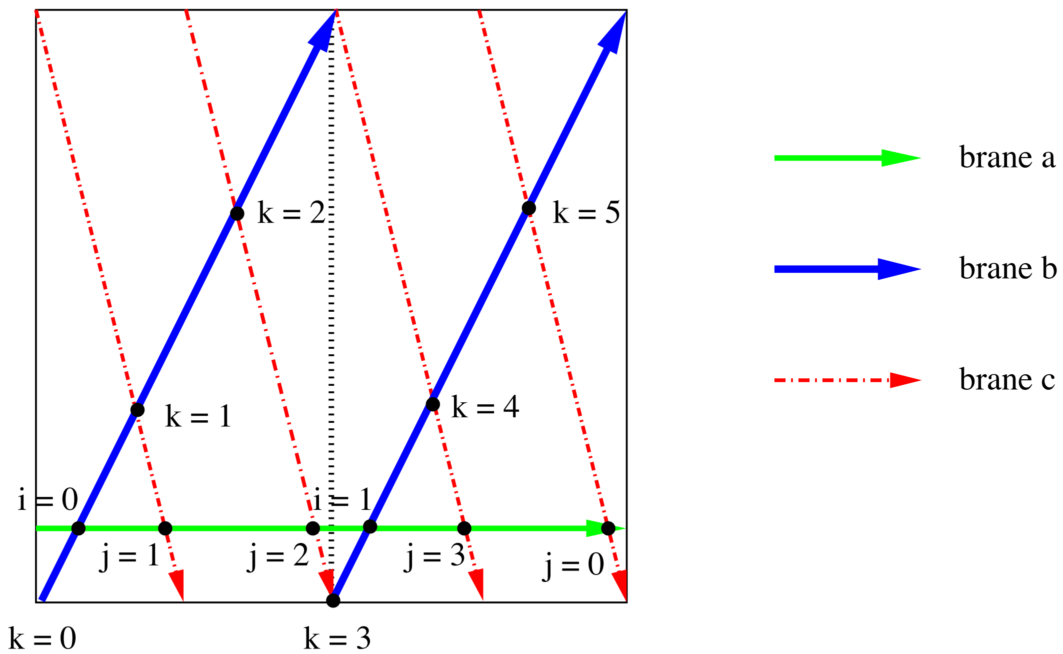
D6-branes on T^6

- This flavor group constrains the Yukawa couplings:

$$Y_{ijk} \neq 0 \iff i + j + k = 0 \bmod d$$

$$Y_{ijk} = Y_{i+\frac{I_{ab}}{d} j+\frac{I_{bc}}{d} k+\frac{I_{ca}}{d}}$$

Cremades, Ibañez, F.M. '03
Abe et al. '09



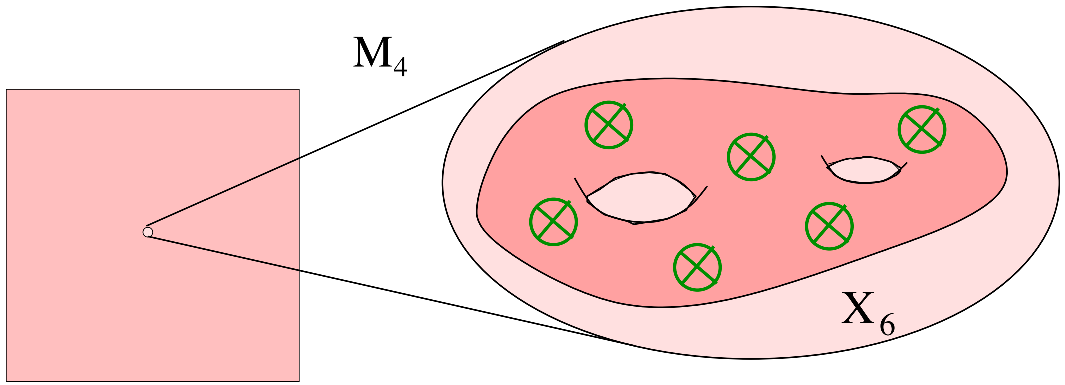
Magnetized D-branes

- Dual example: U(1) gauge theory compactified on **T^2**, with a gauge field strength background in the extra dimensions

$$F_2 = 2\pi M dx \wedge dy \quad \Rightarrow \quad A = \pi M (xdy - ydx)$$

- The magnetization breaks (gauges) the translational isometry, because A changes as we move on **T^2**

$$A(x+\lambda_x, y) = A(x,y) + \pi M \lambda_x dy$$
$$A(x, y+\lambda_y) = A(x,y) - \pi M \lambda_y dx$$



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$$e^{\lambda_j D_j} iD_k e^{-\lambda_j D_j} = iD_k + \lambda_j F_{jk}$$
$$e^{\mu_j B_j} iD_k e^{-\mu_j B_j} = iD_k + \mu_j \delta_{jk}$$

$$\mathcal{L}_{\mathrm{St}} = -\frac{1}{2} \sum_{\alpha=a,b} \left\{ \left(\partial_\mu \xi_{x,\alpha} + m_\alpha V_\mu^y - B_\mu^x\right)^2 + \left(\partial_\mu \xi_{y,\alpha} - m_\alpha V_\mu^x - B_\mu^y\right)^2 \right\}$$

$$e^{\frac{D_x}{M}} \psi^j \rightarrow e^{2\pi i \frac{j}{M}} \psi^j$$
$$e^{\frac{D_y}{M}} \psi^j \rightarrow \psi^{j+1}$$

$\Rightarrow$ discrete Heisenberg flavor symmetry

(twisted torus isometries)

Beyond tori

- This is all very nice, but $\mathbf{T^6}$ is not a good example of compactification manifold because one cannot build stable chiral D-brane models.
  - One should add O-planes
  - One should add curvature → generic CY   (example: $\mathbf{T^6/\Gamma}$)
- Problem: CY's do not have continuous isometries, so we cannot apply our 4d calculations to find the flavor group

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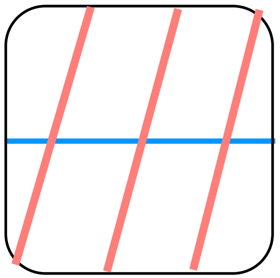
- Idea: Revisit $\mathbf{T^2}$ case and understand flavor group in terms of symmetries



$$U(1) \times U(1) \to \mathbb{Z}_3$$

$$\mathbf{P}^{\text{bulk}} \quad \to \quad \mathbf{P}^{\text{bulk+brane}}$$

- Apply the same for CY or $\mathbf{T^6/\Gamma}$

Discrete Flavor Symmetries in orbifolds

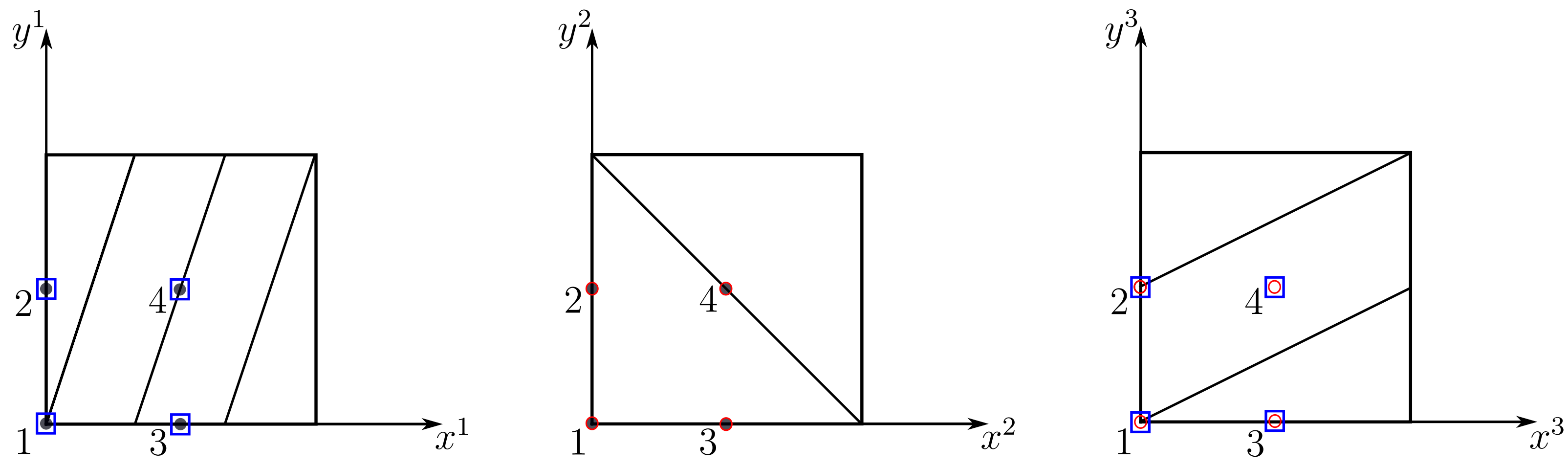
Flavor in T^6/Z_2xZ_2

- Simple example of orbifold: **T^6/Z_2xZ_2**
  - Allows for chiral *N*=1 models
  - Allows for D-branes with no moduli

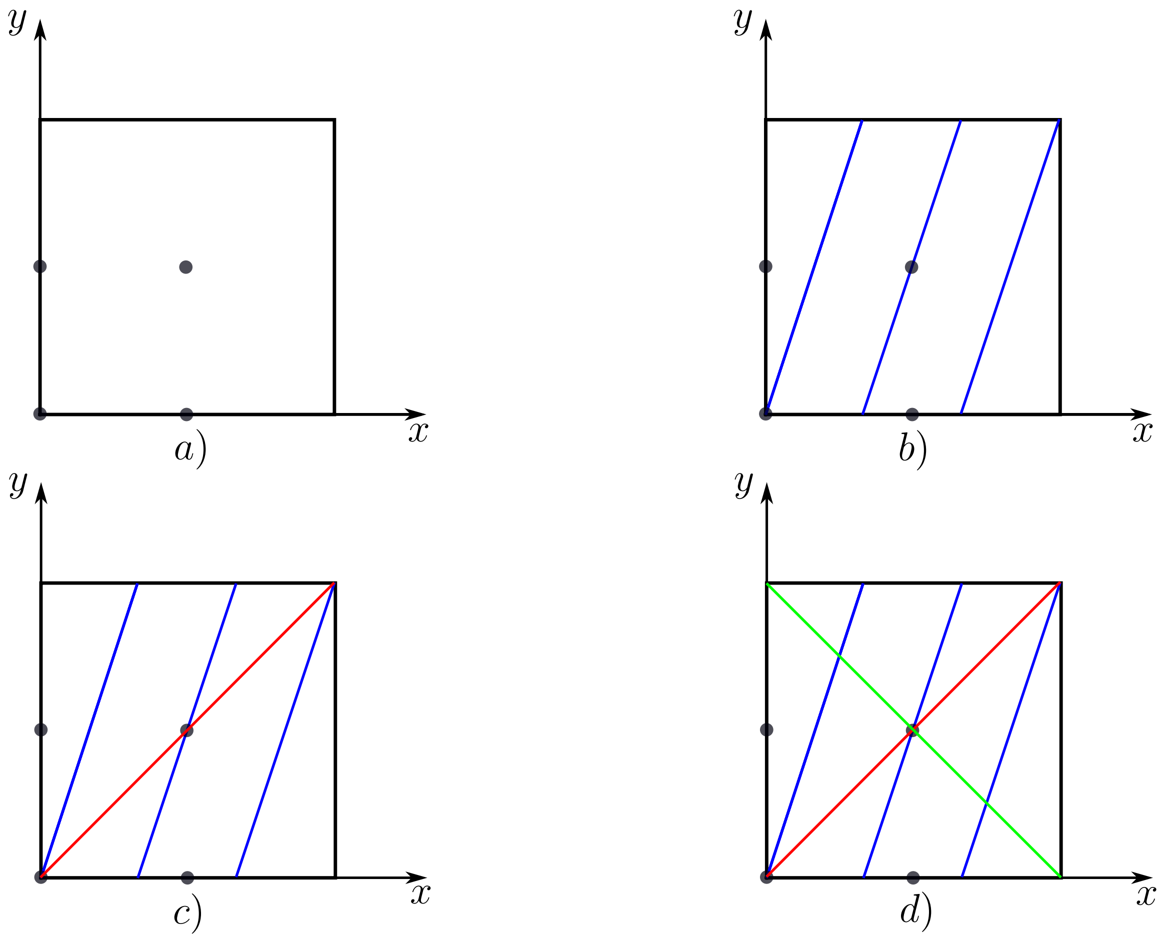
Blumenhagen, Cvetic, F.M., Shiu, '05
Dudas, Tirmigaziu '05

- Isometry group broken to **Z_2^6** by the orbifold action

- Rigid D6-branes go through fixed points



Flavor in T^6/Z_2xZ_2



- Breaking pattern for isometries on $\mathbf{T^2/Z_2}$

$$\mathbb{Z}_2 \times \mathbb{Z}_2 \;\rightarrow\; \mathbb{Z}_2 \;\underbrace{\rightarrow}_{I_{ab}\ \text{even}}\; \mathbb{Z}_2 \;\underbrace{\rightarrow}_{d\ \text{even}}\; \mathbb{Z}_2$$

Flavor in $T^6/Z_2 x Z_2$

- The same applies to the B-field transformations
- Final flavor group: $H_2 \simeq D_4$
- For $\mathbf{T^6/Z_2xZ_2}$:

$$\mathbf{P} = D_4^{[d_1-1]} \times D_4^{[d_2-1]} \times D_4^{[d_3-1]}$$

$$d_i = \text{g.c.d.}(2, I^i_{ab}, I^i_{bc}, I^i_{ca}, \ldots)$$

If in a $\mathbf{T^2}$ all intersection numbers are even we have a D_4 factor

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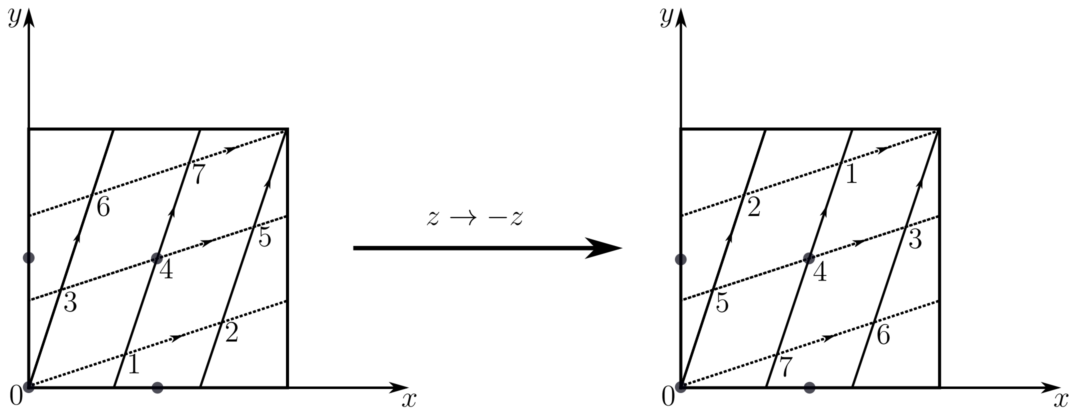
If in a $\mathbf{T^2}$ all intersection numbers are even we have a D_4 factor

- Remarks:
  - D6-branes through same fixed points ⟷ twisted tadpoles
  - I_{ab} = even does not imply even number of families



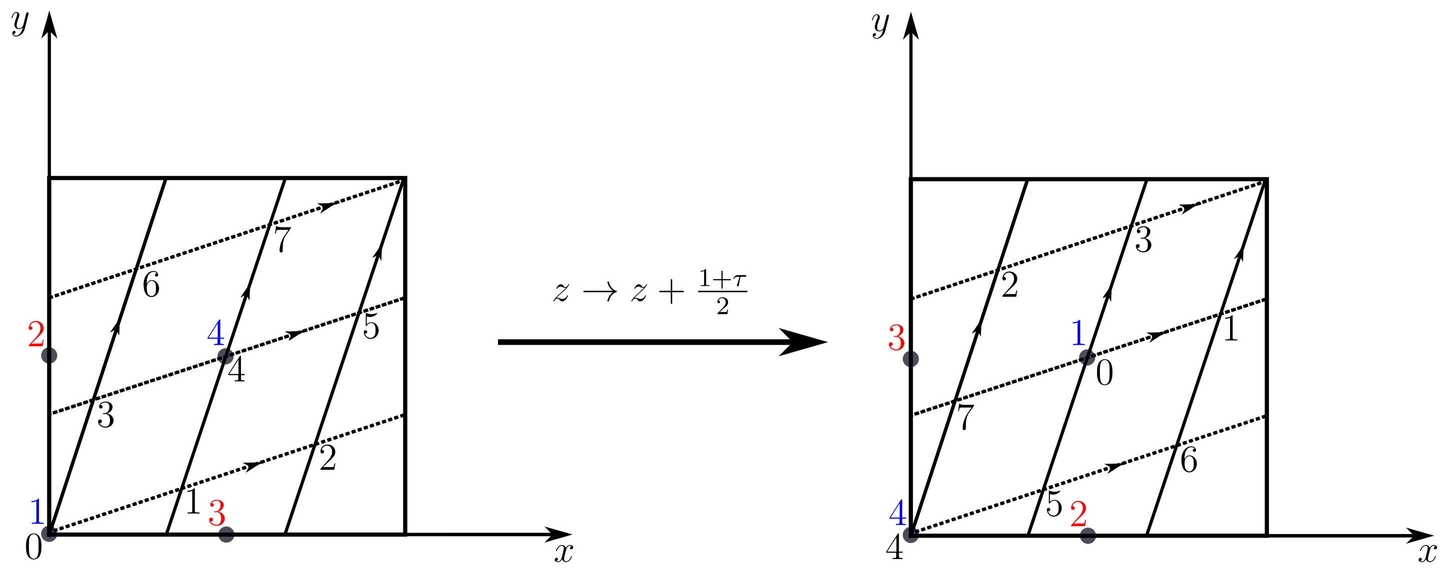
$$\psi^j_{\text{even}} = \psi^{j,N} + \psi^{N-j,N}$$
$$\psi^j_{\text{odd}} = \psi^{j,N} - \psi^{N-j,N}$$

Flavor in $T^6/Z_2 x Z_2$

- Representations:

$$\psi_{ab}^{j_1,j_2,j_3} = \psi_{ab}^{j_1} \cdot \psi_{ab}^{j_2} \cdot \psi_{ab}^{j_3}$$

$\lvert I_{ab}^i \rvert$	$\psi_{\text{even}}^{j_i} \quad \dim = \lvert I_{ab}^i \rvert/2 + 1$	$\psi_{\text{odd}}^{j_i} \quad \dim = \lvert I_{ab}^i \rvert/2 - 1$
$4s+2$	$\overset{s+1}{\oplus}\ \mathbf{R}_2$	$\overset{s}{\oplus}\ \mathbf{R}_2$
$8s+4$	$\overset{s+1}{\oplus}(+,+)\ \overset{s+1}{\oplus}(+,-)\ \overset{s+1}{\oplus}(-,+)\ \overset{s}{\oplus}(-,-)$	$\overset{s}{\oplus}(+,+)\ \overset{s}{\oplus}(+,-)\ \overset{s}{\oplus}(-,+)\ \overset{s+1}{\oplus}(-,-)$
$8s+8$	$\overset{s+2}{\oplus}(+,+)\ \overset{s+1}{\oplus}(+,-)\ \overset{s+1}{\oplus}(-,+)\ \overset{s+1}{\oplus}(-,-)$	$\overset{s}{\oplus}(+,+)\ \overset{s+1}{\oplus}(+,-)\ \overset{s+1}{\oplus}(-,+)\ \overset{s+1}{\oplus}(-,-)$



Examples

- 4-generation Pati-Salam from *Blumenhagen, Cvetic, F.M., Shiu, '05*

N_α	(n_α^1, m_α^1)	(n_α^2, m_α^2)	(n_α^3, m_α^3)
$N_{a_1} = 4$	$(1, 0)$	$(0, 1)$	$(0, -1)$
$N_{a_2} = 2$	$(1, 0)$	$(2, 1)$	$(4, -1)$
$N_{a_3} = 2$	$(-3, 2)$	$(-2, 1)$	$(-4, 1)$

$$U(4) \times U(2)_L \times U(2)_R$$
$$\downarrow$$
$$SU(4) \times SU(2)_L \times SU(2)_R$$

Sector	Field	$D_4^{(1)}$	$D_4^{(2)}$	$D_4^{(3)}$
$a_1 a_2$	$F_L = (\mathbf{4}, \bar{\mathbf{2}}, \mathbf{1})$	$\mathbf{1}$	$\mathbf{R}_2$	$(-,-)$
$a_1 a_2'$	$F_L' = (\mathbf{4}, \mathbf{2}, \mathbf{1})$	$\mathbf{1}$	$(-,-)$	$\mathbf{R}_2$
$a_1 a_3$	$F_R = (\bar{\mathbf{4}}, \mathbf{1}, \mathbf{2})$	$\mathbf{R}_2$	$\mathbf{R}_2$	$(-,-)$
$a_2 a_3$	$H = (\mathbf{1}, \mathbf{2}, \bar{\mathbf{2}})$	$\mathbf{R}_2$	$\mathbf{1} \oplus (+,-) \oplus (-,+)$	$\mathbf{1}$
$a_2 a_3'$	$H' = (\mathbf{1}, \bar{\mathbf{2}}, \bar{\mathbf{2}})$	$\mathbf{R}_2$	$\mathbf{1}$	$\mathbf{1}^2 \oplus (+,-) \oplus (-,+) \oplus (-,-)$

$$Y : (a_1 a_2) \otimes (a_1 a_3) \otimes (a_2 a_3) \longrightarrow (\mathbf{4}, \bar{\mathbf{2}}, \mathbf{1}) \otimes (\bar{\mathbf{4}}, \mathbf{1}, \mathbf{2}) \otimes (\mathbf{1}, \mathbf{2}, \bar{\mathbf{2}})$$
$$Y' : (a_1' a_2) \otimes (a_1 a_3) \otimes (a_2' a_3) \longrightarrow (\mathbf{4}, \mathbf{2}, \mathbf{1}) \otimes (\bar{\mathbf{4}}, \mathbf{1}, \mathbf{2}) \otimes (\mathbf{1}, \bar{\mathbf{2}}, \bar{\mathbf{2}})$$

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$N_{a_1} = 4$	$(1,0)$	$(0,1)$	$(0,-1)$
$N_{a_2} = 2$	$(1,0)$	$(2,1)$	$(4,-1)$
$N_{a_3} = 2$	$(-3,2)$	$(-2,1)$	$(-4,1)$

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Sector	Field	$D_4^{(1)}$	$D_4^{(2)}$	$D_4^{(3)}$
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$a_1 a_2'$	$F_L' = (\mathbf{4}, \mathbf{2}, \mathbf{1})$	$\mathbf{1}$	$(-,-)$	$\mathbf{R}_2$
$a_1 a_3$	$F_R = (\bar{\mathbf{4}}, \mathbf{1}, \mathbf{2})$	$\mathbf{R}_2$	$\mathbf{R}_2$	$(-,-)$
$a_2 a_3$	$H = (\mathbf{1}, \mathbf{2}, \bar{\mathbf{2}})$	$\mathbf{R}_2$	$\mathbf{1} \oplus (+,-) \oplus (-,+)$	$\mathbf{1}$
$a_2 a_3'$	$H' = (\mathbf{1}, \bar{\mathbf{2}}, \bar{\mathbf{2}})$	$\mathbf{R}_2$	$\mathbf{1}$	$\mathbf{1}^2 \oplus (+,-) \oplus (-,+) \oplus (-,-)$

$$Y : (a_1 a_2) \otimes (a_1 a_3) \otimes (a_2 a_3) \longrightarrow (\mathbf{4}, \bar{\mathbf{2}}, \mathbf{1}) \otimes (\bar{\mathbf{4}}, \mathbf{1}, \mathbf{2}) \otimes (\mathbf{1}, \mathbf{2}, \bar{\mathbf{2}})$$ → *only 3 indep couplings*

$$Y' : (a_1' a_2) \otimes (a_1 a_3) \otimes (a_2' a_3) \longrightarrow (\mathbf{4}, \mathbf{2}, \mathbf{1}) \otimes (\bar{\mathbf{4}}, \mathbf{1}, \mathbf{2}) \otimes (\mathbf{1}, \bar{\mathbf{2}}, \bar{\mathbf{2}})$$ *forbidden*

Examples

- 3-generation Pati-Salam

N_α	(n_α^1, m_α^1)	(n_α^2, m_α^2)	(n_α^3, m_α^3)
$N_a = 4$	$(1, 0)$	$(1, 1)$	$(1, \text{-}1)$
$N_b = 2$	$(n, \text{-}3)$	$(0, 1)$	$(3, \text{-}1)$
$N_c = 2$	$(l, \text{-}1)$	$(\text{-}2, 1)$	$(\text{-}1, \text{-}1)$

$$U(4) \times U(2)_L \times U(2)_R$$

$$\downarrow$$

$$SU(4) \times SU(2)_L \times SU(2)_R$$

↙

Sector	Fields	D_4
ab	$F_R = (\bar{\mathbf{4}}, \mathbf{2}, \mathbf{1})$	$\mathbf{R}_2$
ab'	$F_R' = (\bar{\mathbf{4}}, \bar{\mathbf{2}}, \mathbf{1})$	$(-,-)$
ac	$F_L = (\mathbf{4}, \mathbf{1}, \bar{\mathbf{2}})$	$\mathbf{R}_2$
ac'	$F_L' = (\mathbf{4}, \mathbf{1}, \mathbf{2})$	$(+,+)$
bc	$H = (\mathbf{1}, \bar{\mathbf{2}}, \mathbf{2})$	$(-,-) \oplus (-,-)$
bc'	$H' = (\mathbf{1}, \mathbf{2}, \mathbf{2})$	$\overset{6}{\oplus}\, \mathbf{R}_2$

$$Y : ab \otimes ac \otimes bc \longrightarrow (\bar{\mathbf{4}}, \mathbf{2}, \mathbf{1}) \otimes (\mathbf{4}, \mathbf{1}, \bar{\mathbf{2}}) \otimes (\mathbf{1}, \bar{\mathbf{2}}, \mathbf{2})$$

$$Y' : ab' \otimes ac \otimes bc' \longrightarrow (\bar{\mathbf{4}}, \bar{\mathbf{2}}, \mathbf{1}) \otimes (\mathbf{4}, \mathbf{1}, \bar{\mathbf{2}}) \otimes (\mathbf{1}, \mathbf{2}, \mathbf{2}).$$

8 indep couplings

Conclusions

- We have analyzed appearance of discrete flavor symmetries in D-brane models. In toroidal models, they can be read from BF couplings of closed string U(1)'s to open string axions.

- In orbifold models we need a new approach: we first consider the group $\mathbf{P}^{\mathrm{bulk}}$ that leaves the closed string background invariant and then the subgroup $\mathbf{P}$ that also leaves D-branes invariant.

Conclusions

- We have analyzed appearance of discrete flavor symmetries in D-brane models. In toroidal models, they can be read from BF couplings of closed string U(1)'s to open string axions.

- In orbifold models we need a new approach: we first consider the group $\mathbf{P}^{\mathrm{bulk}}$ that leaves the closed string background invariant and then the subgroup **P** that also leaves D-branes invariant.

- **P** will act non-trivially on open string zero modes and generate a non-Abelian flavor group → forbid Yukawa couplings beyond Z_k's. We can also define the approximate discrete symmetry $\mathbf{P}^{\mathrm{abc}}$

- We have analyzed the case of $\mathbf{T^6/Z_2xZ_2}$, obtaining a flavor group given by $\mathbf{D_4}$ and tensor products of it.

- This definition of flavor group is quite general and can be applied to any manifold with discrete symmetries in the closed string sector, like e.g. smooth Calabi-Yau compactifications.