The heterotic string on magnetized orbifolds

Stefan Groot Nibbelink

Arnold Sommerfeld Center, Ludwig-Maximilians-University, Munich

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together with:

Patrick Vaudrevange

DESY, Hamburg

Overview of this talk

1. Heterotic string model building
2. Euler number zero manifolds
3. $\mathbb{Z}_2 \times \mathbb{Z}_2$ orbifolds
4. Orbifold resolution with magnetic flux
5. Example: A semi–realistic MSSM model
6. Conclusion
7. Appendix: Intermediate $T^4/\mathbb{Z}_2$ orbifold

Some goals of string phenomenology

Goals:

- Construct the Standard Model
- Understand moduli stabilization
- Understand supersymmetry breaking
- Cosmological evolution and inflation

Standard Model from String Theory

Generically in this field obtaining the Standard Model for String Theory means getting close to the MSSM, i.e.:

- a 4D $\mathcal{N} = 1$ supersymmetric gauge theory,
- with gauge group containing $SU(3)_C \times SU(2)_L \times U(1)_Y$,
- with a net number of three chiral generations of quarks and leptons,
- and at least one Higgs doublet pair.

Standard Model exotics

In additions essentially all MSSM–like string models suffer from exotics: State that are charged under the SM group but not part of the MSSM:

- additional duplicates of SM states,
- quark or lepton like–states but with different hyper charge.

Since these exotics states are not part of the SM, they should decouple at the low energies (i.e. 1 TeV):

- They have to be vector–like such that they could pair up to become massive,
- Yukawa–like couplings have to present for them for this to actually work.

Different string pheno settings

Possible theories:

- heterotic string
- open strings
- F–theory
- M–theory

Possible exact constructions:

- Orbifold CFTs
- Free-fermionic models
- Gepner models

Different formulations of the heterotic string

- Target space description

- Worldsheet description

Target space formulation of the heterotic string

1. **10D $\mathcal{N} = (1, 0)$ Supergravity:**

   (Graviton, B–field, dilaton; Gravitino, dilatino)

2. **10D $\mathcal{N} = (1, 0)$ Super Yang–Mills:**

   (Gauge fields; Gauginos)

   Gauge group: $E_8 \times E_8$ or $SO(32)$

Worldsheet formulations of the heterotic string

2D $\mathcal{N} = \mathbf{(0,1)}$ Super conformal field theory (in light–cone gauge):

- Coordinate fields $X^M\ , \quad M = 2, \ldots, 9,$
- and their right–moving superpartners ψ_R^M.
- additional left–moving bosons $X_L^I\ , \quad I = 1, \ldots, 16.$

These additional left–moving bosons give rise to the target space gauge degrees of freedom.

Geometrical construction

The basic requirement is that one obtains an effective 4D field theory with $\mathcal{N} = 1$ SUSY: Candelas,Horowitz,Strominger,Witten'85

$$\mathcal{M}^{1,9} \to \mathcal{M}^{1,3} \times \mathcal{M}^{6}$$

- Six internal dimensions form a compact complex manifold $\mathcal{M}^6$ with vanishing first Chern class.
- By the Calabi–Yau theorem such a manifold can be equiped with a Ricci–flat Kähler metric.

Background gauge flux

In order that also the gauge background preserves 4D $\mathcal{N} = 1$ SUSY it has to satisfy the Hermitean Yang–Mills equations

$$\mathcal{F}_{(2,0)} = \mathcal{F}_{(0,2)} = 0 \,, \quad G^{i\underline{i}}\mathcal{F}_{i\underline{i}} = 0$$

These are complicated differential equations involving the explicitly unknown Calabi–Yau metric $G_{i\underline{i}}$.

The Donaldson-Uhlenbeck-Yau theorem provides quasi–topological conditions when these equations can be satisfied.

Toroidal orbifold geometries

The idea of orbifolds is that they are very simple geometries yet shared the main property of Calabi–Yau manifolds namely that only 4D $\mathcal{N} = 1$ SUSY survives. Dixon,Harvey,Vafa,Witten'85

Toroidal orbifolds are defined as

$$T^6/G$$

where T^6 is some six dimensional torus spanned by lattice vectors e_α and G a finite group.

A full classification of all orbifolds compatible with $N \geq 1$ in 4D has recently been obtained: Fischer,Ratz,Torrado,Vaudrevange'12

- There are in total 520 inequivalent toroidal orbifolds
- 162 of them have Abelian point groups (e.g. $\mathbb{Z}_3, \mathbb{Z}_4, \mathbb{Z}_2 \times \mathbb{Z}_2$)

Calabi–Yau model building results

On one particular complete intersection Calabi–Yau, the Schoen manifold, various groups have constructed MSSM–like models using stable $SU(5)$ vector bundles. Donagi,Ovrut,Pantev,Waldram'00, Bouchard,Donagi'05, Braun,He,Ovrut,Pantev'05

A more systematic study is possible if one starts with line bundles on complete intersection Calabi–Yaus which then could be deformed to non–Abelian bundles as well. This has lead to a large set of MSSM–like models. Anderson,Gray,Lukas,Palti'11

Orbifold model building results

Based on such orbifolds various studies have been undertaken to construct MSSM–like models from the heterotic string:

- $T^6/\mathbb{Z}_{\text{6-II}}$ Buchmuller,Hamaguchi,Lebedev,Ratz'05, Lebedev,Nilles,Raby,Ramos-Sanchez,Ratz,Vaudrevange,Wingerter'06
- $T^6/\mathbb{Z}_{12-I}$ Kim,Kim,Kyae'07
- $T^6/\mathbb{Z}_2 \times \mathbb{Z}_2$ Blaszczyk,SGN,Ratz,Ruehle,Trapletti,Vaudrevange'09
- $T^6/\mathbb{Z}_4 \times \mathbb{Z}_2$ Mayorga-Pena,Nilles,Oehlmann'12
- $T^6/\mathbb{Z}_{\text{8-I,II}}$ SGN,Loukas'13

Euler number of orbifolds

Orbifolds that have been used for string model building so far have non–vanishing Euler number, $\chi = 2(h_{11} - h_{21})$, e.g.:

Erler,Klemm'92

- $T^6/\mathbb{Z}_3$: $(h_{11}, h_{21}) = (36, 0)$; $\chi = 72$
- $T^6/\mathbb{Z}_{\text{6-II}}$: $(h_{11}, h_{21}) = (35, 11)$; $\chi = 48$
- $T^6/\mathbb{Z}_{\text{12-I}}$: $(h_{11}, h_{21}) = (29, 5)$; $\chi = 48$
- $T^6/\mathbb{Z}_2 \times \mathbb{Z}_2$: $(h_{11}, h_{21}) = (51, 3)$; $\chi = 96$

Vanishing Euler number manifolds

Calabi–Yau manifolds with vanishing Euler number are interesting as they exhibit properties of enhanced supersymmetry.

Kashani–Poor,Minasian,Triendl'03

However, orbifolds with vanishing Euler number are not considered for phenomenology since they always give rise to a non–chiral spectrum.

Recent classification reveals that 23 of the 138 orbifolds with Abelian point group given *exactly* $N = 1$ 4D supersymmetry have vanishing Euler number.

Fischer,Ratz,Torrado,Vaudrevange'12

They are all variants of $T^6/\mathbb{Z}_2 \times \mathbb{Z}_2$ orbifolds.

Donagi,Wendland'08

Strategy of this talk

In this talk we will see that one should not write off all these orbifolds with vanishing Euler number just yet.

To this end, we:

- consider a concrete orbifold with vanishing Euler number,
- show that by putting magnetic flux on its tori, it is possible to obtain 4D chirality,
- and construct an MSSM-like model in this way.

Some $\mathbb{Z}_2 \times \mathbb{Z}_2$ orbifolds

- Generalities of heterotic $\mathbb{Z}_2 \times \mathbb{Z}_2$ orbifolds
- The standard $\mathbb{Z}_2 \times \mathbb{Z}_2$ orbifold
- A roto–translational $\mathbb{Z}_2 \times \mathbb{Z}_2$ orbifold

Generalities of heterotic $\mathbb{Z}_2 \times \mathbb{Z}_2$ orbifolds

$\mathbb{Z}_2 \times \mathbb{Z}_2$ orbifolds are defined as

$$T^6/\mathbb{Z}_2 \times \mathbb{Z}_2 = \mathbb{C}^3/S$$

where the *space group* $S \ni g = (\text{rot}, \text{trans})$ is generated by:

- translations $g_i = (1, e_i)$ over basis vectors

$$e_1 = (1,0,0)\ , \quad e_3 = (0,1,0)\ , \quad e_5 = (0,0,1)\ ,$$
$$e_2 = (i,0,0)\ , \quad e_4 = (0,i,0)\ , \quad e_6 = (0,0,i)\ ,$$

- elements $g_\theta = (\theta, t_\theta)$ and $g_\omega = (\omega, t_\omega)$, involve $\mathbb{Z}_2 \times \mathbb{Z}_2$ rotations,

$$\theta\ :\ (z_1, z_2, z_3) \rightarrow (z_1, -z_2, -z_3)\ , \quad \omega\ :\ (z_1, z_2, z_3) \rightarrow (-z_1, z_2, -z_3)$$

possibly combined with some translations: t_θ, t_ω.

Gauge embedding

The gauge embedding in the bosonic formulation with 16 left–moving coordinates X_L^I ($I = 1, \ldots, 16$) is defined as:

$$g\, X_L^I = X_L^I + V_g^I \;, \qquad V_g = k\, V_\theta + \ell\, V_\omega + n_i\, W_i \;,$$

for $g = g_\theta^k\, g_\omega^\ell\, g_1^{n_1} \cdot \ldots \cdot g_6^{n_6}$ in terms of:

- gauge shift vectors V_θ and V_ω,
- and discrete Wilson lines W_i,

satisfying appropriate conditions for modular invariance.

The standard $\mathbb{Z}_2 \times \mathbb{Z}_2$ orbifold

The space group S is generated by the elements:

$$g_\theta = \big(\theta, 0\big) \ , \quad g_\omega = \big(\omega, 0\big) \ , \quad g_i = \big(1, e_i\big) \ .$$

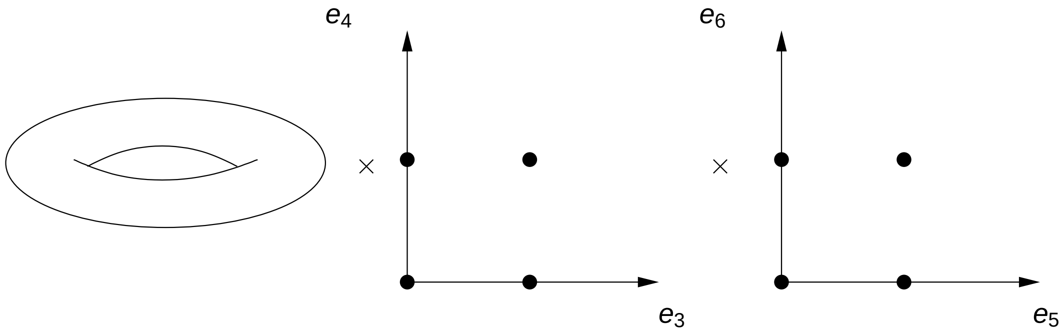
This is the DW(0–1) orbifold in the classification Donagi,Wendland'08 .

In detail, the action of g_θ and g_ω is given by

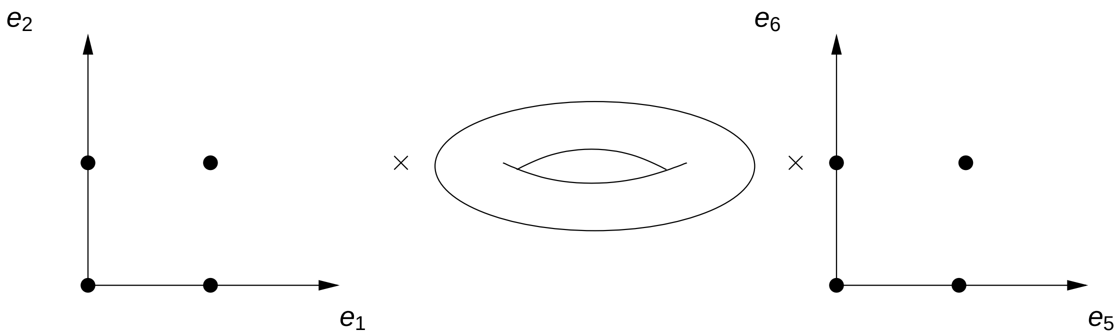
$$g_\theta\big(z_1, z_2, z_3\big) = \big(z_1, -z_2, -z_3\big) \ ,$$
$$g_\omega\big(z_1, z_2, z_3\big) = \big(-z_1, z_2, -z_3\big) \ ,$$
$$g_\theta g_\omega\big(z_1, z_2, z_3\big) = \big(-z_1, -z_2, z_3\big) \ .$$

The standard $\mathbb{Z}_2 \times \mathbb{Z}_2$ orbifold

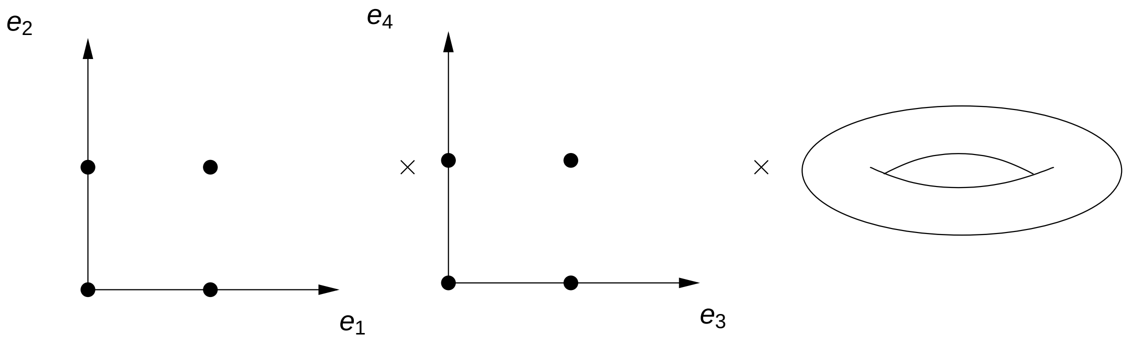
θ–sector:



ω–sector:



$\theta\omega$–sector:



The standard $\mathbb{Z}_2 \times \mathbb{Z}_2$ orbifold

For the orbifold standard embedding,

$$V_\theta = \big(0, \frac{1}{2}, \text{-}\frac{1}{2}, 0^5\big)\big(0^8\big)\ , \qquad V_\omega = \big(\text{-}\frac{1}{2}, 0, \frac{1}{2}, 0^5\big)\big(0^8\big)\ ,$$

we obtain:

- 51 (= 3 untwisted and $3 \cdot 16$ twisted) **27**–plets,
- and 3 untwisted $\overline{\mathbf{27}}$–plets of E_6.

Consequently, the Hodge / Euler numbers of the DW(0–1) orbifold read:

$$(h_{11}, h_{21}) = (51, 3)\ , \qquad \chi = 2(h_{11} - h_{21}) = 96$$

A roto–translational $\mathbb{Z}_2 \times \mathbb{Z}_{2,\text{rototrans}}$ orbifold

The space group S is generated by the elements:

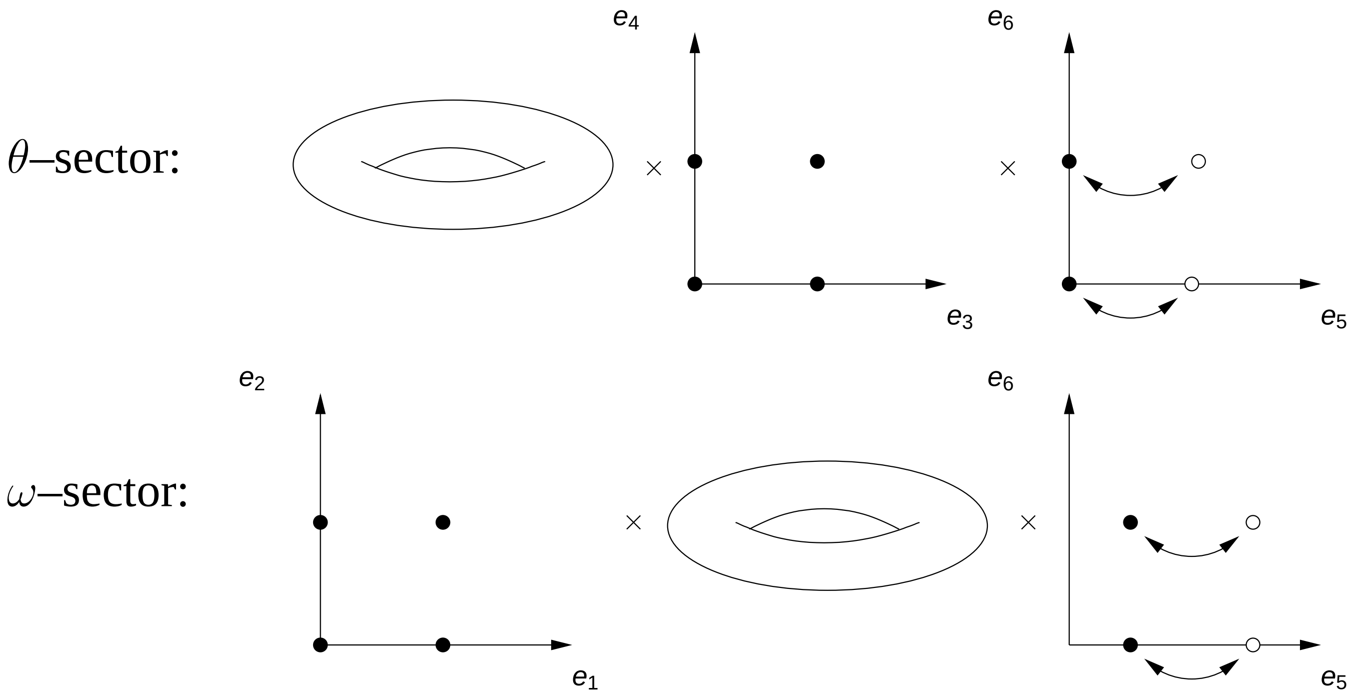
$$g_\theta = \big(\theta, 0\big)~, \quad g_\omega = \big(\omega, \tfrac{1}{2}e_5\big)~, \quad g_i = \big(1, e_i\big)~.$$

This is the DW(0–2) orbifold in the classification of Donagi,Wendland'08
.

In detail, the action of g_θ and g_ω is given by

$$g_\theta\big(z_1, z_2, z_3\big) = \big(z_1, -z_2, -z_3\big)~,$$
$$g_\omega\big(z_1, z_2, z_3\big) = \big(-z_1, z_2, -z_3 + \tfrac{1}{2}\big)~,$$
$$g_\theta g_\omega\big(z_1, z_2, z_3\big) = \big(-z_1, -z_2, z_3 - \tfrac{1}{2}\big)~.$$

A roto–translational $\mathbb{Z}_2 \times \mathbb{Z}_{2,\text{rototrans}}$ orbifold



$$g_\theta(z_1, z_2, z_3) = (z_1, -z_2, -z_3) \ ,$$

$$g_\omega(z_1, z_2, z_3) = (-z_1, z_2, -z_3 + \tfrac{1}{2})$$

A roto–translational $\mathbb{Z}_2 \times \mathbb{Z}_{2,\text{rototrans}}$ orbifold

In this case the orbifold standard embedding gives us

- 19 (= 3 untwisted and $2 \cdot 8$ twisted) **27**–plets,
- and 19 (= 3 untwisted and $2 \cdot 8$ twisted) $\overline{\mathbf{27}}$–plets of E_6.

Consequently, the Hodge / Euler numbers of the DW(0–2) orbifold read:

$$(h_{11}, h_{21}) = (19, 19)\ , \qquad \chi = 2(h_{11} - h_{21}) = 0$$

Orbifold resolution

Now we want to put magnetic fluxes on the tori of this orbifold. But as far as we are aware there is no exact CFT description in this case.

To overcome this, we

- construct the orbifold resolution
- put an Abelian gauge flux background
- compute the spectrum

Constructing the orbifold resolution

To construct an orbifold resolution we have to identify:

Denef,Douglas,Florea'04, Luest,Reffert,Scheidegger,Stieberger'06, SGN,Held,Ruehle,Trapletti,Vaudrevange'09

- a complete set of divisors of the resolution,
- the set of linear equivalence relations among them,
- their intersection numbers.

For an orbifold resolution there are three types of divisors:

- inherited divisors (four–tori within the orbifold),
- ordinary divisors (vanishing loci of local coordinates near the orbifold singularities),
- exceptional divisors (blow–up cycles).

Constructing the orbifold resolution

For the resolution of the $T^6/\mathbb{Z}_2 \times \mathbb{Z}_{2,\text{rototrans}}$ we have:

SGN,Vaudrevange'12

- Three inherited divisors:

$$R_1 := \{z_1 = c_1\} \cup \{z_1 = -c_1\}\ ,\ R_2 := \{z_2 = c_2\} \cup \{z_2 = -c_2\}$$

$$R_3 := \{z_3 = c_3\} \cup \{z_3 = -c_3\} \cup \{z_3 = \tfrac{1}{2} + c_3\} \cup \{z_1 = \tfrac{1}{2} - c_1\}$$

- 12 ordinary divisors:

$$D_{1,n_1 n_2} := \{z_1 = \tfrac{1}{2} n_1 + \tfrac{i}{2} n_2\}\ ,\ D_{3,n_6} := \{z_3 = \tfrac{i}{2} n_6\} \cup \{z_3 = \tfrac{1}{2} + \tfrac{i}{2} n_6\}$$

$$D_{2,n_3 n_4} := \{z_2 = \tfrac{1}{2} n_3 + \tfrac{i}{2} n_4\}\ ,\ D'_{3,n'_6} := \{z_3 = \tfrac{1}{4} + \tfrac{i}{2} n'_6\} \cup \{z_3 = \tfrac{3}{4} + \tfrac{i}{2} n'_6\}$$

- 16 exceptional divisors:

$$\theta\text{–sector: } E_r = E_{n_3 n_4 n_6}\ , \qquad \omega\text{–sector: } E'_{r'} = E'_{n_1 n_2 n'_6}$$

Constructing the orbifold resolution

The linear equivalence relations read:

$$2\,D_{1,n_1n_2} = R_1 - \sum_{n'_6} E'_{n_1n_2n'_6}\ , \quad 2\,D'_{3,n'_6} = R_3 - \sum_{n_1,n_2} E'_{n_1n_2n'_6}\ ,$$
$$2\,D_{2,n_3n_4} = R_2 - \sum_{n_6} E_{n_3n_4n_6}\ , \quad 2\,D_{3,n_6} = R_3 - \sum_{n_3,n_4} E_{n_3n_3n_6}\ .$$

The non–vanishing self–intersections between these divisors are:

$$R_1R_2R_3 = 4\ , \quad R_2(E'_{n_1n_2n'_6})^2 = R_1(E_{n_3n_4n_6})^2 = -4\ .$$

The total Chern class $\mathrm{c}(TX)$ is computed from the splitting principle:

$$\mathrm{c}(TX) = \prod(1+D)\prod(1+E)\prod(1-R)^2\ ,$$

In this way we have found an alternative description of the Schoen manifold, which was used to construct MSSM–like models with bundles. Bouchard,Donagi'05 Braun,He,Ovrut,Pantev'05

Abelian gauge flux background

The gauge flux is expanded as:

$$\frac{\mathcal{F}}{2\pi} = \sum_a R_a H_{B_a} + \sum_r E_r H_{V_r} + \sum_{r'} E'_{r'} H_{V'_{r'}}$$

where $H_A = A_I H_I$, with H_I are the Cartan generators of $\mathrm{E}_8 \times \mathrm{E}_8{}'$.

This embedding is characterized by 16-dimensional vectors:

- line bundle vectors $V_r, V'_{r'}$

  (analogous to shift vectors and Wilson lines on the orbifold)

- and the magnetic fluxes B_a.

They are subject to sets of flux quantization and Bianchi identities.

Determining the massless spectrum

The spectrum in four dimensions is determined by the multiplicity operator SGN,Trapletti,Walter'07

$$N_{4D} = \int_X \Big\{ \frac{1}{6} \Big(\frac{\mathcal{F}}{2\pi} \Big)^3 + \frac{1}{12} c_2(TX) \frac{\mathcal{F}}{2\pi} \Big\} \, .$$

This operator counts the number of chiral states arise for each of the $248 + 248$ gaugino components.

For the orbifold resolution of in interest in this talk, it is readily computed:

$$N_{4D} = 2 \Big(1 - \sum_r H_{V_r}^2 \Big) H_{B_1} + 2 \Big(1 - \sum_{r'} H_{V'_{r'}}^2 \Big) H_{B_2} + 4 \, H_{B_1} H_{B_2} H_{B_3} \, .$$

This result shows that without magnetized tori, i.e. $B_a = 0$, no chiral states in four dimensions.

Schoen line bundle MSSM

- Input data: gauge fluxes
- double GUT spectra
- Wilson line GUT $\rightarrow$ MSSM breaking
- Detailed matching

Choice gauge fluxes

We define a line bundle model on the Schoen manifold with the flux vectors

$$B_1 = \big(3, -3, 0^6\big)\big(3, 3, 0^6\big) \quad \text{and} \quad B_2 = B_3 = 0 \ ,$$

on the ordinary divisors R_a,

$$V_{(0,0,0)} = V_{(0,1,0)} = -V_{(0,0,1)} = -V_{(0,1,1)} = \big(\tfrac{1}{4}^8\big)\big(0, 0, 0, \tfrac{1}{2}, 0, -\tfrac{1}{2}, -\tfrac{1}{2}, -\tfrac{1}{2}\big) \ ,$$

$$V_{(1,0,0)} = V_{(1,1,0)} = -V_{(1,0,1)} = -V_{(1,1,1)} = \big(0, \tfrac{1}{2}, \tfrac{1}{2}, 0^5\big)\big(0, \tfrac{1}{2}, 0, 0, 0, -\tfrac{1}{2}, -\tfrac{1}{2}, -\tfrac{1}{2}\big) \ ,$$

on the exceptional divisors E_r, and finally,

$$V'_{(0,0,0)} = -V'_{(0,1,1)} \ = \ \big(0, -\tfrac{1}{2}, -\tfrac{1}{2}, 0^5\big)\big(\tfrac{1}{2}, \tfrac{1}{2}, \tfrac{1}{2}, 0, -\tfrac{1}{2}, 0, 0, 0\big) \ ,$$

$$V'_{(0,1,0)} = -V'_{(0,0,1)} \ = \ \big(0, -\tfrac{1}{2}, -\tfrac{1}{2}, 0^5\big)\big(\tfrac{1}{2}, \tfrac{1}{2}, -\tfrac{1}{2}, 0, \tfrac{1}{2}, 0, 0, 0\big) \ ,$$

$$V'_{(1,0,0)} = V'_{(1,1,0)} \ = \ \big(0, 1, 0, 0^5\big)\big(-\tfrac{1}{2}, -\tfrac{1}{2}, 0, 0, 0, 0, 0, 0\big) \ ,$$

$$V'_{(1,1,1)} = V'_{(1,0,1)} \ = \ \big(-1, 0^7\big)\big(-\tfrac{1}{2}, -\tfrac{1}{2}, 0^6\big) \ ,$$

on the exceptional divisors $E'_{r'}$.

Double six generation GUT

Superfield multiplicity	Representation $\mathrm{SU}(5) \times \mathrm{SU}(5)'$	U(1) charges q_0	q_1	q_2	q_3	q_4	q_5	q_6	q_7
6	$(\overline{\mathbf{10}}, \mathbf{1})$	0	0	0	0	1	0	-3	0
6	$(\mathbf{5}, \mathbf{1})$	0	0	0	0	0	0	-6	0
6	$(\overline{\mathbf{5}}, \mathbf{1})$	1	0	1	0	-1	0	1	0
6	$(\mathbf{5}, \mathbf{1})$	1	0	1	0	0	0	4	0
24	$(\mathbf{1}, \mathbf{1})$	2	0	0	0	0	0	0	0
6	$(\mathbf{1}, \mathbf{1})$	-1	0	-1	0	-1	0	5	0
6	$(\mathbf{1}, \mathbf{1})$	1	0	-3	0	0	0	0	0
6	$(\mathbf{1}, \mathbf{1})$	0	0	0	0	2	0	0	0
6	$(\mathbf{1}, \overline{\mathbf{10}})$	0	0	0	2	0	0	0	-6
24	$(\mathbf{1}, \mathbf{5})$	0	1	0	3	0	0	0	-2
6	$(\mathbf{1}, \overline{\mathbf{5}})$	0	0	0	-2	0	0	0	-8
6	$(\mathbf{1}, \overline{\mathbf{5}})$	0	0	0	0	0	1	0	7
6	$(\mathbf{1}, \overline{\mathbf{5}})$	0	0	0	0	0	-1	0	7
42	$(\mathbf{1}, \mathbf{1})$	0	0	0	4	0	1	0	-5
42	$(\mathbf{1}, \mathbf{1})$	0	0	0	4	0	-1	0	-5
24	$(\mathbf{1}, \mathbf{1})$	0	1	0	-3	0	1	0	-5
24	$(\mathbf{1}, \mathbf{1})$	0	1	0	-3	0	-1	0	-5
6	$(\mathbf{1}, \mathbf{1})$	0	2	0	0	0	0	0	0

Wilson line GUT $\to$ MSSM breaking

Both the orbifold and the resolution admit a freely acting involution:

$$\big(z_1, z_2, z_3\big) \longrightarrow \big(z_1 + \tfrac{i}{2}, z_2 + \tfrac{i}{2}, z_3 + \tfrac{i}{2}\big) \ .$$

The freely acting involution can be embedded as a Wilson line

$$W_{\text{free}} = \big(0^3, 1, 1, 1, -\tfrac{3}{2}, -\tfrac{3}{2}\big)\big(0^8\big) \ ,$$

that breaks $\mathrm{SU}(5)$ to $\mathrm{SU}(3) \times \mathrm{SU}(2) \times \mathrm{U}(1)_Y$ non–locally.

This choice leads to an MSSM–like model with three generations.

Summary

We have studied a specific class of heterotic orbifolds with vanishing Euler number:

- these do not give chirality in 4D,
- unless the tori become magnetized.

To determine the spectrum one can consider:

- the full resolution of the orbifold with gauge fluxes,
- or intermediate 6D models on magnetized tori.

A concrete example of this all was provided by a orbifold $T^6/\mathbb{Z}_2 \times \mathbb{Z}_{2,\text{rototrans}}$. We construct a MSSM–like model based on it.

Route II: Two–step procedure

- Intermediate $T^4/\mathbb{Z}_2$ models
- 6D field theories on magnetized two–tori
- Consistency of routes I & II

Magnetized intermediate 6D models

Intermediate $T^4/\mathbb{Z}_2$ models:

The divisor R_1 can itself be viewed as the resolution of the sub–orbifold $T^4/\mathbb{Z}_2$ inside $T^6/\mathbb{Z}_2 \times \mathbb{Z}_{2,\text{rototrans}}$.

Using the CFT techniques discussed above we can determine the massless spectrum in 6D.

6D field theories on magnetized two–tori:

Next one takes this 6D theory as the starting point for a dimensional reduction on a two–tori with a magnetic flux B_1.

By analyzing the Dirac equation in 6D one can show that the number of 4D chiral states is proportional to the the magnetic flux Cremades,Inbanez,Marchesano'04, Abe,Choi,Kobayashi,Ohki'09 .

Consistency of routes I & II

The 6D multiplicity operator:

$$N_{6D}(R_1) = 2\Big(1 - \sum_r H_{V_r}^2\Big) \ ,$$

determines the spectrum on the resolution of $T^4/\mathbb{Z}_2$.

Notice that the 4D multiplicity operator can expressed as:

$$N_{4D} = H_{B_1} N_{6D}(R_1) \ ,$$

assuming that $B_2 = B_3 = 0$ for simplicity.

Hence we see that the 4D multiplicity is given as the 6D multiplicity times the contribution due to the magnetic flux.

SGN,Vaudrevange'12

Detailed spectrum matching

From the full blow down limit we can identify the input data for the intermediate orbifold $T^4/\mathbb{Z}_2$:

$$V_\theta = \left(\tfrac{1}{4}^8\right)\left(\tfrac{1}{2}, -\tfrac{1}{2}, \tfrac{1}{2}, 0, \tfrac{1}{2}, 0^3\right)~, \quad W_3 = \left(-\tfrac{1}{4}, \tfrac{1}{4}, \tfrac{1}{4}, -\tfrac{1}{4}^5\right)\left(0, \tfrac{1}{2}, 0, \tfrac{1}{2}, -1, 0^3\right)$$

Blowing up the intermediate orbifold requires:

SGN,Held,Ruhle,Trapletti,Vaudrevange'09

- selecting the blow–up modes,
- performing field redefinitions using the blow–up modes.

Finally, one needs to take into account the consequences of the magnetic flux.

6D $\mathcal{N}=1$ super multiplet on $T^4/\mathbb{Z}_2$ $(\mathrm{E}_6 \times \mathrm{SU}(8)' \times \mathrm{U}(1)^3)$	Blow–up induced redefinitions of its chiral superfield component(s) $(\mathrm{E}_6 \times \mathrm{SU}(7)' \times \mathrm{U}(1)^4)$	Surviving 4D chiral superfields $(\mathrm{SU}(5) \times \mathrm{SU}(5)' \times \mathrm{U}(1)^8)$	4D multi-plicity $\tilde{N}_{4D}$
untwisted gauge sector			
$(\mathbf{78},\mathbf{1})_{(0,0,0)}$ (vector)	$(\mathbf{78},\mathbf{1})_{(0,0,0)}$	$(\overline{\mathbf{10}},\mathbf{1})_{(0,0,0,0,1,0,-3,0)}$ $(\mathbf{5},\mathbf{1})_{(0,0,0,0,0,0,-6,0)}$ $(\mathbf{1},\mathbf{1})_{(0,0,0,0,2,0,0,0)}$	6 6 6
$(\mathbf{1},\mathbf{63})_{(0,0,0)}$ (vector)	$(\mathbf{1},\mathbf{48})_{(0,0,0,0)}$	$(\mathbf{1},\overline{\mathbf{5}})_{(0,0,0,0,0,1,0,7)}$ $(\mathbf{1},\overline{\mathbf{5}})_{(0,0,0,0,0,-1,0,7)}$	6 6
	$(\mathbf{1},\overline{\mathbf{7}})_{(0,0,0,4)}$	–	–
	$(\mathbf{1},\mathbf{7})_{(0,0,0,-4)}$	$(\mathbf{1},\mathbf{1})_{(0,0,0,-4,0,1,0,5)}$ $(\mathbf{1},\mathbf{1})_{(0,0,0,-4,0,-1,0,5)}$	6 6
	$(\mathbf{1},\mathbf{1})_{(0,0,0,0)}$	–	–
untwisted matter sectors: U_a, $a=2,3$			
$(\mathbf{27},\mathbf{1})_{(-1,0,-1)}$ (hyper)	$(\mathbf{27},\mathbf{1})_{(-1,0,-1,0)}$	$(\mathbf{1},\mathbf{1})_{(-1,0,-1,0,-1,0,5,0)}$	6
	$(\overline{\mathbf{27}},\mathbf{1})_{(1,0,1,0)}$	$(\mathbf{5},\mathbf{1})_{(1,0,1,0,0,0,4,0)}$ $(\overline{\mathbf{5}},\mathbf{1})_{(1,0,1,0,-1,0,1,0)}$	6 6
$(\mathbf{1},\mathbf{70})_{(0,0,0)}$ (half–hyper)	$(\mathbf{1},\overline{\mathbf{35}})_{(0,0,0,-2)}$	$(\mathbf{1},\overline{\mathbf{5}})_{(0,0,0,-2,0,0,0,-8)}$	6
	$(\mathbf{1},\mathbf{35})_{(0,0,0,2)}$	$(\mathbf{1},\overline{\mathbf{10}})_{(0,0,0,2,0,0,0,-6)}$	6
$(\mathbf{1},\mathbf{1})_{(1,0,-3)}$ (hyper)	$(\mathbf{1},\mathbf{1})_{(1,0,-3,0)}$	$(\mathbf{1},\mathbf{1})_{(1,0,-3,0,0,0,0,0)}$	6
	$(\mathbf{1},\mathbf{1})_{(-1,0,3,0)}$	–	–
$(\mathbf{1},\mathbf{1})_{(0,2,0)}$ (hyper)	$(\mathbf{1},\mathbf{1})_{(0,2,0,0)}$	$(\mathbf{1},\mathbf{1})_{(0,2,0,0,0,0,0,0)}$	6
	$(\mathbf{1},\mathbf{1})_{(0,-2,0,0)}$	–	–

6D $\mathcal{N}=1$ super multiplet on $T^4/\mathbb{Z}_2$ $(\mathrm{E}_6 \times \mathrm{SU}(8)' \times \mathrm{U}(1)^3)$	Blow–up induced redefinitions of its chiral superfield component(s) $(\mathrm{E}_6 \times \mathrm{SU}(7)' \times \mathrm{U}(1)^4)$	Surviving 4D chiral superfields $(\mathrm{SU}(5) \times \mathrm{SU}(5)' \times \mathrm{U}(1)^8)$	4D multiplicity $\tilde{N}_{4D}$
twisted matter sector at the fixed tori: $r=(0,n_4,n_5,0)$, $n_4,n_5=0,1$			
$(\mathbf{1},\mathbf{8})_{\left(-\frac{1}{2},-\frac{1}{2},-\frac{3}{2}\right)}$ (hyper)	$(\mathbf{1},\mathbf{1})_{\left(\frac{1}{2},\frac{1}{2},\frac{3}{2},-\frac{7}{2}\right)} = e^{+b_r}$	blow–up mode	axion
	$(\mathbf{1},\mathbf{1})_{\left(-\frac{1}{2},-\frac{1}{2},-\frac{3}{2},\frac{7}{2}\right)} = e^{+b_r}\,(\mathbf{1},\mathbf{1})_{(0,0,0,0)}$	–	–
	$(\mathbf{1},\mathbf{7})_{\left(-\frac{1}{2},-\frac{1}{2},-\frac{3}{2},-\frac{1}{2}\right)} = e^{+b_r}\,(\mathbf{1},\mathbf{7})_{(0,0,0,-4)}$	–	–
	$(\mathbf{1},\overline{\mathbf{7}})_{\left(\frac{1}{2},\frac{1}{2},\frac{3}{2},\frac{1}{2}\right)} = e^{-b_r}\,(\mathbf{1},\overline{\mathbf{7}})_{(0,0,0,4)}$	$(\mathbf{1},\mathbf{1})_{(0,0,0,4,0,1,0,-5)}$ $(\mathbf{1},\mathbf{1})_{(0,0,0,4,0,-1,0,-5)}$	6 6
$(\mathbf{1},\mathbf{8})_{\left(\frac{1}{2},-\frac{1}{2},\frac{3}{2}\right)}$ (hyper)	$(\mathbf{1},\mathbf{1})_{\left(\frac{1}{2},-\frac{1}{2},\frac{3}{2},\frac{7}{2}\right)} = e^{+b_r}\,(\mathbf{1},\mathbf{1})_{(1,0,3,0)}$	–	–
	$(\mathbf{1},\mathbf{1})_{\left(-\frac{1}{2},\frac{1}{2},-\frac{3}{2},-\frac{7}{2}\right)} = e^{-b_r}\,(\mathbf{1},\mathbf{1})_{(-1,0,-3,0)}$	–	–
	$(\mathbf{1},\overline{\mathbf{7}})_{\left(-\frac{1}{2},\frac{1}{2},-\frac{3}{2},\frac{1}{2}\right)} = e^{+b_r}\,(\mathbf{1},\overline{\mathbf{7}})_{(0,1,0,-3)}$	$(\mathbf{1},\mathbf{1})_{(0,1,0,-3,0,1,0,-5)}$ $(\mathbf{1},\mathbf{1})_{(0,1,0,-3,0,-1,0,-5)}$	6 6
	$(\mathbf{1},\mathbf{7})_{\left(\frac{1}{2},-\frac{1}{2},\frac{3}{2},-\frac{1}{2}\right)} = e^{-b_r}\,(\mathbf{1},\mathbf{7})_{(0,-1,0,3)}$	–	–

And similar for the other twisted sectors.

Heterotic CFT on magnetized orbifolds

Following the logic of Aldazabal,Font,Ibanez,Uranga,Violero'97 taking into account the Bianchi identities in the presence of magnetic fluxes, we propose that the local modular invariance conditions are modified to

$$V_{g_r}^2 \equiv \frac{3}{2} + \frac{1}{4} B_2 \cdot B_3 \,, \qquad V_{g_{r'}}^2 \equiv \frac{3}{2} + \frac{1}{4} B_1 \cdot B_3 \,.$$

In addition, we expect that the left–moving mass is modified to

$$M_L^2 = \frac{1}{2}(P + V_{g_r})^2 + \widetilde{N} - \frac{3}{4} - \frac{1}{8} B_2 \cdot B_3 \,,$$

$$M_L^2 = \frac{1}{2}(P + V_{g_{r'}})^2 + \widetilde{N} - \frac{3}{4} - \frac{1}{8} B_1 \cdot B_3 \,.$$